On the second-order asymptotics of the partially-smoothed conditional min-entropy & Applications to quantum compression

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Joint work with Dina Abdelhadi (EFPL)

arXiv:1905.08268 [quant-ph]

Optimal quantum compression

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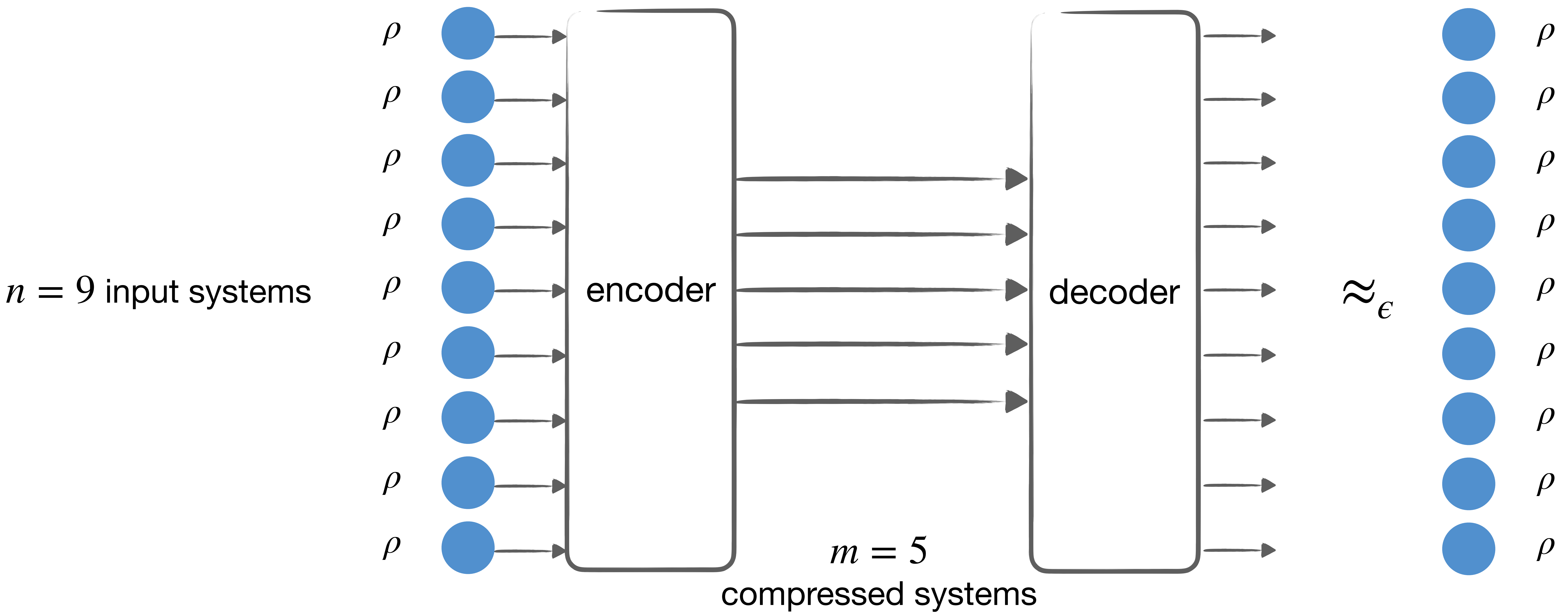
Isn't quantum compression already understood?

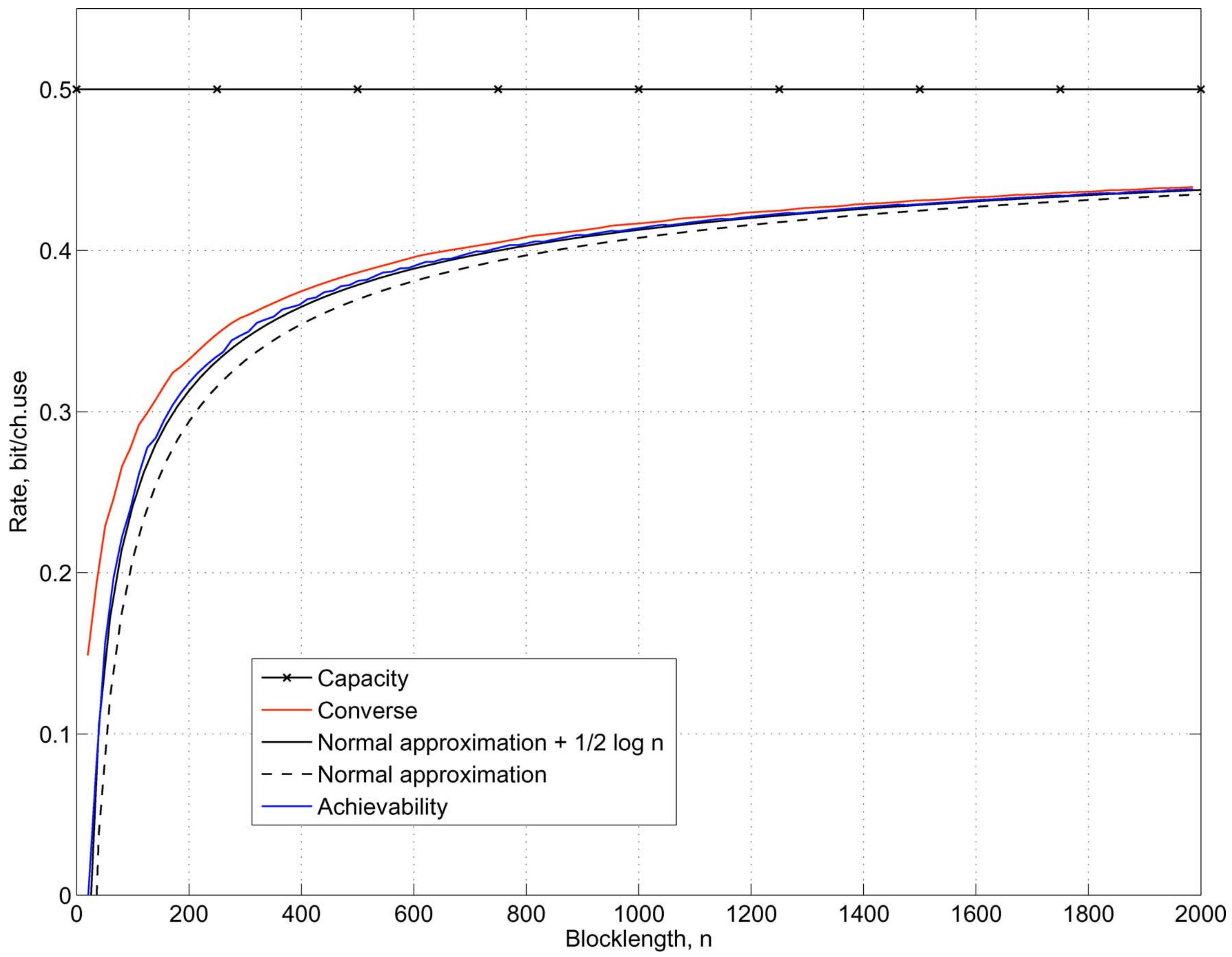


Rate $R = \frac{m}{n}$

$\lim_{n\to\infty} R = H(\rho)$, the von Neuman entropy

Schumacher 1995

Finite blocklength & second-order asymptotics



Rate-blocklength tradeoff for the BSC with crossover probability $\delta = 0.11$ and maximal block error rate $\epsilon = 10^{-3}$.

Polyanskiy, Poor, and Verdú, IEEE Transactions Info Theory, 2010

Can we make more refined statements?

- What about a fixed n and ϵ?
- What about the next order for large n?

Breakthrough in classical information theory:
Tight finite-size bounds & matching 2nd order

At $n = 500$ and $\epsilon = 10^{-3}$ for BSC(0.11):

lower bound: $m \geq 190$

upper bound: $m \leq 194$

$$R \approx nC - \sqrt{nV}Q^{-1}(\epsilon) + O(\log n)$$

Known bounds for compression

Datta and Leditzky, IEEE Transactions Info Theory 2015

$$m \geq nH(\rho) + \sqrt{nV(\rho)}Q(1-\epsilon) + O(\log n)$$

$$m \leq nH(\rho) + \sqrt{nV(\rho)}Q(\sqrt{1-\epsilon}) + O(\log n)$$

Converse: using hypothesis testing

Protocol: cut off small probabilities

Which one (if any) is tight?

Our Result: The direct part (upper bound) is tight!

Qubit example

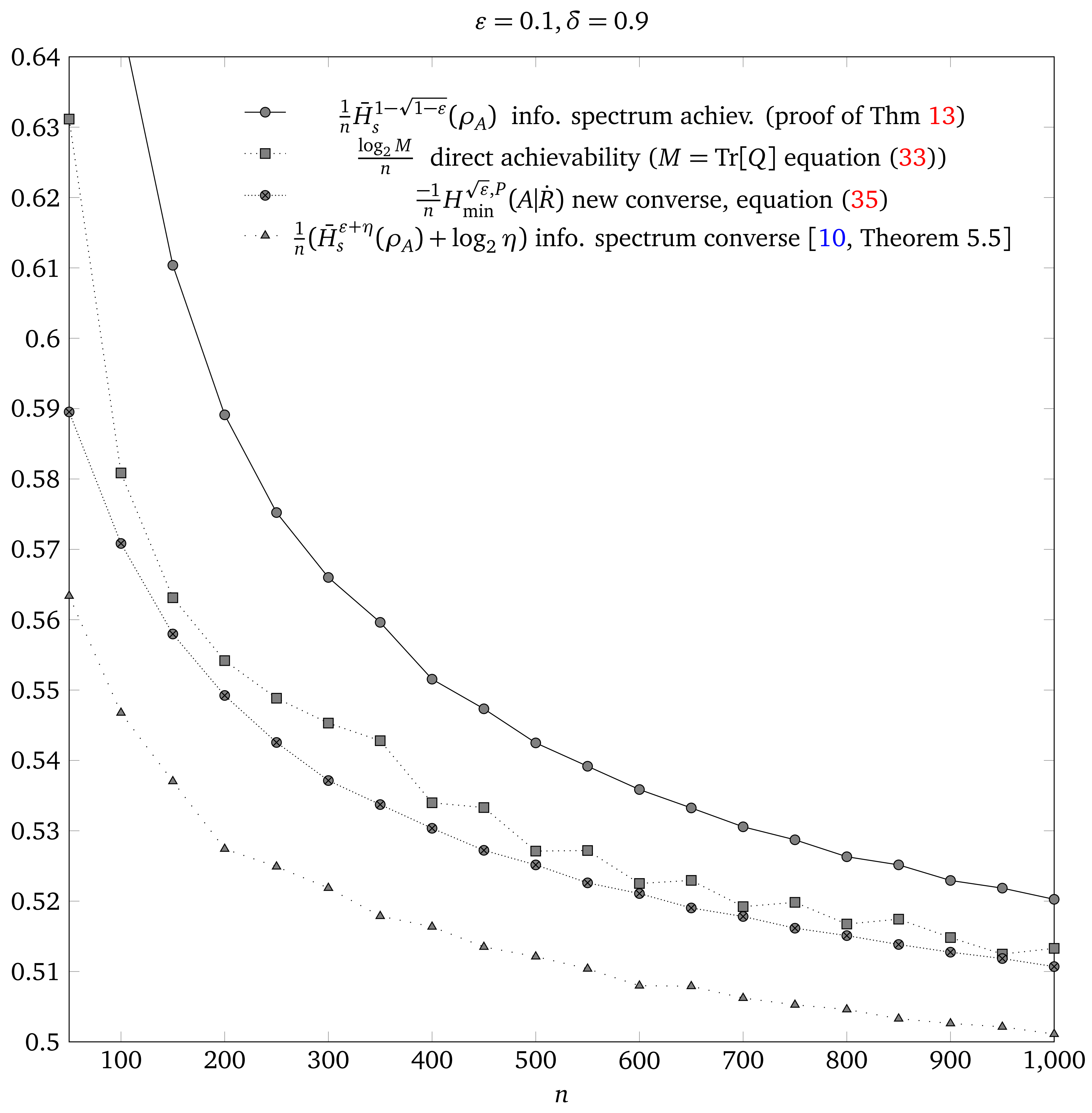


Figure 1: Compression bounds for the qubit case $\rho_A = \delta|0\rangle\langle 0| + (1-\delta)|1\rangle\langle 1|$

Blocksize $n = 250$:

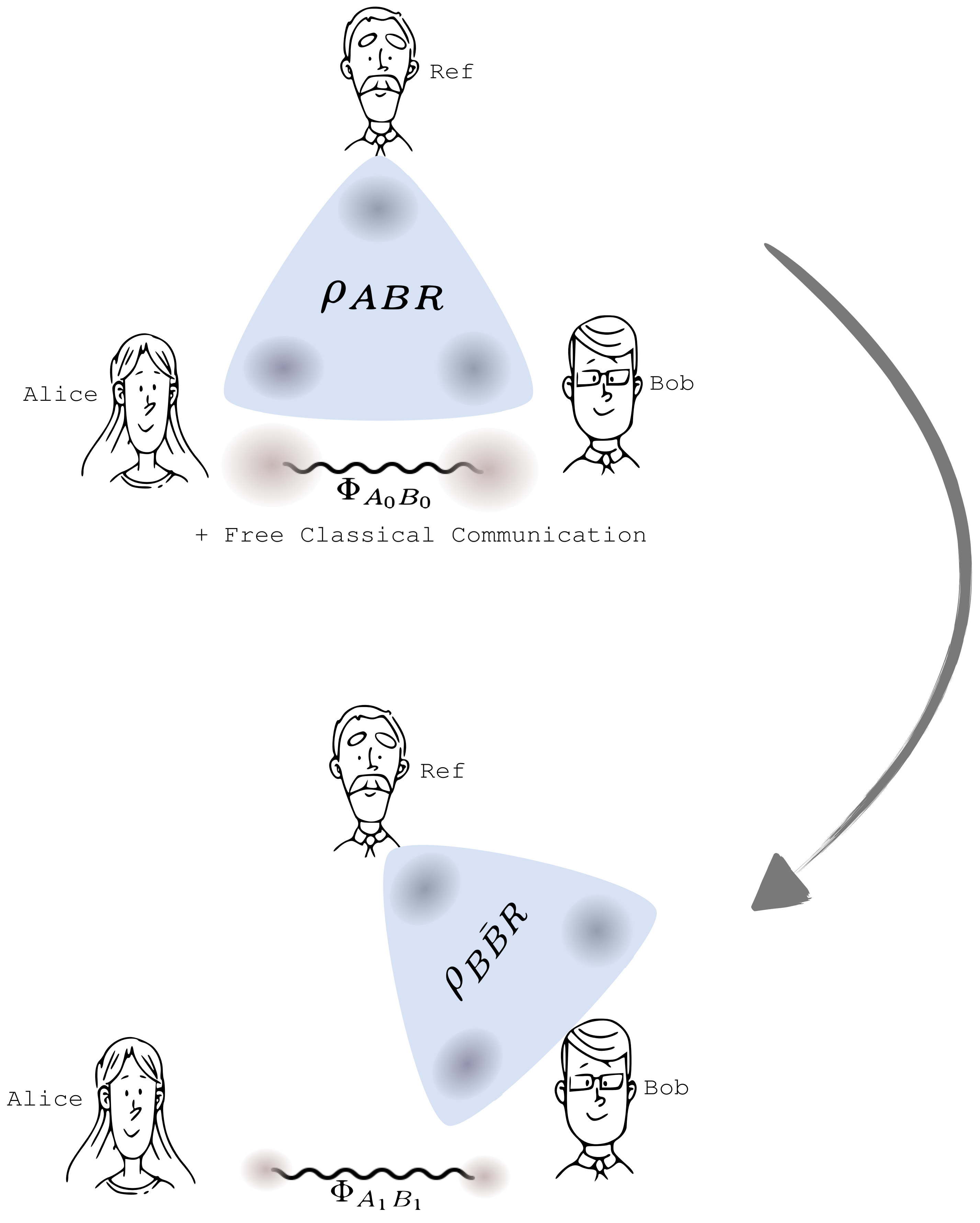
lower bound $m \geq 135$

upper bound $m \leq 137$

(old lower bound: $m \geq 130$)

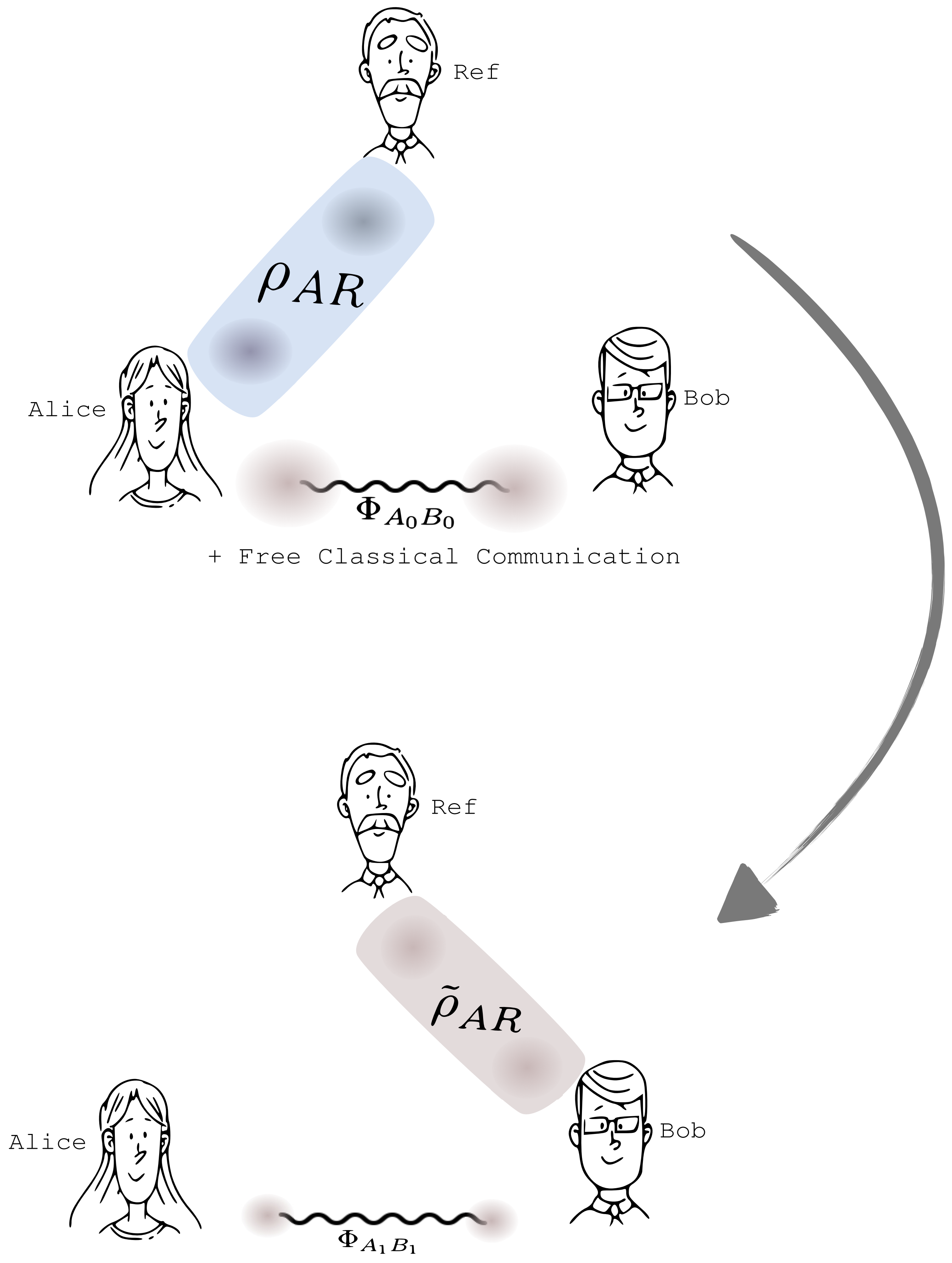
A look at the proof

First: Relate compression & state merging



Minimize entanglement used

For trivial Bob:
compress
and
teleport



Trivial Bob

A look at the proof

Second: Find a converse bound for state merging

Anshu et al., arXiv:1807.05630:
"partially-smoothed" conditional min-entropy…

… can be formulated as a semidefinite program.

Third: Specialize SDP to pure states,
play around with numerics till you see the form of the optimizer,
then prove it.

Fourth: Work out the asymptotics of the resulting quantity.

Summary and open questions

Tight finite-size bounds for quantum compression

Tight second-order (even third order) asymptotics

Make use of “partially-smoothed” quantity

Can we extend to general state merging?

Second order of partially smoothed quantity not uniform in the state!

For pure states the second order term is: $\sqrt{nV(\rho)}Q(\sqrt{1-\epsilon})$,

but other states have: $\sqrt{nV(\rho)}Q(1-\epsilon)$

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