The need for parton showers:

- Accelerated charges radiate. The cascade of emission is a "parton shower."
- Jets are defined experimentally with respect to hadronic final states.
- How to pass from few hardy emitted partons to a multitude of hadrons?
- Soft and collinear legs need to be factorized (modelled).
Probability in Radioactivity.

Survival probability after time \( t \): \( S(t) = P_{\text{decay}}(t) = e^{-t/\tau} = e^{-\tau t} \)

This is the cumulative probability. The decay probability distribution is:

\[
\frac{dP_{\text{decay}}}{dt} = -\frac{dP_{\text{decay}}}{dt} = \Gamma e^{-\Gamma t}
\]

Rewrite\( \Gamma t \rightarrow \int_{0}^{t} \dt' \Gamma(t') \)

\[
\frac{dP_{\text{decay}}}{dt} = \Gamma(t) \frac{\mathcal{L}}{\Gamma(t)}
\]

Probability of no decay up to \( t \)!

Remember:\( \frac{d\mathcal{Q}(x, Q^2)}{d(\log Q^2)} = \int \frac{d\mathcal{L}}{x \gamma} \mathcal{A}(Q^2) P_{\gamma\gamma} \left( \frac{x}{\gamma} \right) Q(y, Q^2). \) DELAP.

Remember: Collinear emission comes with a price!

\[
d\sigma_{\text{collinear}} = d\sigma \frac{Q_0}{2\pi} \frac{d\mathcal{K}_T^2}{\mathcal{K}_T^2} P(z) dz.
\]

You can write the collinear variable \( \rightarrow \frac{-\mathcal{K}_T^2}{\mathcal{K}_T^2} \frac{-\theta^2}{\theta^2} \). \( \mathcal{K}_T^2 = 2(1-\gamma) E^2 \theta^2. \)

\( \rightarrow \mathcal{K}_T^2 \) integration diverges!

\( \rightarrow \) Introduce a cutoff \( \rightarrow \) divergence becomes a log to

\( \rightarrow z \rightarrow 0 \) divergence cured by cutoff.
Resolution criterion: $k_T > Q_0$ (≈ 16 GeV)

Combine virtual contributions with unresolved emissions would cause divergences.

The differential probability for emission between $q^2$ and $q^2 + dq^2$ is:

$$dP = \frac{\alpha_s}{\pi} \frac{dq^2}{q^2} \int_{z_{\min}}^{z_{\max}} d\alpha P(\alpha) \equiv dq^2 \frac{\alpha_s}{\pi} \frac{i}{q^2} \int_{z_{\min}}^{z_{\max}} dz \frac{P(\alpha)}{\Gamma(q^2)}$$

No emission up to $q^2$: Sudakov form factor $\Delta(q^2, q^2) = e^{-\int_{q^2}^{q^2} \frac{dk^2}{k^2} \Gamma(k^2)}$
Multiple emissions

Highest contribution comes from ordered emissions:

\[ d\sigma = \sigma_0 \int_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \cdots \int_{Q_0^2}^{t_{n-1}} \frac{dt_n}{t_n} \sim \log \left( \frac{Q^2}{Q_0^2} \right) \]

\[ Q^2 \text{ is process dependent!} \]

Improvement: instead of fixing \( Q^2 \) once and for all, one can introduce a \( k_t^2 \) dependent \( \kappa_s(k_t^2) \) for every branching. This incorporates some virtual pieces!

\[ \Rightarrow \text{Must avoid Landau pole} \quad k_t^2 > Q_0^2 \Rightarrow \lambda_{QCD}^2 \]

{\text{Soft logarithms}}

\[ \Rightarrow \text{Problem: they come from all over!} \]

\[ \Rightarrow \text{But angular ordering takes care of soft limit! (Marchesini and Webber)} \]

\[ \Rightarrow k_t^2 \text{ ordering doesn't automatically!} \]
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**Hadronization**

- Dipoles in QCD

- Assuming a linear QCD potential describes "well" Quarkonia spectrum

- Feynman - Field fragmentation
  - Recursively fragment \( q \rightarrow q' + \text{hadron} \)
  - With \( p_T \) fitted Gaussian
  - \( p_T \) arbitrary
  - Flavour from symmetry arguments

Problems: what to do with last quark?

- Infrared unsafe
- No direct link to pert. theory
- No predictability!

- Yo-yo strings

\[ M^2 \sim \text{area}. \]

Mesons only! Should we treat baryons as \((q\bar{q}) - q\) bound states?
Ignoring gluon radiation (q\bar{q} pairs pop out of vacuum)

String tension is a parameter!

\[ \rightarrow \text{Gluons: kinks on strings!} \]

Clustering - Preconfinement

Large $N_c$ limit \[ \text{only planar graphs!} \]

The mass of the singlets peaked at $Q_0^2$!

Preconfinement \(\rightarrow\) gluons in $q\bar{q}$ \(\rightarrow\) clusters (excited hadrons) \(\rightarrow\) decay to hadrons
**Underlying event** (beyond factorization)

- Protons are actually extended objects
- No guarantee for one scattering only
- Running of $\alpha_s$: preference for soft scattering!

- Evidence for multiple scattering, in $g+3$ jets.
- Definition problem: everything apart from hard+slowest
  - remnants
  - remnants interactions

- There is no first-principle approach yet.
- Models are usually based on collinear factorization x-sections, which
  makes them sensitive to $p_T^{\text{min}}$
- Tuning to data very aggressive!