Particle Physics in the Early Universe (FS 17)

Exercise Series 5

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The goal of this exercise is to derive the relation between the dipole anisotropy of the Cosmic Microwave Background radiation and the movement of the earth in the reference frame defined by the CMB photon gas. We will consider the earth at rest in an inertial frame (unprimed) that is moving with constant speed β with respect to the inertial frame (primed) defined by the photon gas. We will also set both coordinate systems such that the motion is along the z-axis of both systems. Hence, a photon with momentum p^μ in the CMB frame has momentum $p^{\mu\prime}$ in the earth frame.

Exercise 1 Convince yourself that, contrary to the number density, the number of photons with momentum in the infinitesimal phase space volume between $\vec{p}$ and $\vec{p}+d\vec{p}$, **divided by** this infinitesimal phase-space volume, is a Lorentz scalar. In order to do this, start with

$$\frac{1}{8\pi V}dn = \frac{1}{e^{E/KT}-1}d^3\vec{p} \tag{1}$$

and construct an argument (or more than one) showing that $\frac{1}{8\pi V}\frac{dn}{d^3\vec{p}}$ is a Lorentz scalar. As a consequence

$$\frac{1}{e^{E/KT}-1} = \frac{1}{e^{E'/KT'}-1} \tag{2}$$

under a Lorentz boost sending E and T in E' and T'.

Exercise 2 Show that

$$\frac{E'}{E} = \frac{T'}{T} = \frac{1}{\gamma(1+\beta\cos\theta)} \tag{3}$$

where $\gamma = 1/\sqrt{1-\beta^2}$ and θ is the angle between the photon's momentum and the direction of earth's movement. In order to do this, given that $E=\hbar\nu$, where ν is a frequency, one has to show that in two different reference systems

$$\nu' = \frac{\nu}{\gamma(1 + \beta\cos\theta)}. \tag{4}$$

This is called the relativistic Doppler effect. It can be derived by going in the rest frame of the CMB (considered as a source of radiation), and calculate the frequency of radiation received by the observer as measured in this frame. This should yield the classical Doppler effect. By transforming to the reference of frame of the observer, the final result should be obtained.

Exercise 3 Show that, for $\beta \ll 1$ (i.e. for speeds much smaller than the speed of light), photons coming from the direction of earth's movement have

$$\frac{\delta T}{T} \simeq \beta \tag{5}$$

while photons coming from the opposite direction have

$$\frac{\delta T}{T} \simeq -\beta \tag{6}$$

The WMAP experiment has measured a maximum $\delta T = 3.346 mK$. Compute the speed of earth ($T_0 = 2.725K$) that this measurement yields.

note: the earth, along with the solar system, moves with respect to the Galaxy with $\simeq 230 km/s$ in the direction opposite to the movement with respect to the CMB frame! This implies that the Milky Way (and the local group of galaxies) move with a speed of approximately $600 km/s$.

note: The fractional anisotropy $\delta T/T$, being a function of $\cos\theta$, can be expanded in terms of Legendre polynomials[1]:

$$\frac{\delta T}{T} = -\frac{\beta^2}{6} - \beta P_1(\cos\theta) + \frac{2\beta^2}{3} P_2(\cos\theta) + \ldots \tag{7}$$

The constant term is called the monopole term, the $P_1(\cos\theta)$ term is called the dipole term, while the $P_2(\cos\theta)$ is the quadruple term etc.

Since $\beta \simeq 10^{-3}$, only the dipole term practically survives. However the quadruple term, although very small, is not much smaller than the intrinsic CMB anisotropies and has to be taken into account in a careful analysis of data.

[1] In principle the expansion is in spherical harmonics, the so-called multipole expansion. But in our case there is no dependence in the azimuthal angle ϕ, so that the expansion can be written in terms of Legendre polynomials alone.