Approximate quantum nondemolition

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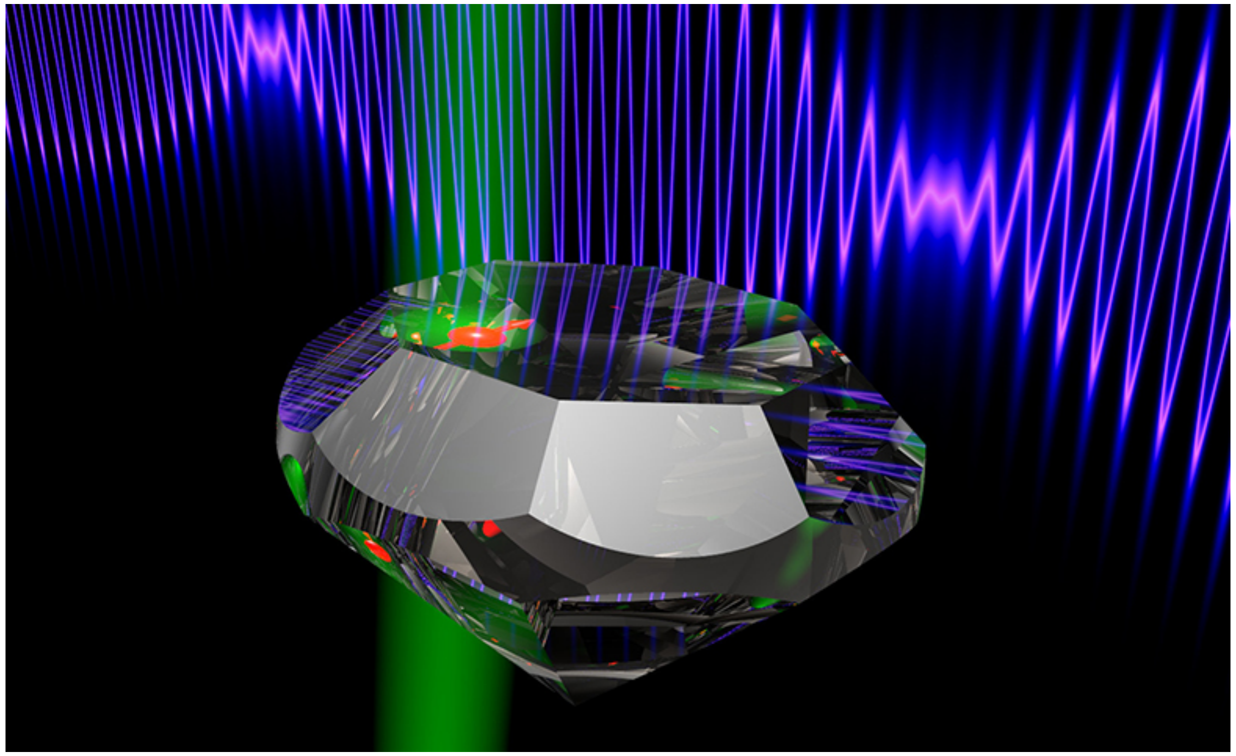
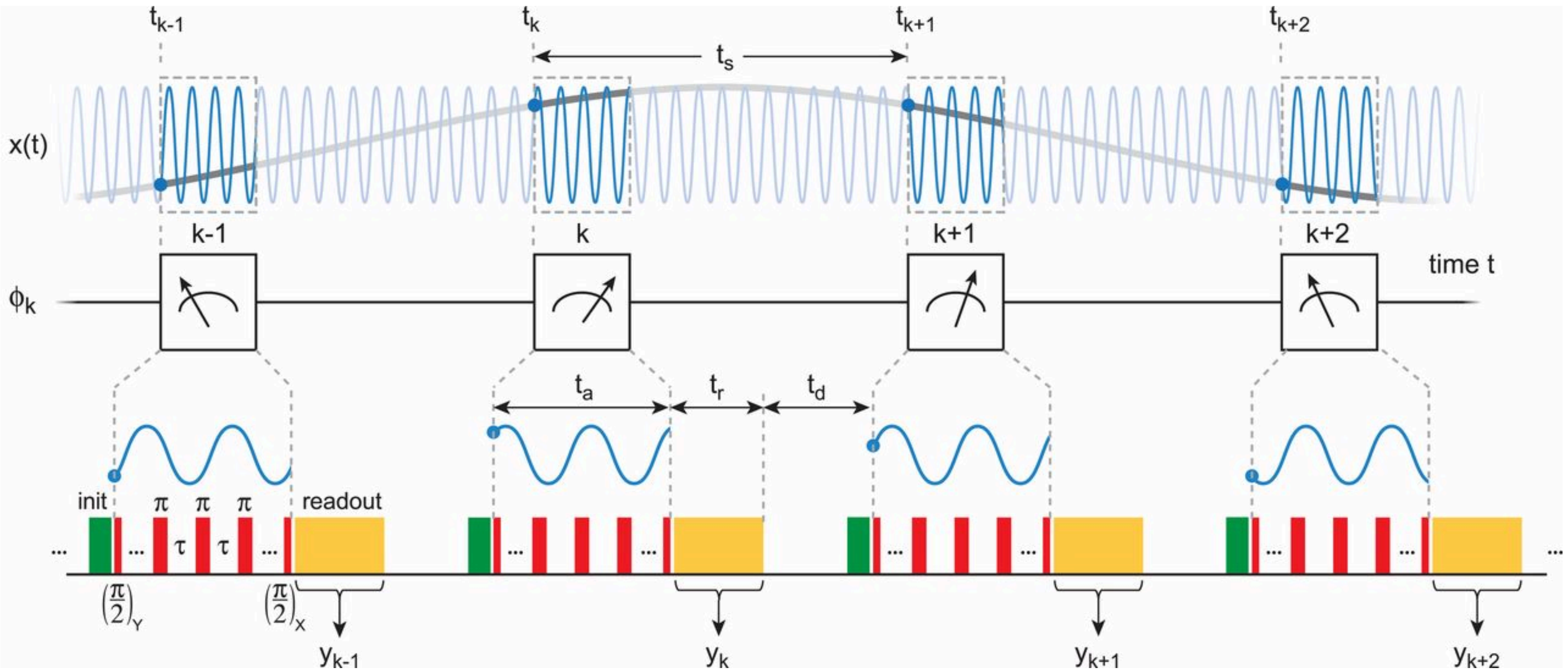
arXiv:1909.05265 [quant-ph]



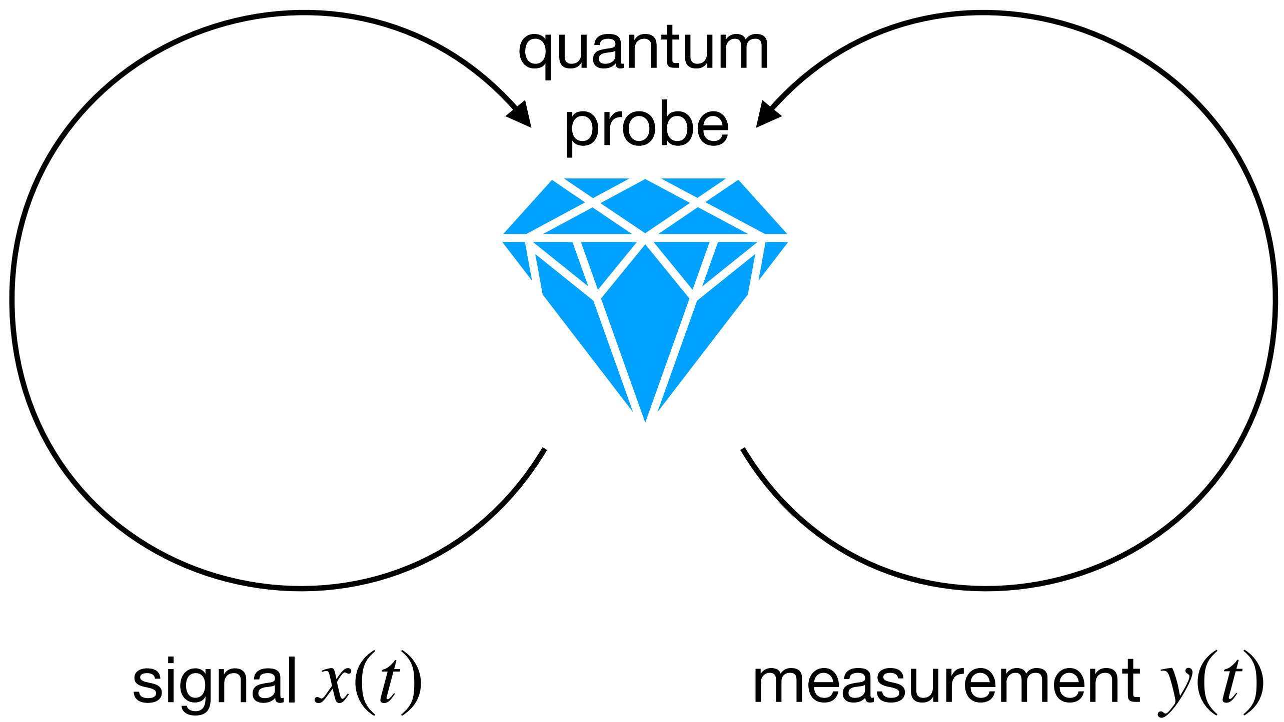
Quantum metrology: Small probes



Boss et al., Science 2017


World’s largest quantum sensor: LIGO

Problem of waveform estimation: backaction



Quantum backaction of the measurement process can destroy the probe sensitivity to the signal

One solution: Repeatedly reprepare the probe state

Another solution: nondemolition measurement

Quantum nondemolition measurement

$\hat{O}(t)$ is "nondemolition" if $[\hat{O}(t), \hat{O}(t')] = 0$

Measurement now does not disturb measurement later

Tsang & Caves 2012: Can construct QND observables satisfying *any* desired equations of motion

Example: Classical harmonic oscillator

$$\hat{Q} = \frac{1}{2}(\hat{q}_1 + \hat{q}_2) \text{ and } \hat{P} = \hat{p}_1 - \hat{p}_2$$

for

$$\hat{H} = \frac{1}{2m}(\hat{p}_1^2 + \omega^2 \hat{q}_1^2) - \frac{1}{2m}(\hat{p}_2^2 + \omega^2 \hat{q}_2^2)$$

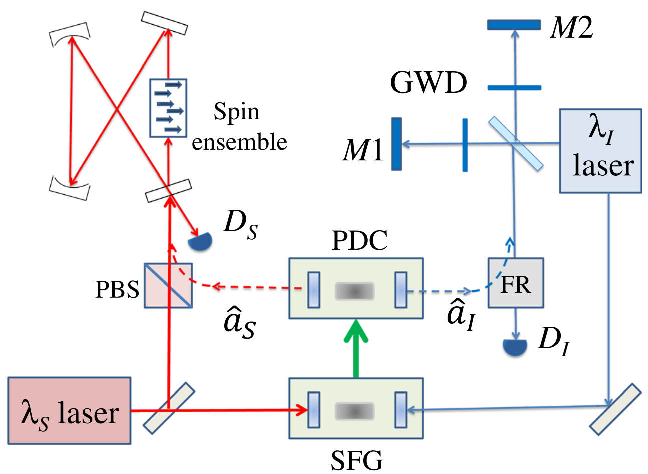
Approximate QND condition with finite energy

QND nominally requires infinite energy:
Hamiltonian unbounded above and below

Generic to QND: S Boulebnane, MP Woods, JMR, in preparation

Try to approximate with finite energy / dimension:

- spin coherent states
- red/blue sidebands
- just truncate



Khalili & Polzik, PRL 2018

How large an energy is needed for given estimation error?

We find that precision scales as $E^{-1/4}$

Quasi-ideal clock

Approximate version of a different QND system:

$$\hat{H} = \hat{p}$$

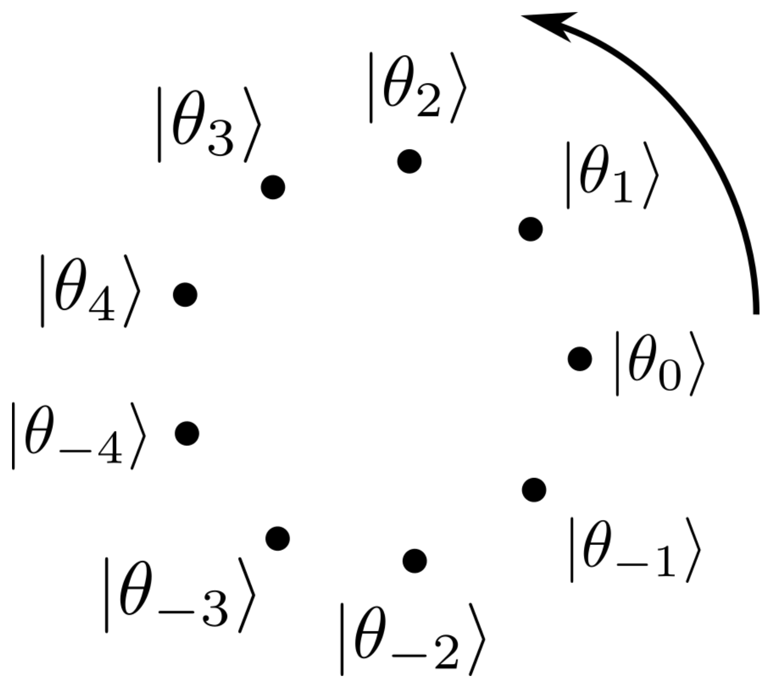
(Also arises from Tsang/Caves using two uncoupled free particles)

For odd dimension d,

Hamiltonian: $\hat{H} = \frac{2\pi}{\sqrt{d}} \sum_{n=\mathbb{Z}_d} |n\rangle\langle n|$

time states: $|\theta_k\rangle = \frac{1}{\sqrt{d}} \sum_{n\in\mathbb{Z}_d} e^{-2\pi i n k/d} |n\rangle$

evolve as: $e^{-i\hat{H}m/\sqrt{d}} |\theta_k\rangle = |\theta_{k+m}\rangle$



Waveform estimation with quasi-ideal clock

$$\hat{H}(t) = (1 + x(t))\hat{H}$$

Ideally, $\hat{q}(t) = \hat{q}(0) + \int_0^t \mathrm{d}\tau\, x(\tau)$

Measuring time $\hat{T}$ gives waveform estimate:

For $\hat{\xi}_j = \hat{T}(j/\sqrt{d})/\sqrt{d}$, we have

$$\hat{\xi}_j - \hat{\xi}_{j-1} \approx \int_{j-1}^{j} \mathrm{d}\tau\, x(\tau)$$

Start in a "Gaussian" state of width σ_s

Measure with "Gaussian" precision σ_m

As with linear measurements, $\langle \hat{\xi}_j \hat{\xi}_k \rangle$ has three contributions:

$\langle \hat{\xi}_j \hat{\xi}_k \rangle$ = (state correlations) + (measurement imprecision) + (backaction)

σ_s σ_m $1/\sigma_s$ $1/\sigma_m$

Result: minimum detectable $|x| \propto d^{-1/4} = E^{-1/4}$

Details on the calculation

Not easy to work with $\langle \hat{\xi}_j \hat{\xi}_k \rangle$ due to periodicity;

instead consider $\langle e^{\frac{2\pi i \ell \hat{\xi}_j}{\sqrt{d}}} e^{\frac{2\pi i m \hat{\xi}_k}{\sqrt{d}}} \rangle$ for integer ℓ and m

Close enough: For $X \sim \mathcal{N}(\mu, \sigma^2)$,

we have $\langle e^{i\alpha X} \rangle = e^{i\alpha\mu} e^{-\frac{1}{2}\alpha^2\sigma^2}$

Evolution between $j-1$ and j is $e^{-i\hat{H}(t)\Delta t_j/\sqrt{d}}$, with $\Delta t_j = 1 + \int_{j-1}^{j} \mathrm{d}\tau \, x(\tau/\sqrt{d})$

For example, with $\ell = -1, m = 0$, we find $\langle e^{-\frac{2\pi i \tilde{\xi}_n}{\sqrt{d}}} \rangle = e^{-\frac{2\pi i}{d} \sum_{j=0}^{n} \Delta t_j} C_1 C_2 C_3$

$$C_1 = e^{-\frac{\pi\sigma_s^2}{2d}}$$

$$C_2 = e^{-\frac{\pi\sigma_m^2}{2d}}$$

C_3 more complicated:

Random walk on $\mathbb{Z}_d$ of step size $\mathcal{N}(0, d/4\pi\sigma_m^2)$;

contributions when landing on the last position.

Conclusion and open questions

- Estimation errors scale “decently well” with energy
- Cannot expect really fast decay, , e.g. exponential

- How does this relate to errors in timekeeping
- or covariant quantum error correction?

- What impact does this scaling have in actual setups?

- Can we improve the 1/4 exponent by more sophisticated analysis?