

Achieving the Capacity of any DMC using only Polar Codes

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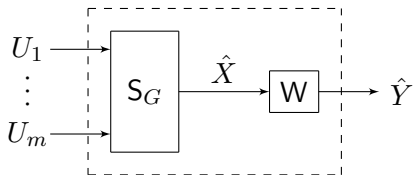


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- Polar codes achieve the *symmetric* capacity of any DMC W having $O(N \log N)$ encoding and decoding complexity, where N denotes the blocklength [Arıkan'09].

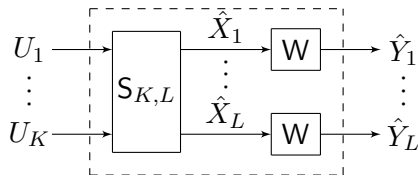
Background

- Polar codes achieve the *symmetric* capacity of any DMC W having $O(N \log N)$ encoding and decoding complexity, where N denotes the blocklength [Arıkan'09].
- Using Gallager's shaper idea, polar codes can be used to achieve the capacity of any DMC at the cost of a slightly higher encoding and decoding complexity [Sasoglu, Telatar & Arıkan'09].

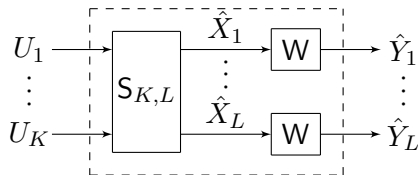


- A q -ary input U^m (with $q = 2^m$ for $m \in \mathbb{Z}^+$) whose elements are i.i.d. Bernoulli $(\frac{1}{2})$ distributed is shaped into a rational approximation to $X \sim \text{Bernoulli}(p)$, i.e. $\hat{X} \sim \text{Bernoulli}(k/q)$ where $k \in \mathbb{Z}^+$ and $k/q \approx p$.

- We construct a different shaper $S_{K,L}$ using polarization-based randomness extraction to simulate the capacity-achieving input vector X^L from a uniformly-random input U^K .



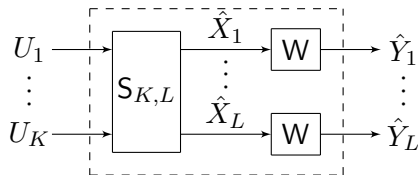
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- We use source polarization to obtain an efficient encoder and decoder.

	Gallager's scheme	New scheme
Rate	$C - O\left(\frac{1}{q} \log q\right)$	$C - \frac{o(N)}{N}$
Complexity	$O(q \log q \cdot N \log N)$	$O(N \log N)$
Error probability	$O\left(\log q \cdot 2^{-N^\beta}\right)$	$O\left(\sqrt{N} 2^{-\frac{1}{2} N^{\frac{\beta}{2}}}\right)$

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- An application to the AWGN channel is discussed.