Particle Physics in the Early Universe (FS 17)

Exercise Series 6
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We will derive, in this exercise series, the surface tension that a bubble had while expanding during a first order Electroweak phase transition in the Early Universe. The tension, μ, was defined in the lecture from

$$\Delta F = 4\pi R^2 \mu - \frac{4}{3}\pi R^3 \Delta V \tag{1}$$

where ΔF is the difference in the free energy between having and not having a bubble, and ΔV is considered small and positive.

We will consider a bubble with a very thin wall separating it from the outside. We will ignore the curvature of the surface wall, so that the problem becomes one-dimensional. The bubble has radius R, and we denote by r the radial coordinate in a frame that is centred at the centre of the bubble. Inside the bubble, i.e. for $r \ll R$

$$h(r) = h_0 \quad , \quad V(h) = V_{\texttt{true}} \tag{2}$$

while far outside the bubble, $r \gg R$

$$h(r) = 0 \quad , \quad V(h) = V_{\texttt{false}} \tag{3}$$

where $h(r)$ is the Higgs field and $V(h)$ is the Higgs potential[1]. In this notation $\Delta V = V_{\texttt{false}} - V_{\texttt{true}}$

The free energy of the bubble depends on the configuration of the Higgs field. For a given configuration it is

$$F[h] = \int_0^\infty 4\pi r^2 dr \left\{ \frac{1}{2}(\frac{dh}{dr})^2 + V(h) \right\} \tag{4}$$

Exercise 1 Show that the difference in free energy between a configuration with a bubble and a configuration without one is

$$\Delta F = F_{bub} - F_{nobub} = \int_0^\infty 4\pi r^2 dr \left\{ \frac{1}{2}(\frac{dh}{dr})^2 + V(h) - V_{\texttt{false}} \right\} \tag{5}$$

[1] In fact the effective Higgs potential, including possible thermal terms.

Exercise 2 The wall is assumed to be thin. It starts at $R-\delta$ and ends at $R+\delta$. Show that

$$\Delta F = -\frac{4}{3}\pi R^3 (V_{\texttt{false}} - V_{\texttt{true}}) + \int_{R-\delta}^{R+\delta} 4\pi r^2 dr \left\{ \frac{1}{2}(\frac{dh}{dr})^2 + V(h) - V_{\texttt{false}} \right\} \tag{6}$$

We will call the second term ΔF_T for 'tension'.

For small values of δ we can consider $r^2 \simeq R^2$ and get

$$\Delta F_T = 4\pi R^2 \int_{R-\delta}^{R+\delta} dr \left\{ \frac{1}{2}(\frac{dh}{dr})^2 + V(h) - V_{\texttt{false}} \right\} \tag{7}$$

Exercise 3 The field will assume the configuration that minimizes ΔF_T. This is found by solving the Euler-Lagrange equation

$$\frac{d}{dr}\frac{\vartheta L}{\vartheta \frac{dh}{dr}} = \frac{\vartheta L}{\vartheta h} \tag{8}$$

with L being the integrand of the functional ΔF_T. Show that the resulting equation is

$$\frac{d^2 h}{dr^2} = \frac{\vartheta V}{\vartheta h} \tag{9}$$

Exercise 4 Multiply the above equation with $\frac{dh}{dr}$ and integrate over dr to obtain

$$\frac{dh}{dr} = -\sqrt{2(V(h) - V_{\texttt{false}})} \tag{10}$$

Convince yourself that the minus sign has to be picked by drawing a diagram of $h(r)$ compatible with the constraints (i.e. see that the derivative of $h(r)$ is negative in the wall region).

Exercise 5 Insert the above expression into eq. 7, change the integration variable from r to $h(r)$ and show that

$$\Delta F_T = 4\pi R^2 \int_0^{h_0} dh \sqrt{2(V(h) - V_{\texttt{false}})} \tag{11}$$

which is equivalent to

$$\mu = \int_0^{h_0} dh \sqrt{2(V(h) - V_{\texttt{false}})} \tag{12}$$