## The Top quark mass at Hadron Colliders

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## Outline

## Talk based upon

#### PN, arXiv:1712.02796;

#### Ferrario Ravasio, Ježo, Oleari, PN, arXiv:1801.03944.

- General issues on the top mass measurement at hadron colliders;
  - ► Top, precision physics, vacuum stability
  - Current measurements
  - Issues on the theoretical interpretation of the results
  - A discussion focussing upon the Pole Mass Renormalon problem.
- Addressing the problem with NLO+PS Monte Carlos
  - Available NLO+PS Monte Carlos
  - Comparison within the same shower model (Pythia8)
  - Comparison using Herwig7.
- Conclusions

## Top and precision physics



$$\Delta G_{\mu}/G_{\mu} = 5 \cdot 10^{-7}; \quad \Delta M_Z/M_Z = 2 \cdot 10^{-5};$$
  
$$\Delta \alpha(M_Z)/\alpha(M_Z) = \begin{cases} 1 \cdot 10^{-4} (\text{Davier et al.; PDG}) \\ 3.3 \cdot 10^{-4} (\text{Burkhardt, Pietrzyk}) \end{cases}$$

Now that  $M_H$  is known, tight constraint on  $M_W-m_t$ , (depending on how aggressive is the error on  $\alpha(M_Z)$ ).

But: precision on  $M_W$  is more important now ...

## Top and vacuum stability



With current value of  $M_t$  and  $M_H$  the vacuum is metastable. No indication of new physics up to the Plank scale from this.

## Top and vacuum stability



The quartic coupling  $\lambda_H$  becomes tiny at very high field values, and may turn negative, leading to vacuum instability.  $M_t$  as low as 171 GeV leads to  $\lambda_H \rightarrow 0$  at the Plank scale.

## Top Mass Measurements



DIRECT MEASUREMENTS

(roughly, from the mass of the system of decay products). The most precise method as of now.

Add: CMS 13 TeV, 172.25±0.08 (stat+JSF) ±0.62 (syst) GeV

## Theory issues

- The measurement is performed by reconstructing a top mass peak out of a reconstructed W and a b-jet.
- The reconstructed mass is only loosely related to the top mass (i.e. it cannot be identified with the top mass, for obvious reasons, since it is a colourless system).
- ► The extracted mass is the mass parameter in the Monte Carlo that yields the best fit to the reconstructed mass distribution.

So:

- $\diamond\,$  in which renormalization scheme is the MC mass parameter? Pole mass?  $\overline{\rm MS}$  mass?
- $\diamond\,$  It has been argued that since MC are Leading-Order, they can't distinguish between Pole and  $\overline{\rm MS}$  mass (the difference is around 10 GeV ...).

#### Selected Th. results relevant to top mass measurements

- Narrow width tt production and decay at NLO, Bernreuther, Brandenbourg, Si, Uwer 2004, Melnikov, Schulze 2009.
- IvIvbb final states with massive b, Frederix, 2013, Cascioli,Kallweit,Maierhöfer,Pozzorini, 2013.
- ▶ NNLO differential top decay, Brucherseifer, Caola, Melnikof 2013.
- ► NLO+PS in production and decay, Campbell, Ellis, Re, PN
- ▶ NNLO production, Czakon, Heymes, Mitov, 2015.
- ►  $l\nu l\nu b\bar{b} + jet$  Bevilacqua, Hartanto, Kraus, Worek 2016.
- ► Approx. NNLO in production and exact NNLO in decay for tt. Gao,Papanastasiou 2017.
- Resonance aware formalism for NLO+PS: Ježo, PN 2015;
- Off shell + interference effects+PS, Single top, Frederix, Frixione, Papanastasiou, Prestel, Torielli, 2016
- ► Off shell + interference effects+PS, *lvlvbb*, Jeo,Lindert,Oleari,Pozzorini,PN, 2016.

### Alternative mass-sensitive observables

- Butenschoen, Dehnadi, Hoang, Mateu, Preisser, Stewart, 2016 Use boosted top jet mass + SCET.
- Agashe, Franceschini, Kim, Schulze, 2016: peak of *b*-jet energy insensitive to production dynamics.
- Kawabata,Shimizu,Sumino,Yokoya,2014: shape of lepton spectrum. Insensitive to production dynamics and claimed to have reduced sensitivity to strong interaction effects.
- ► Frixione, Mitov: Selected lepton observables.
- ► Alioli, Fernandez, Fuster, Irles, Moch, Uwer, Vos ,2013; Bayu etal: M<sub>t</sub> from tt̄j kinematics.
- *tt* threshold in γγ spectrum (needs very high luminosity), Kawabata, Yokoya, 2015

## From total cross section and $t\bar{t}j$ kinematics



It is claimed that since higher order calculations (NNLO for total cross section, NLO for  $t\bar{t}j$  shape variables) are used in this determination, one is entitle to specify the scheme used for the mass.

In the figure they are quoted as "pole mass measurement".

- ► The "pole mass" attribute is not given to direct measurement.
- In some experimental papers and talk, direct measurements are reported as "Monte Carlo Mass" measurements, often stating that they need some theoretical interpretation.
- "Monte Carlo Mass" measurements are often interpreted as pole mass measurements by theorists. See for example
  - Degrassi etal, 2012 on the EW vacuum stability, adding a further 250 MeV error to direct measurements.
  - Ciuchini etal, 2017 in Global EW fits, adding a further 500 MeV error to direct measurements.
- Theorist have done work in proposing alternative methods to avoid the issues on direct measurements; however, the alternative methods are generally inferior in precision. Illuminating theoretical progress that saves the direct measurements is not in sight.

Tick the correct statements:

- □ Direct top mass measurements measure the Pole Mass.
- □ Direct top mass measurements measure the Monte Carlo Mass.
- □ Direct top mass measurements measure the Monte Carlo Mass. but you can pretend that it is the pole mass, just inflate the error a bit.
- $\Box$  The top is the only SM particle with more than one mass.
- □ You should use only leptons to avoid hadronization uncertainty.
- $\Box$  You should use at least NLO calculations to measure the pole mass.
- □ The top pole mass has renormalons, you should stay away from it.
- The MC mass differs from the pole mass by
   □ terms of order mα<sub>s</sub>; □ terms of order Λ<sub>QCD</sub>; □ terms of order α<sub>s</sub>Γ<sub>t</sub>.
- The Pole Mass renormalon ambiguity is  $\Box \approx 1 \text{GeV}; \ \Box \approx 250 \text{ MeV}; \ \Box \approx 200 \text{ MeV}; \ \Box \approx 110 \text{ MeV}.$



Thinking in terms of the Pole Mass renormalon can serve as a guideline for sorting out the mass measurement problem. Correction to the Pole Mass:

$$\delta m = \mathcal{N} \int_0^m \mathrm{d}k \, \alpha_s(k^2) + \mathrm{UV} \text{ divergent terms.} \tag{1}$$

(Linear sensitivity to the momentum scale flowing in the graph.)

The integration region  $k \approx \Lambda$  is uncertain, leading to an ambiguity of order  $\Lambda$  in the mass correction.

The uncertainty can be associated to the factorial growth of the perturbative expansion:

$$\int_0^m \mathrm{d}k \,\alpha_s(k^2) = \int_0^m \mathrm{d}k \frac{\alpha_s(m^2)}{1 + b_0 \alpha_s(m^2) \log \frac{k^2}{m^2}}$$
$$= \alpha_s(m^2) \sum_{n=0}^\infty (2b_0 \alpha_s(m^2))^n \int_0^m \underbrace{\mathrm{d}k \log^n \frac{m}{k}}_{n!}.$$

Asymptotic expansion.

- Minimal term at  $n_{\min} \approx \frac{1}{2b_0 \alpha_s(m^2)}$ .
- Size of minimal term:  $\alpha_s(m^2)\sqrt{2\pi n_{\min}}e^{\frac{1}{n_{\min}}} \approx \Lambda_{\rm QCD}$ .
- Typical scale dominating at order  $\alpha_s^{n+1}$ :  $m \exp(-n)$ .

The mass divergence needs a UV counterterm.

- $\overline{\text{MS}}$ : only subtract high-scale effects.
  - The mass of initial or final state (on-shell) top quarks is shifted at each order in perturbation theory
  - ► The factorial growth appears in the self energy as it approaches the mass shell.
  - If the top width is accounted for, the top dominant off-shellness is Γ, and the factorial growth is visible up to the order n ≈ log(m/Γ), and should disappears for larger orders.
- Pole Mass Scheme: subtract everthing, leading to no mass correction when on-shell.
  - The factorial growth appears in radiative corrections, since they involve off-shell top quarks
  - ► In processes with dominant off-shellness Γ the factorial growth is determined solely by the mass counterterm.

Should we use the Pole Mass scheme in all circumstances when we have on-shell tops in the final state?



Inclusive cross sections: they can be written as

$$\int_0^Q \mathrm{d}k^2 G(k^2) \mathrm{Im}\left[\frac{1}{k^2 - m^2 + i\epsilon} \left(\Sigma(k^2 + i\epsilon) - m_{\mathrm{ct}}\right) \frac{1}{k^2 - m^2 + i\epsilon}\right]$$

The imaginary part can be written as half a contour integral, that can be moved away from the pole. So:  $\overline{\mathrm{MS}}$  is more appropriate.



#### Reminder

The relation of the pole mass  $m_p$  to the  $\overline{\text{MS}}$  mass m is (Marquard, A.V.Smirnov, V.A.Smirnov, Steinhauser, 2015)

 $m_p = m(1 + 0.4244\alpha_s + 0.8345\alpha_s^2 + 2.375\alpha_s^3 + (8.49 \pm 0.25)\alpha_s^4)$ 

The renormalon asymptotic formula (Beneke, Braun+Beneke, 1994)

$$r_n \rightarrow N \ m_t (2b_0)^n \Gamma(n+1+b) \left(1+\sum_{k=1}^\infty \frac{s_k}{n^k}\right), \qquad b=\frac{b_1}{b_0^2},$$

yields a good fit to the exact result, so that higher order terms can be estimated, yielding a very accurate conversion formula, with typical size

$$m_{p} = m + \underbrace{7.557}_{NL0} + \underbrace{1.617}_{N^{2}L0} + \underbrace{0.501}_{N^{3}L0} + \underbrace{0.195}_{N^{4}L0} + \underbrace{0.300}_{N^{5.6,..}L0} \text{ GeV}$$
(Pineda etal, 2001,2014 for bottom;  
Beneke,Marquard,Steinhause,PN, 2016 and  
Hoang,Lepenik,Preisser, 2017 for top)

The asymptotic nature of the relation between the  $\overline{\rm MS}$  and the pole mass leads to an irreducible ambiguity of the order of typical hadronic scales.

Some authors have quoted an ambiguity of 1 GeV.

Recent calculations give much smaller results:

- ▶ Beneke, Marquard, Steinhauser, PN 2016: 110 MeV.
- ► Hoang, Lepenik, Preisser, 26 Jun 2017: 250 MeV.
- Pineda etal, 2001,2013,2014 190,200 MeV. (in a bottom context, but valid also for top.)

Safely below currently quoted errors.

In calculations involving off-shell top quarks, as in the Vacuum Stability calculations, or in electroweak fits, it is more appropriate to use the  $\overline{\rm MS}$  mass scheme. If the pole mass is used, we expect higher order corrections (arising from the mass counterterm) with factorially growing coefficients.

When converting the Pole to the  $\overline{\rm MS}$  mass or viceversa, it is safe to include the missing higher order terms, fitted from the renormalon formula, rather than truncate it to the known fourth order terms.

Mass from the total cross section: the MS mass should be more appropriate. At the NNLO level, using the Pole mass should amount to a difference of about 1 GEV, well below present errors.

Should the error decrease, one should also worry about the fact that the cross section is measured in a fiducial region (i.e. not fully inclusive) ...

- Mass from tt
  j kinematic distributions: no particular reason to pick either scheme, not fully inclusive.
- Mass measurements from decay products (insensitive to production dynamics): Pole Mass measurements.

## Mass from decay products



If we DO NOT use the pole mass, the term in the round bracket differs from zero near the mass peak. This leads to an NLO correction of the form

$$1 + \delta m \frac{\partial}{\partial m} \tag{2}$$

to be applied to the amplitude, i.e. a shift in mass.

Thus, even when using LO Monte Carlo, we better think of it as using the pole mass, as far as measurements of the mass of the decay products are concerned. In processes dominated by a top off-shellness  $m_t \gg \delta k \gg \Lambda_{\rm QCD}$  it makes sense to use intermediate mass schemes, such that the mass counterterm il close to the Pole Scheme for orders  $n \approx \log(m_t/\delta k)$ , but such that the factorial growth stops at higher orders.

- ► Schemes of this sort are used in calculations of  $e^+e^- \rightarrow t\bar{t}$  production near threshold like the PS mass (Beneke 98) and the 1S mass (Hoang,Ligeti,Manohar 98).
- Since the top has a width  $\Gamma \gg \Lambda_{\rm QCD}$ , that screens infrared effects in self-energy insertions, using one such scheme may become appropriate even for direct measurements (if it ever became possible to reach accuracy below the pole mass renormalon uncertainty)

### Top and precision physics

Rather than  $e^+e^- \rightarrow t\bar{t}$  at threshold, we may also look at the  $\gamma\gamma$  spectrum at LHC Kawabata, Yokoya, 2016. It is unclear whether this can be done even at the High Luminosity LHC ...



It avoids theoretical problems present in direct measurements.

In Hoang, Stewart, 2008 it is stated that "It is not the pole mass that is measured at the Tevatron".

In a sequel of papers they have advocated the use of boosted top jet, claiming that for these observables the non-perturbative effects can be reliably modeled.

It is clear that for this method to be useful one should show that a mass measurement using boosted top jets can achieve the GeV precision (at the moment, the error is near 10 GeV).

## A weaker objection to direct measurements

- In order to avoid the renormalon problem, they make use of an intermediate mass scheme (the MSR mass) evaluated at a scale near the top width.
- They extract a relation between this MSR mass and the mass parameter in the Monte Carlo by comparing their SCET calculations to Shower MC output (always in the context of highly boosted top quarks).
- When translated into the Pole Mass language, the difference they find with respect to the Monte Carlo mass parameter is few hundred MeV.
- ◊ This "definition" of Monte Carlo mass is clearly process dependent.
- It leads to shifts of the order of typical hadronic scales. We might as well say that direct measurements do measure the pole mass, with a non-perturbative error that needs to be quantified.

## Calibration of the "Monte Carlo mass"

 $m_t^{\rm MSR}(1\,{\rm GeV})[{\rm GeV}]$ 175 Calibration :  $(\tau_2^{e^+e^-})$ Pythia 8.205, tune 7 174 NNLL,  $\Gamma_t = 1.4 \,\mathrm{GeV}$ 173 172 171 170  $(m_t^{\text{MSR}}(1 \text{ GeV}) - m_t^{\text{MC}})[\text{GeV}]$ 0.2 0.0 -0.2 -0.4 170 171 172 173 174 175  $m_{\star}^{\rm MC}[{\rm GeV}]$ 

Butenschoen, Dehnadi, Hoang, Mateu, Preisser, Stewart 2016 Esimating  $m_t^{\text{pole}}$  from  $m_t^{\text{MSR}}(R)$ requires resumming the asymptotic series. But the difference is positive, going in the opposite direction with respect to the calibration plot. Hard to argue for large differences.

In all cases, this is for boosted top. No reason to believe that it should apply to direct measurements On Calibration: see also Kieseler,Lipka,Moch 2015. Deals with eventual perturbatuive inaccuracy of the MC (unlike Hoang's paper), and addresses cross sections and distributions that can be computed at NLO or NNLO (i.e. does not consider direct measurements.)

- Strictly theoretical issue: How to understand the uncertainties of order Λ on the top mass from a theoretical viewpoint. Must rely upon simplified theoretical models, or simpler kinematic regions.
- More practical issue: How to use Monte Carlo generators to estimate order Λ uncertainties.

- Linear power suppressed corrections:  $\delta m = m \times \frac{\Lambda}{m}$ .
- They can be associated to IR renormalon with linear IR sensitivity: ∫ dk α<sub>s</sub>(k<sup>2</sup>).
- Those associated to the mass renormalon can yield systematic shifts of the mass value for processes dominated by off-shell tops?
- How about renormalons associated with jets?

For example, Hoang and collaborators have looked at boosted tops in  $e^+e^-$  annihilation, where renormalons associated with jets should be the same as for standard jets.

## Issue #2

Can we use Monte Carlo generators to assess (linear) power-suppressed corrections? (usually the method of choice in collider physics). What shall we require?

- The Monte Carlo should implement the correct perturbative physics, as accurately as possible, and model non-perturbative physics in a coherent way.
- The errors associated to power-suppressed effects should be determined by varying parameters, hadronization models, or even Monte Carlo implementations.
- One should impose the following restriction: the range of parameters variations, the hadronization models, and in general the Monte Carlo should give a good description of relevant data.

If, on one side, it is difficult to accept the result of this procedure as an upper bound on the error, it can certainly provide a lower bound, i.e. it can give an indication of the level of accuracy that we may conceivably aim to.

## Modeling perturbative physics as accurately as possible

- MC@NLO Frixione, Webber, PN and POWHEG-hvq Frixione, Ridolfi, PN. Include NLO radiation in production.
   hvq: User-Processes-V2/hvq
- The above with Shower Monte Carlo that do MEC corrections to top decay (Pythia8, Herwig7).
- tt\_dec Campbell,Ellis,Re,PN. Includes exact spin correlations and NLO corrections in decay in NWA. User-Processes-V2/ttb\_NLO\_dec
- ▶ bb41 Ježo,Lindert,Oleari,Pozzorini,PN 2016 Includes exact NLO matrix element for pp → lv
  <sub>l</sub>ℓv<sub>ℓ</sub>bb
  , thus finite width effects and interference between radiation in production and decay is included (Ježo,PN, 2015). User-Processes-RES/b\_bbar\_41

Experimental collaborations use typically the POWHEG-hvq generator interfaced to Pythia8 for the shower.

Comparing with the two more accurate generators, i.e.  $t\bar{t}\_dec$  and  $b\bar{b}41,$  we can understand whether

- modeling of radiation from the *b*-jet is adequate (comparing hvq and tt\_dec).
- ► interference effects in radiation from production and decay play a relevant role (comparing bb41 and tt\_dec).

(Ferrario-Ravasio, Ježo, Oleari, PN, arXiv:1801.03944)

- ▶ We focus upon the  $pp \rightarrow l\bar{\nu}_l \bar{\ell} \nu_\ell b\bar{b}$  process. Can be studied with the hvq, tt¯\_dec, and bb̃41 generators.
- ► We make the simplifying assumption that the *W* can be fully reconstructed.
- We consider the top mass determination from mass distribution of the system comprising the W and a (charge matched) b jet. (we also considered the b-jet energy spetrum, and the leptonic observables proposed by Frixione and Mitov.)
- We studied the effect of scale variation, PDF and α<sub>s</sub> sensitivity, and the differences between the Pythia8 and Herwig7 shower interface, as a first rough estimate of non-perturbative errors.

Assuming we have an observable O sensitive to the top mass, we will have in general

$$O = O_{\rm c} + B(m_t - m_{t,{\rm c}}) + O((m_t - m_{t,{\rm c}})^2)$$

where  $m_{t,c} = 172.5$  GeV is our central value for the top mass.  $O_c$  and B differ for different generator setup. Given an experimental result for O, the extracted mass value is

$$m_t = m_{t,\mathrm{c}} + (O_{\mathrm{exp}} - O_{\mathrm{c}})/B$$

By changing the generator setup  ${\it O}_{
m c}, {\it B} 
ightarrow {\it O}_{
m c}', {\it B}'$ :

$$m_t - m'_t = -\frac{O_c - O'_c}{B} - (O_{exp} - O'_c)(B - B')/(BB') \approx -\frac{O_c - O'_c}{B}$$

Thus:

- ► Compute the *B* coefficient using a single setup for the generator.
- ► Compute the O<sub>c</sub> coefficient (i.e. the value of the observable for m<sub>t</sub> = m<sub>t,c</sub>) for all different setup we want to explore.
- ► Extract the difference in the extracted *m<sub>t</sub>* between different setups, according to the equation

$$\Delta m_t = -rac{\Delta O_c}{B}.$$

W - bj is defined in the following way:

- ▶ Jets are defined using the anti- $k_T$  algorithm with R = 0.5. The  $b/\bar{b}$  jet is defined as the jet containing the hardest  $b/\bar{b}$ .
- ► W<sup>±</sup> is defined as the hardest I<sup>±</sup> paired with the hardest matching neutrino.
- ► The W bj system is obtained by matching a W<sup>+/-</sup> with a b/b̄ jet (i.e. we assume we know the sign of the b).

A difference  $\delta m_{rec}$  in the reconstructed mass peak between two generators with the same  $m_t$  parameter will lead to a  $\delta m_t = -\delta m_{rec}$  in the mass extracted by fitting a given data set (i.e.  $B \approx 1$ )

## Impact of finite width

Both  $b\bar{b}41$  and  $t\bar{t}_{-}dec$  include NLO radiation in decay.  $b\bar{b}41$  also includes finite width, non-resonant effects, interference of radiation in production and decay. Comparison of the two indicates that these effects, although not negligible, are not large.



$M_{ m rec}$ (GeV), $bar{b}$ 41-t $ar{t}_{-}$ dec					
bare smeared					
Py8	-0.03	-0.14			
Hw7	-0.046	-0.052			
Hw6	-0.012	-0.1			

Focus upon bb41-hvq comparison.

## Pythia8, POWHEG-hvq - POWHEG-bb41 comparison

We compare the new  $b\bar{b}41$  NLO+PS generator with the old hvq, using Pythia8 for the shower.



## Pythia8, POWHEG-hvq - POWHEG-bb41 comparison

Same, accounting for experimental errors by smearing the peak with a gaussian distribution with a width of 15 GeV.



## Pythia8, hvq, $t\bar{t}_dec, b\bar{b}41$ comparison

	PS	only	full		
	No smearing 15 GeV smearin		No smearing	15 GeV smearing	
$b\bar{b}4\ell$	$172.522 \pm 0.002~{\rm GeV}$	$171.403 \pm 0.002~{\rm GeV}$	$172.793 \pm 0.004~{\rm GeV}$	$172.717 \pm 0.002~{\rm GeV}$	
$t\bar{t}dec - b\bar{b}4\ell$	$-18\pm2~{\rm MeV}$	$+191 \pm 2 \text{ MeV}$	$+21\pm 6~{\rm MeV}$	$+140 \pm 2 \text{ MeV}$	
$hvq - b\bar{b}4\ell$	$-24\pm2~{\rm MeV}$	$-89\pm2~{\rm MeV}$	$+10\pm 6~{\rm MeV}$	$-147\pm2~{\rm MeV}$	

**Table 3.** Differences in the  $m_{Wb_j}$  peak position for  $m_t=172.5$  GeV for  $t\bar{t}dec$  and hvq with respect to  $b\bar{b}4\ell$ , showered with Pythia8.2, at the NLO+PS level and at the full hadron level.

	No sme	earing	$15 \mathrm{GeV}$ smearing		
	MEC	MEC - no MEC	MEC	MEC - no MEC	
$b\bar{b}4\ell$	$172.793 \pm 0.004  {\rm GeV}$	$-12\pm 6~{\rm MeV}$	$172.717 \pm 0.002~{\rm GeV}$	$+55\pm2~{\rm MeV}$	
$t\bar{t}dec$	$172.814 \pm 0.003~{\rm GeV}$	$-4 \pm 5 \text{ MeV}$	$172.857 \pm 0.001~{\rm GeV}$	$-26\pm2~{\rm MeV}$	
hvq	$172.803 \pm 0.003 \ {\rm GeV}$	$+61\pm5~{\rm MeV}$	$172.570 \pm 0.001 ~{\rm GeV}$	$+916\pm2~{\rm MeV}$	

**Table 4.**  $m_{Wb_j}$  peak position for  $m_t=172.5$  GeV obtained with the three different generators, showered with Pythia8.2+MEC (default). We also show the differences between Pythia8.2+MEC and Pythia8.2 without MEC.

Small differences in the smeared peak. Larger differences when smearing is included (i.e. modeling differences).

# Pythia8, hvq, tt\_dec,bb41 comparison

	$\begin{tabular}{ c c c c c } \hline $R = 0.4$ \\ \hline $No $ smearing $ 15 GeV $ smearing $ $ \end{tabular} \end{tabular} \end{tabular}$		R = 0.5		R = 0.6	
			No smearing	15 GeV smearing	No smearing	15 GeV smearing
$b\bar{b}4\ell$ [GeV]	$172.156 \pm 0.004$	$171.018 \pm 0.002$	$172.793 \pm 0.004$	$172.717 \pm 0.002$	$173.436 \pm 0.005$	$174.378 \pm 0.002$
$t\bar{t}dec - b\bar{b}4\ell$	$+35\pm5~{\rm MeV}$	$+195 \pm 2 \text{ MeV}$	$+21\pm 6~{ m MeV}$	$+140\pm2~{\rm MeV}$	$+1 \pm 7 {\rm ~MeV}$	$+97\pm2~{\rm MeV}$
$hvq - b\bar{b}4\ell$	$+47\pm5~{ m MeV}$	$-113\pm2~{\rm MeV}$	$+10\pm 6~{\rm MeV}$	$-147\pm2~{\rm MeV}$	$-7\pm 6~{\rm MeV}$	$-174\pm2~{\rm MeV}$

#### Jet radius dependence:

**Table 7**.  $m_{Wb_j}$  peak position obtained with the  $b\bar{b}4\ell$  generator for three choices of the jet radius. The differences with the  $t\bar{t}dec$  and the hvq generators are also shown.

#### Summary of theoretical uncertainties:

	No smearing			15 GeV smearing				
	$\% - b\bar{b}4\ell$	$(\mu_{ m R},\mu_{ m F})$	PDF	$\alpha_{\rm s}$	$\% - b\bar{b}4\ell$	$(\mu_{ m R},\mu_{ m F})$	PDF	$\alpha_{\rm s}$
$b\bar{b}4\ell$	+0  MeV	$^{+26}_{-17} {\rm MeV}$	-	$\pm 8 \text{ MeV}$	$+0 {\rm MeV}$	$^{+86}_{-53} {\rm MeV}$	-	$\pm 64 \ {\rm MeV}$
$t\bar{t}dec$	$+21 { m MeV}$	$^{+2}_{-10}$ MeV	-	$\pm 8 \text{ MeV}$	$+140~{\rm MeV}$	$^{+6}_{-6} {\rm MeV}$	-	$\pm 54 \ {\rm MeV}$
hvq	$+10 {\rm MeV}$	$^{+2}_{-6} {\rm MeV}$	$\pm 3 \text{ MeV}$	$\pm 2 \text{ MeV}$	$-147~{ m MeV}$	$^{+7}_{-7} {\rm MeV}$	$\pm 5 \text{ MeV}$	$\pm 9 \text{ MeV}$

**Table 6.** Theoretical uncertainties associated with the  $m_{Wb_j}$  peak position extraction for  $m_t=172.5$  GeV for the three different generators, showered with Pythia8.2. The PDF uncertainty on the  $b\bar{b}4\ell$  and  $t\bar{t}dec$  generators is assumed to be equal to the hvq one, as explained in Sec. 6.1.2.

We can summarize the comparison with Pythia8 by saying that we find a fairly consistent picture.

- The matrix element corrections (MEC) in Pythia work as well as the NLO corrections in decays, as expected.
- ► The smallness of scale variations in tt\_dec and hvq with respect to the bb41 can be explained as being due to the way in which the two generators implement off-shell effects.
- Hadronization effects have a consistent impact on the three generators.
- ► The shift in mass associated to the use of the bb41 generator with respect to the other two is around 150 MeV, with opposite signs. Although not totally negligible, this shift is well below presently quoted errors.

## POWHEG-bb41, Herwig7 - Pythia8 comparison



Figure 12.  $d\sigma/dm_{Wb_j}$  distribution obtained by showering the  $b\bar{b}4\ell$  results with Pythia8.2 and Herwig7.1, at parton-shower level (left) and with hadronization and underlying events (right).

No large difference in the peak position (i.e. no indication here of large NP effects that displace the peak.). However, the marked difference in shape is bound to lead to problems when the experimental resolution is taken into account.

## POWHEG-bb41, Herwig7 - Pythia8 comparison



When the resolution is accounted for, we find a 1.1 GeV difference between Herwig7 and Pythia8.

# POWHEG-bb41, Herwig7 - Pythia8 comparison

		No smea	ring	15 GeV smearing		
		Hw7.1	Py8.2 - Hw7.1	Hw7.1	Py8.2 - Hw7.1	
	$b\bar{b}4\ell$	$172.727 \pm 0.005~{\rm GeV}$	$+66\pm7~{\rm MeV}$	$171.626 \pm 0.002~{\rm GeV}$	$+1091\pm2~{\rm MeV}$	
ſ	$t\bar{t}dec$	$172.775 \pm 0.004~{\rm GeV}$	$+39\pm5~{ m MeV}$	$171.678 \pm 0.001~{\rm GeV}$	$+1179 \pm 2 \text{ MeV}$	
	hvq	$173.038 \pm 0.004~{\rm GeV}$	$-235\pm5~{\rm MeV}$	$172.319 \pm 0.001~{\rm GeV}$	$+251 \pm 2 \text{ MeV}$	

**Table 8.**  $m_{Wb_j}$  peak position for  $m_t=172.5$  GeV obtained with the three different generators, showered with Herwig7.1 (Hw7.1). The differences with Pythia8.2 (Py8.2) are also shown.

	Pythia8.2 - Herwig7.1						
	PS	5 only	full				
	No smearing 15 GeV smearing		No smearing	15 GeV smearing			
$b\bar{b}4\ell$	$+10\pm 2~{\rm MeV} \qquad +984\pm 2~{\rm MeV}$		$+66\pm7~{\rm MeV}$	$+1091\pm2~{\rm MeV}$			
$t\bar{t}dec$	$+5\pm 2~{\rm MeV}$	$+1083\pm2~{\rm MeV}$	$+39\pm5~{\rm MeV}$	$+1179\pm2~{\rm MeV}$			
hvq	$-0 \pm 2 \text{ MeV}$ $+113 \pm 2 \text{ M}$		$-235\pm5~{\rm MeV}$	$+251\pm2~{\rm MeV}$			

**Table 9.** Differences between Pythia8.2 and Herwig7.1 in the extracted  $m_{Wb_j}$  peak position for  $m_t=172.5$  GeV obtained with the three different generators, at the NLO+PS level (PS only) and including also the underlying events, the multi-parton interactions and the hadronization (full).

# POWHEG-bb41

While in the Pythia8 case we found a fully consistent picture, we cannot say the same for Herwig7. Several results are hard to understand:

- ▶ While the new generators bb41 and tt\_dec behave consistently with Herwig7, they display a large difference with respect to hvq.
- This means that MEC in Herwig7 do not have the same (expected) effect as in Pythia8

Can we dismiss Herwig7 on this ground? Consider that

- ▶ MEC in Pythia8 are also *technically* very similar to POWHEG.
- MEC in Herwig, being an angular ordered shower, are technically very different, since they are applied to the hardest emission found at each step of the shower.

So, the difference may well be beyond NLO effects, and thus may have to be considered as an uncertainty.

# Including Herwig6

With the collaboration of Bryan Webber, we have also included Herwig6 in our study.



At the shower level, Hw7 and Hw6 are very similar. Glitch right before the peak absent in Hw6. After hadronization and MPI, Hw6 becomes more symmetric with respect to Py8. As a consequence of that:

$M_{Wj}$ (GeV)							
	P	y8	Нwб		Hw7		
	bare smeared		bare	smeared	bare	smeared	
bb41	172.793	172.717	172.59	172.384	172.727	171.626	
tt_dec	172.814	172.857	172.602	172.484	172.775	171.678	
hvq	172.803	172.570	172.803	172.95	173.038	172.552	

as a fortuitous consequence of compensation due to hadronization and MPI in Herwig6.

This findings also suggest that shower and hadronization uncertainties may be dominant in direct measurements.

Agashe, Franceschini, Kim, Schulze, 2016

With Pythia8:

- ▶  $t\bar{t}_{dec}$  and  $b\bar{b}41$  differ by less than 200 MeV
- hvq differs from the other two by more than 500 MeV

▶ hvq NO MEC differs from the others by more than 1.9 GeV. Obviously more sensitive to radiation from the *b* quark. Since  $\delta m_t \approx \delta E_{\rm bjet}^{(\rm max)}/0.45$ , using hvq can cause a 1 GeV shift in mass (well below current uncertainties). With Herwig7:

- ▶ tt̄\_dec and bb̄41 differ by 20 MeV
- hvq differs from the other two by more than 660 MeV
- ▶ bb41+Py8 and bb41+Hw7 differ by more than 2 GeV

Switching from Pythia8 to Herwig7 leads to large differences, that would impact the mass measurement by more than 4 GeV.

#### Frixione, Mitov, 2014



Looking only at Pythia8: only  $p_T(\ell^+\ell^-)$  and  $m(\ell^+\ell^-)$  differ, presumably because of their sensitivity to spin correlations. Nearly 3 GeV difference between Pythia8 and Herwig7.

#### ► Try Pythia6.

- Try Sherpa? (unfortunately, no POWHEG-BOX interface is given there ...)
- Include also fully hadronic decay in a bb41 style generator, and perform more realistic studies of direct measurements.

#### Caveat:

Our results cannot be directly translated into an error in standard measurement. This can only be done within the experimental collaborations. However, it strongly suggests to consider using other shower generators in the analysis to assess the errors.

## Conclusions

- Unsatisfactory status of top mass measurements.
- The most precise measurements (i.e. the Direct Measurements) are being discredited in favour of much less precise methods.
- Two direction of research need to be explored:
  - Theoretical issues with direct top mass measurements should be addressed, also in simplified theoretical contexts, in order to understand whether there are phenomena that lead to systematic shifts of the measured mass.
  - The study of ambiguities in the top mass determination related to the use of different, or differently tuned NLO+PS combinations, may evidence problems in current measurements that are even more important, since they are manifestly practical problems that are likely to impact the accuracy of the measurements.

## **BACKUP MATERIAL**

## Renormalon Issue: Simplified (1-loop $\alpha_s$ ) illustration

$$m_P = m + N\alpha_s \sum_{n=0}^{\infty} c_n(\mu, m)\alpha_s^n,$$

where  $m_p$  is the pole mass, m is the  $\overline{\text{MS}}$  mass, and  $\alpha_s = \alpha_s(\mu)$ . The asymptotic behaviour of the expansion is (in 1-loop  $\alpha_s$ )

$$\begin{array}{rcl} \alpha_s^n c_n & \stackrel{n \to \infty}{\longrightarrow} & \mu t_a^{(n)}, \\ t_a^{(n)} & \equiv & (2b_0 \alpha_s)^n n! \approx \sqrt{2\pi} e^{(n+1/2)\log n - n + n\log(2b_0 \alpha_s)}, \end{array}$$
(3)

Minimum at  $n_{\rm m} \approx 1/(2b_0\alpha_s)$ . Using  $\alpha_s = 1/(b_0\log[\mu^2/\Lambda^2])$ :

$$t_{a}^{(n_{\rm m})} = \sqrt{2\pi n_{\rm m}} e^{-n_{\rm m}} = \sqrt{2\pi n_{\rm m}} \frac{\Lambda}{\mu}$$

The ambiguity of the asymptotic formula should be  $\mu$  independent. But the minimal term goes like

$$N\mu\alpha_{s}t_{a}^{(n_{\rm m})} = N\,\alpha_{s}\sqrt{2\pi n_{\rm m}}\,\Lambda\tag{4}$$

Needs an extra factor of  $\sqrt{n_{\rm m}}$  to be  $\mu$  independent.

Around the minimum

$$t_{a}^{(n)} \approx t_{a}^{(n_{\rm m})} \left( 1 + \frac{1}{2n} \left( n - n_{\rm m} \right)^2 \right)$$
 (5)

We can supplement the minimal term by a factor quantifying how many terms are close to the minimum

$$\frac{1}{2n}\left(n-n_{\rm m}\right)^2$$

 $\Delta_n$  times the minimal term is in fact  $\mu$  independent, and equal to

$$N\frac{\sqrt{4\pi p}}{2b_0}$$

#### Borel sum approach

We transform the series in the inverse Borel transform of a convergent series. Order by order in  $\alpha_s$  we have the identity

$$N\alpha_s \sum_{n=0}^{a} c_n(\mu, m) \alpha_s^n = N \int_0^{\infty} \mathrm{d}r \, e^{-\frac{r}{\alpha_s}} \sum_{n=0}^{a} c_n(\mu, m) \, \frac{r^n}{n!}.$$

Plugging in the asymptotic value for the coefficients:

$$N\mu\int_0^\infty \mathrm{d}r\,e^{-\frac{r}{\alpha_s}}\sum_{n=0}^a (2b_0r)^n = N\mu\int_0^\infty \mathrm{d}r\,\frac{e^{-\frac{r}{\alpha_s}}}{1-2b_0r}$$

The singularity in  $r = 1/(2b_0)$  is due to the renormalon. One can define the sum as the principal value for the integral, and the ambiguity as the imaginary part of the integral divided by  $\pi$  (Beneke, 1999)

$$N\mu \frac{1}{2b_0} e^{-\frac{1}{2b_0\alpha_s}} = \frac{N}{2b_0} \Lambda$$

#### Mass Renormalon: size of the ambiguity

The Pole Mass  $m_P$  is given in terms of the  $\overline{MS}$  mass m by an expansion of the form

$$m_P = m + N\alpha_s \sum_{n=0}^{\infty} c_n(\mu, m) \alpha_s^n.$$
(6)

The coefficients grow as the factorial of n. Can also be written as

$$m_P = m + N \int_0^\infty \mathrm{d}r \, e^{-\frac{r}{\alpha_s}} \sum_{n=0}^\infty \frac{c_n(\mu, m)}{n!} r^n. \tag{7}$$

 $c_n \propto n! \longrightarrow c_n/n! \propto \text{const.}$ , i.e.: geometric divergence for some r.

Prescription used by Beneke etal: take the principal value of the integral as its central value, and (the absolute value of) its imaginary part divided by Pi as the estimate of the error.



Hoang etal prescription:

Take as error half of the sum of all terms that do not exceed the smallest term by more than a factor f.

*f* is defined to be "a number larger but close to unity" and f = 5/4 = 1 + 0.25 is chosen.

It is not difficult to make contact among the two procedures, and show that the Beneke etal method roughly corresponds to the above with  $f = 1 + 1/(4\pi)$  (which explains a good part of the difference).

A remaining part is due to the way scale variation uncertainties are estimated in Hoang etal: they truncate the expansion at the minimal term and then perform scale variation. When the scale is varied, the position of the minimal term changes, leading to scale compensation, see PN, arXiv:1712.02796. It is clear that the choice of f in Hoang etal, as well as the choice of the factor in front of Im/Pi in Beneke etal, are rather arbitrary (and slightly reminiscent of scale variation issues).

The motivation for the Im/Pi choice in Beneke etal is that it works well in context where the renormalon effect can be related to some physical observable (Beneke 1999).

In all cases, the message is: the renormalon problem cannot be used as an excuse to abandon pole mass measurements.