Particle Physics in the Early Universe (FS 17)

Exercise Series 4

Dr. Achilleas Lazopoulos

December 20, 2019

The goal of this exercise is to show that the total rate for the reaction of a positron by neutrons in the Early Universe, before electron-positron annihilation, according to

$$n + e^+ \to p + \overline{\nu}_e \tag{1}$$

is

$$\lambda(n + e^+ \to p + \overline{\nu}) = \int_{-\infty}^{m_e - Q} \sqrt{1 - \frac{m_e^2}{(q+Q)^2}} \frac{q^2(q+Q)^2}{(1 + e^{-(q+Q)/k_B T})(1 + e^{q/k_B T_\nu})} \tag{2}$$

Exercise 1 Consider the reaction as a quark-level scattering process

$$d + e^+ \to u + \overline{\nu} \tag{3}$$

and show that the cross section is

$$\sigma = \frac{2\pi^2 A' E_\nu^2}{v_{e^+}} \tag{4}$$

where

$$A' = \frac{2G_F^2 \cos^2\theta_c}{\pi^3} \tag{5}$$

note: as a reminder, the coupling of the W boson to quarks is $g\gamma^\mu \frac{1-\gamma_5}{2} V_{ud}$, where g is the SU(2) coupling constant. Also $m_W = \frac{gv}{2}$ where v is the vacuum expectation value of the Higgs boson, and $G_F = \frac{1}{v^2\sqrt{2}}$. Finally, the CKM matrix V_{ud} is also written in therms of the Cabbibo angle as $\cos\theta_c$.

note: this approximation is missing the fact that the neutron, as a whole, has a different axial coupling than the down quark. This induces a correction factor of

$$\frac{(1 + 3g_A^2)}{4} \tag{6}$$

where g_A has to be measured from neutron's life time.

Hence we will use

$$\sigma = \frac{2\pi^2 A E_\nu^2}{v_{e^+}} \tag{7}$$

with

$$A = \frac{G_F^2(1+3g_A^2)\cos^2\theta_c}{2\pi^3} \tag{8}$$

Exercise 2 Show that the density of positrons in the medium, that have 3-momentum between p_{e^+} and $p_{e^+} + dp_{e^+}$ is

$$n_{e^+} = 4\pi \frac{p_{e^+}^2 dp_{e^+}}{(2\pi)^3} \frac{1}{1+e^{E_{e^+}/k_B T}} \tag{9}$$

Exercise 3 The produced neutrino has to occupy an energy state of energy E_ν. In the medium there are many other neutrinos that, being in thermal equilibrium with temperature T_ν, occupy already a fraction of energy levels equal to

$$\frac{1}{1+e^{E_\nu/k_B T_\nu}} \tag{10}$$

Since neutrinos are fermions (with one helicity state) the produced neutrino has to occupy one of the free energy states available. Show that their fraction is

$$\frac{1}{1+e^{-E_\nu/k_B T_\nu}} \tag{11}$$

Exercise 4 Assembling everything together show that

$$\lambda(n + e^+ \to p + \overline{\nu}_e) = A \int_0^\infty \frac{E_\nu^2}{v_{e^+}} \frac{p_{e^+}^2 dp_{e^+}}{(1+e^{E_{e^+}/k_B T})(1+e^{-E_\nu/k_B T_\nu})} \tag{12}$$

The protons and the neutrons are non-relativistic, to the point that they can be considered at rest. Then define

$$E_n - E_p = m_n - m_p \equiv Q \tag{13}$$

Also define the variable

$$q = -E_\nu \tag{14}$$

and show that the energy of the neutrino has to be larger than $Q + m_e$, so that

$$q < -Q + m_e \tag{15}$$

Then, using $v_{e^+} = p_{e^+}/E_{e^+}$, and changing variables from p_{e^+} to q, show that

$$\lambda(n + e^+ \to p + \overline{\nu}) = \int_{-\infty}^{m_e - Q} \sqrt{1 - \frac{m_e^2}{(q+Q)^2}} \frac{q^2(q+Q)^2}{(1+e^{-(q+Q)/k_B T})(1+e^{q/k_B T_\nu})} \tag{16}$$