Particle Physics in the Early Universe (FS 17)

Exercise Series 8

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The presence of the sphaleron transitions, in equilibrium, much above the energy range of the EW phase transition works as a wash-out mechanism of any pre-existing baryon or lepton asymmetry that might have been created in an earlier stage of the Universe evolution. Let's assume that there is such an asymmetry from some unspecified mechanism. The chemical potential of each fermion will then be non-zero, in general. However the chemical potential of a particle and its antiparticle are always of equal value and opposite in sign.

At equilibrium, each SM particle, i, has a chemical potential μ_i. In every reaction

$$A + B + \ldots \leftrightarrow A' + B' + \ldots \tag{1}$$

the sum of chemical potential at each side are equal

$$\mu_A + \mu_B + \ldots = \mu_{A'} + \mu_{B'} + \ldots \tag{2}$$

Moreover, all conserved quantum numbers, Q_a are also the same at each side. We can express this efficiently, by assigning a "chemical potential" to every conserved quantum number, μ_a and letting

$$\mu_i = \sum_a \mu_a Q_a^i \tag{3}$$

Note that there is only one chemical potential for every quantum number. The conservation of these quantum numbers $\sum_i Q_a^i = \sum_j Q_a^j$ between the two sides of any reaction, then, translates automatically to eq.2.

In our case there are two[1] quantum numbers relevant, $B - L$, and Y.

Hence

$$\mu_i = \mu(B_i - L_i) + \mu_Y Y_i/2 \tag{4}$$

[1]One would think that also T_3, the quantum number associated with $\mathrm{SU}(2)_L$ would be relevant, but it turns out that its contribution would drop out of the computation here, i.e. $\mu_{T_3} = 0$ at equilibrium.

where $i \in \{u_L, d_L, u_R, d_R, e_L, \nu_e, e_R, h^0, h^+\}$.

Their hypercharges[2] are respectively $Y_i = \{1/3, 1/3, 4/3, -2/3, -1, -1, -2, 1, 1\}$.

Exercise 1 Show that (for temperatures much larger than the masses of all particles and also much larger than the chemical potentials themselves), the difference in number density between particles and antiparticles of any species is related to their chemical potential by

$$\Delta n_i = n_i - n_{\bar{i}} = 8\pi^3 g_i \mu_i \frac{T^2}{3} \quad : Bosons \tag{5}$$

and

$$\Delta n_i = n_i - n_{\bar{i}} = 8\pi^3 g_i \mu_i \frac{T^2}{6} \quad : Fermions \tag{6}$$

(where μ_i is the chemical potential of the particles of type i and $n_{\bar{i}}$ is the number density of the anti-particles of type i).

note: Use the familiar equation for the number density as an integral over the Fermi-Dirac or Bose-Einstein distributions

$$n_i = g_i 4\pi \int dE\ E^2 \frac{1}{e^{(E-\mu)/T} \mp 1} \tag{7}$$

and expand in terms of μ, noting that only the linear terms in μ survive in Δn_i. Use

$$\int_0^\infty dx \frac{x^2 e^x}{e^x - 1} = \frac{\pi^2}{6} \quad , \quad \int_0^\infty dx \frac{x^2 e^x}{e^x + 1} = \frac{\pi^2}{3} \tag{8}$$

Exercise 2 Based on the previous result, compute $\Delta n(h^+) + \Delta n(h^0)$, $\Delta n(e_L) + \Delta n(\nu_e)$,$\Delta n(u_L) + \Delta n(d_L)$,$\Delta n(e_R)$, $\Delta n(u_R)$, $\Delta n(d_R)$ as a function of μ and μ_Y. Note that there are 3 quark versions of every type, due to color, and account for ν_f different families of quarks and leptons.

Exercise 3 Impose that the system is neutral under Hypercharge, i.e. that

$$\sum_i \Delta n(i) Y_i = 0 \tag{9}$$

and find

$$\nu_f \left(\frac{5}{3}\mu_Y + \frac{4}{3}\mu \right) + \frac{1}{2}\mu_Y = 0 \tag{10}$$

[2] In case of doubt, you can always find the hypercharges using $Q = T_3 + Y/2$ where T_3 is the eigenvalue of the weak isospin, e.g. $+1/2$ for particles that are upper components in SU(2)$_L$ doublets.

Solve this with respect to μ and substitute in Δn_i so that they are only a function of μ_Y (and T and ν_f).

Exercise 4 The baryon number at equilibrium is

$$B = \frac{1}{3}\left(\Delta n(u_L) + \Delta n(u_R) + \Delta n(d_L) + \Delta n(d_R)\right) \tag{11}$$

and the Lepton number is

$$L = \left(\Delta n(e_L) + \Delta n(e_R) + \Delta n(\nu_e)\right) \tag{12}$$

Compute them in terms of μ_Y, T and ν_f.

Write B as

$$B = C(B - L) \tag{13}$$

and find

$$C = \frac{8\nu_f + 4}{22\nu_f + 13} \tag{14}$$

which, for three families, $\nu_f = 3$ gives $C = 28/79$.