Particle Physics in the Early Universe (FS 17)

Exercise Series 12

Dr. Achilleas Lazopoulos

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Exercise 1 Let us estimate the total duration of inflation, in the chaotic inflation models, for the two cases examined in class:

(i) $V(\phi) = \frac{m^2}{2}\phi^2$

(ii) $V(\phi) = \frac{\lambda}{4}\phi^4$

We will assume that the inflationary period starts when the field ϕ_i is such that the potential energy is at the order of M_{PL}, i.e. we will assume that $V(\phi_i) \sim M_{PL}^4$.

We can write

$$\Delta t = \int_{t_i}^{t_f} dt = \int_{\phi_i}^{\phi_e} \frac{d\phi}{\dot{\phi}} \tag{1}$$

Use the slow-roll equations (that are valid throughout inflation)

$$\dot{\phi} = -\frac{1}{3H} V'(\phi) \tag{2}$$

$$H = \frac{1}{M_{PL}} \sqrt{\frac{8\pi V}{3}} \tag{3}$$

to get

$$\Delta t = -\sqrt{24\pi} \frac{1}{M_{PL}} \int_{\phi_i}^{\phi_e} d\phi \frac{\sqrt{V}}{V'} \tag{4}$$

In order to get the amplitude of primordial density perturbations correct, in this model of inflation, we need for case (1) $m \sim 10^{-6} M_{PL}$ and for case (2) $\lambda \sim 10^{-13}$. Show that in both cases the dependence of Δt on the precise value of t_e is negligible, and arrive at the estimates:

(i)

$$\Delta t \sim \frac{M_{PL}}{m^2} \sim 10^{-31} s \tag{5}$$

(ii)

$$\Delta t \sim \frac{1}{M_{PL}} \frac{1}{\sqrt{\lambda}} \log \frac{1}{\lambda} \sim 10^{-35} s \tag{6}$$

Hint: $M_{PL} = 10^{19} GeV$ and $1 GeV = 10^{24} s^{-1}$, so $\frac{1}{M_{PL}} = 10^{-43} s$

Exercise 2 Solve the two slow roll equations, eq.(2)-(3), explicitly, for the two cases of $V(\phi)$ above and find the time dependence of $\phi(t)$ and $H(t)$ in each case. Using $m \sim 10^{-6} M_{PL}$ and $\lambda \sim 10^{-13}$, verify the estimates for the duration of inflation of the previous exercise.

Exercise 3 The number of e-foldings was defined to be

$$N_e = \log \frac{a(t_e)}{a(t_i)} \tag{7}$$

and since

$$a(t) = a(t_i) e^{\int_{t_i}^{t} dt' H(t')} \tag{8}$$

we have

$$N_e = \int_{t_i}^{t_e} dt H(t) \tag{9}$$

Show that in the case of slow roll inflation, i.e. when eq.(2)-(3) are valid,

$$N_e = -\frac{8\pi}{M_{PL}} \int_{\phi_i}^{\phi_e} d\phi \frac{V}{V'} \tag{10}$$

For the two cases of exercise 1, show that

$$N_e \sim 4\pi \frac{M_{PL}^2}{m^2} \quad , \quad N_e \sim \frac{2\pi}{\sqrt{\lambda}} \tag{11}$$

Using the estimates $m \sim 10^{-6} M_{PL}$ and $\lambda \sim 10^{-13}$, find how many times is a region of size L magnified during the 10^{-31}s (10^{-35}s) that inflation lasts in case 1 (2).

answer: $e^{10^{13}} \sim 10^{10^{13}}$ times in case 1, $e^{10^7} \sim 10^{10^7}$ times in case 2. It's not always easy to realise how large this magnification is: the size of the observable Universe today is, by comparison, 'only' a factor of $\sim 10^{42}$ times the size of the proton...