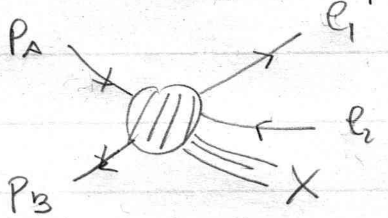
Drell-Yan process

We look for a totally inclusive process in hadron-hadron collisions which is dominated by large momentum scales.
A suitable candidate is hadronic dilepton production

$$A + B \rightarrow \ell^+ + \ell^- + X$$



ℓ^+ and ℓ^- are a lepton-antilepton pair which we detect and we leave all hadronic final states unmeasured

The hadronic c.o.m frame kinematics is

$$P_A = \frac{\sqrt{s}}{2}(1,0,0,1) \qquad \ell_1 = p_{T1}(\cosh y_1, \vec{n}_1, \sinh y_1)$$

$$P_B = \frac{\sqrt{s}}{2}(1,0,0,-1) \qquad \ell_2 = p_{T2}(\cosh y_2, \vec{n}_2, \sinh y_2)$$

The quantities p_{T1} and p_{T2} are the transverse momenta of the two leptons and $\vec{n}_1$ and $\vec{n}_2$ their directions in the plane transverse to the beam. These quantities are invariant under boosts on the longitudinal direction (i.e. the direction of the beam)

The other the quantities y_1 and y_2 are the rapidities of the two leptons. The rapidity y of a particle $p = (E, \vec{p}_T, p_z)$ is defined as

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$

For massless particles $y = -\ln \tan \frac{\theta}{2}$ where θ is the angle with respect to the beam axis.
For massive particles $\eta = -\ln \tan \frac{\theta}{2}$ is referred to as "pseudorapidity"

Dilepton invariant mass distribution

In the naive parton model the total cross section for dilepton production can be obtained by considering the partonic cross section $\hat{\sigma}_{q\bar{q}\to e^+e^-}$ with $p_q \equiv p_1 = x_1 p_A$ and $p_{\bar{q}} \equiv p_2 = x_2 p_B$ and convoluting it with parton densities

$$\sigma_{AB\to e^+e^-} = \sum_{q\bar{q}} \int dx_1\, dx_2 \left[f_{q/A}(x_1)\, f_{\bar{q}/B}(x_2) + f_{\bar{q}/A}(x_1)\, f_{q/B}(x_2) \right] \hat{\sigma}_{q\bar{q}\to e^+e^-}$$

The analogous of Bjorken limit $Q^2, \nu \to +\infty$ with $x = Q^2/2\nu$ fixed is $(\ell_1+\ell_2)^2 \equiv M^2$, $S \to +\infty$ with $\tau = M^2/S$ fixed.

At tree level we can get $\hat{\sigma}_{q\bar{q}\to e^+e^-}$ from e^+e^- annihilation

$$\hat{\sigma}_{q\bar{q}\to e^+e^-} = \frac{4}{3}\pi \frac{\alpha^2}{\hat{s}} \frac{1}{N_c} \cdot Q_q^2$$

At Born level $M^2 = (\ell_1+\ell_2)^2 = (p_1+p_2)^2 = \hat{s}$, so that

$$\frac{d\hat{\sigma}}{dM^2} = \frac{\sigma_0}{N_c} Q_q^2 \cdot \delta(\hat{s} - M^2) \quad \text{where} \quad \sigma_0 = \frac{4}{3}\frac{\pi\alpha^2}{M^2}$$

Introducing $\tau = M^2/S$ we extract the "scaling" of the partonic cross section

$$M^4 \frac{d\hat{\sigma}}{dM^2} = \frac{4}{3}\pi \frac{\alpha^2}{N_c} \cdot \tau\, \delta(x_1 x_2 - \tau) = \frac{4}{3}\pi \frac{\alpha^2}{N_c}\, \delta(1-z) \qquad z = \frac{M^2}{\hat{s}}$$

We now plug this expression in the total hadronic cross section

$$M^4 \frac{d\sigma}{dM^2} = \sum_q \int dx_1\, dx_2 \left[f_{q/A}(x_1) f_{\bar{q}/B}(x_2) + f_{\bar{q}/A}(x_1) f_{q/B}(x_2) \right] M^2 \frac{d\hat{\sigma}}{dM^2} =$$

$$= \frac{4}{3}\pi \frac{\alpha^2}{N_c} \cdot \tau\, F(\tau)$$

At Born level $F(\tau)$ depends only on combination of parton distributions

$$F(\tau) = \sum_q Q_q^2 \int_0^1 dx_1 \int_0^1 dx_2 \left[f_{q/A}(x_1) f_{\bar{q}/B}(x_2) + f_{\bar{q}/A}(x_1) f_{q/B}(x_2) \right] \cdot \delta(x_1 x_2 - \tau)$$

It is now extremely useful to change variables using

$$\frac{dx_1}{x_1}\frac{dx_2}{x_2} = \frac{d\tau}{\tau}\, dy = \frac{d\Pi^2}{\Pi^2}\, dy$$

where $y = \frac{1}{2}\ln\frac{x_1}{x_2}$ is the total rapidity of the lepton system and

$$x_1 = \sqrt{\tau}\, e^{y} \qquad x_2 = \sqrt{\tau}\, e^{-y}$$

Note that $x_1, x_2 < 1 \Rightarrow -\frac{1}{2}\ln\frac{1}{\tau} < y < \frac{1}{2}\ln\frac{1}{\tau}$

We can then integrate over τ and obtain

$$F(\tau) = \sum_q Q_q^2 \int_{-\frac{1}{2}\ln\frac{1}{\tau}}^{\frac{1}{2}\ln\frac{1}{\tau}} dy \cdot \tau \cdot \left[f_{q/A}(\sqrt{\tau}e^{y}) f_{\bar{q}/B}(\sqrt{\tau}e^{-y}) + f_{\bar{q}/A}(\sqrt{\tau}e^{+y}) f_{q/B}(\sqrt{\tau}e^{-y}) \right]$$

It is now possible to set the double differential cross section $d\sigma/d\Pi^2 dy$

$$\frac{d\sigma}{d\Pi^2 dy} = \frac{4}{3}\pi \frac{\alpha^2}{N_c} \frac{1}{S^2} \sum_q Q_q^2 \left[f_{q/A}(\sqrt{\tau}e^{y}) f_{\bar{q}/B}(\sqrt{\tau}e^{-y}) + f_{\bar{q}/A}(\sqrt{\tau}e^{y}) f_{q/B}(\sqrt{\tau}e^{-y}) \right]$$

This measurement gives direct access to antiquark pdf's inside the proton! Notice the different pdf combinations

- pp scattering mixture of valence and sea pdf's

$$\frac{d\sigma}{d\Pi^2 dy} \propto \sum_q Q_q^2 \left[q(\sqrt{\tau}e^{y})\,\bar{q}(\sqrt{\tau}e^{-y}) + \bar{q}(\sqrt{\tau}e^{y})\, q(\sqrt{\tau}e^{-y}) \right]$$

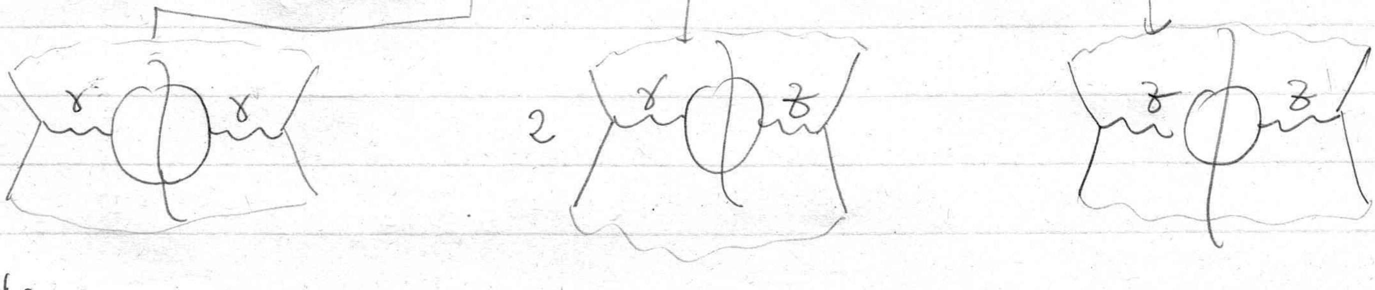
- $p\bar{p}$ scattering dominance of valence pdf's $f_{\bar{q}/\bar{p}}(x) = f_{q/p}(x) \equiv q(x)$

$$\frac{d\sigma}{d\Pi^2 dy} \propto \sum_q Q_q^2 \left[q(\sqrt{\tau}e^{y}) \cdot q(\sqrt{\tau}e^{-y}) + \bar{q}(\sqrt{\tau}e^{y})\,\bar{q}(\sqrt{\tau}e^{-y}) \right]$$

Vector boson production

At fixed target experiments pCu we have $\sqrt{s} = 38.8$ GeV, but at the Tevatron Run I $\sqrt{s} = 1.8$ TeV and Run II $\sqrt{s} = 1.96$ TeV so that we can see the peak of the Z boson.
We have then to modify the total cross section as follows

$$\hat{\sigma}_{q\bar{q} \to e^+e^-} = \frac{4\pi\alpha^2}{4\hat{s}} \frac{1}{N} \left(Q_q^2 - 2 Q_q V_e V_q \, \chi_1(\hat{s}) + (A_e^2 + V_e^2)(A_q^2 + V_q^2) \, \chi_2(\hat{s}) \right)$$



where

$$\chi_1(\hat{s}) \propto \mathrm{Re}\left(\frac{1}{\hat{s} - M_Z^2 + i M_Z \Gamma_Z} \right) = \frac{\hat{s} - M_Z^2}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \to 0 \qquad \hat{s} \to M_Z^2$$

$$\chi_2(\hat{s}) \propto \left| \frac{1}{\hat{s} - M_Z^2 + i M_Z \Gamma_Z} \right| = \frac{1}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \to \frac{1}{M_Z^2 \Gamma_Z^2} \qquad \hat{s} \to M_Z^2$$

In the naive parton model $\hat{s} = M^2$, so that we can expect that for $M^2 \sim M_Z^2$ only the term containing $\chi_2(\hat{s})$ dominates.
In that region one can approximate the squared Z propagator with a δ function

$$\frac{1}{(\hat{s} - M_Z^2) + M_Z^2 \Gamma_Z^2} \simeq \frac{\pi}{M_Z \Gamma_Z} \delta(\hat{s} - M_Z^2)$$

where the normalization is fixed by the fact that the two distributions have the same integral

$$\int_{-\infty}^{+\infty} d\hat{s} \frac{1}{(\hat{s} - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} = \frac{\pi}{M_Z \Gamma_Z}$$

We can then compute the total cross section by integrating $d\sigma/dM^2$ over M^2 in the range $(M_Z-\Delta)^2 < M^2 < (M_Z+\Delta)^2$

$$\int_{(M_Z-\Delta)^2}^{(M_Z+\Delta)^2} dM^2 \frac{d\hat\sigma}{dM^2} = \frac{4\pi\alpha^2}{3\hat s}\frac{1}{N_c}(A_e^2+V_e^2)(A_q^2+V_q^2)\left[\underbrace{\left(\frac{\sqrt2 G_F M_Z^2}{4\pi\alpha}\right)^2}_{\chi_2(\hat s)}\frac{\pi \hat s^2}{M_Z\Gamma_Z}\delta(\hat s - M_Z^2)\right]$$

We now relate this quantity to the total cross section for producing a Z boson and the Z boson decay width in e^+e^-

$$\hat\sigma_{q\bar q\to Z} = \frac{\pi}{N_c}\left(\sqrt2 G_F M_Z^2\right)(V_q^2+A_q^2)\ \delta(\hat s - M_Z^2)$$

$$\Gamma_{Z\to e^+e^-} = \frac{M_Z}{12\pi}\left(\sqrt2 G_F M_Z^2\right)(V_e^2+A_e^2)$$

We then have

$$\int_{(M_Z-\Delta)^2}^{(M_Z+\Delta)^2} dM^2 \frac{d\hat\sigma}{dM^2} = \frac{1}{12N_c}(A_e^2+V_e^2)(A_q^2+V_q^2)\left(\sqrt2 G_F M_Z^2\right)\cdot\frac{M_Z}{\Gamma_Z}\delta(\hat s - M_Z^2) =$$

$$= \hat\sigma_{q\bar q\to Z}\ \frac{\Gamma_{Z\to e^+e^-}}{\Gamma_Z} = \hat\sigma_{q\bar q\to Z}\ Br(Z\to e^+e^-)$$

We can now study hadronic Z_0 production in the naive parton model

$$\int_{(M_Z-\Delta)^2}^{(M_Z+\Delta)^2} dM^2 \frac{d\sigma_{p\bar p\to Z}}{dM^2} = \frac{\pi}{N_c}\sqrt2 G_F \frac{M_Z^2}{S}(A_q^2+V_q^2)$$
$$\times\int_{+\frac12\ln\frac{M^2}{S}}^{-\frac12\ln\frac{M^2}{S}} dy\left[q\left(\sqrt{\frac{M_Z^2}{S}}e^{y}\right)q\left(\sqrt{\frac{M_Z^2}{S}}e^{-y}\right)+\bar q\left(\sqrt{\frac{M_Z^2}{S}}e^{y}\right)\bar q\left(\sqrt{\frac{M_Z^2}{S}}e^{-y}\right)\right]$$

Changing the centre of mass energy we probe different values of x in the parton densities

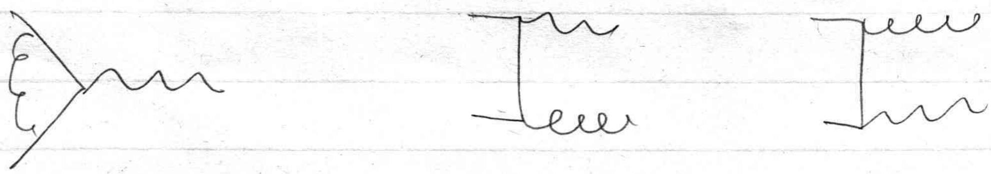
One-loop corrections

In order to see whether the naive parton model is stable under radiative corrections we compute corrections to the total cross section at order α_s.

$$\hat{\sigma}_{q\bar{q}\to Z} = \frac{\pi}{N_c}\left(\sqrt{2}G_F M_Z^2\right)\cdot\frac{(V_q^2+A_q^2)}{M^2}\, z\,\delta(1-z) = \qquad z = \frac{M^2}{\hat{s}}$$

$$= \sigma_0 \cdot z \cdot F(z)$$

At one loop we have the following corrections to the $q\bar{q}$ amplitude



and we have the contribution of a new partonic channel $\hat{\sigma}_{qg\to Zq}$

Let us consider first the real contribution $\hat{\sigma}_{q\bar{q}\to Zg}$

$$\hat{\sigma}_{q\bar{q}\to Zg} = \frac{1}{2\hat{s}}\,|M|^2\cdot\underset{\text{gluon}}{[dk]}\,\underset{Z\text{ boson}}{[dq]}\,(2\pi)^4\,\delta(p_1+p_2-k-q) =$$

$$= \frac{1}{2\hat{s}}\,|M^2|\cdot[dk]\,(2\pi)\,\delta\left(M^2-(p_1+p_2-k)^2\right) =$$

$$= \frac{1}{2\hat{s}}\,|M^2|\,[dk]\,(2\pi)\,\delta(\hat{s}+\hat{t}+\hat{u}-M^2)$$

In the partonic c.o.m. frame $k = \omega(1,0,\sin\theta,\cos\theta)$

$\hat{t} = -\sqrt{\hat{s}}\cdot\omega(1-\cos\theta) \qquad \hat{u} = -\sqrt{\hat{s}}\,\omega(1+\cos\theta)$

One considers the $D = 4 - 2\varepsilon$ phase space,

$$[dk](2\pi)\,\delta(\hat{s}+\hat{t}+\hat{u}-M^2) = \frac{1}{4\pi}\frac{(4\pi)^{\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^{\infty} d\omega\, \omega^{1-2\varepsilon}\int_{-1}^{1} d\cos\theta\,(1-\cos^2\theta)^{-\varepsilon} \times$$

$$\times\, \delta(\hat{s}-M^2-2\sqrt{\hat{s}}\cdot\omega)$$

Notice that the measurement poses a constraint on the <u>energy</u> of the emitted gluon.

Using $z = M^2/\hat{s}$ and defining $y = \frac{1}{2}(1+\cos\theta)$ we get

$$[dk](2\pi)\,\delta(\hat{s}+\hat{t}+\hat{u}-M^2) = \frac{1}{8\pi}\left(\frac{4\pi}{M^2}\right)^{\varepsilon}\frac{1}{\Gamma(1-\varepsilon)}\, z^{\varepsilon}(1-z)^{1-2\varepsilon}\int_0^1 dy\,[y(1-y)]^{\varepsilon}$$

The matrix element squared reads

$$\frac{|M^2|}{2\hat{s}} = \sigma_0 \cdot 4\alpha_s C_F(\mu^2)^{\varepsilon}\left\{(1-\varepsilon)\left(\frac{\hat{u}}{\hat{t}}+\frac{\hat{t}}{\hat{u}}\right)+\frac{2M^2\hat{s}}{\hat{u}\hat{t}}-2\varepsilon\right\}$$

which together with phase space gives

$$\frac{\hat{\sigma}_{q\bar{q}\to Zg}}{\sigma_0} = \frac{\alpha_s}{2\pi}\cdot C_F\left(\frac{4\pi\mu^2}{Q^2}\right)\frac{1}{\Gamma(1-\varepsilon)}(1-z)^{1-2\varepsilon} z^{\varepsilon}\int_0^1 dy\, y^{-\varepsilon}(1-y)^{-\varepsilon} \times$$

$$\times\left[(1-\varepsilon)\left(\frac{1-y}{y}+\frac{y}{1-y}\right)+\frac{2z}{(1-z)^2\,y(1-y)}-2\varepsilon\right]$$

The integrand contains two singular regions

$z \to 1$ corresponds to $\omega = M \cdot \frac{\sqrt{z}}{2}(1-z) \to 0$

$y \to 0$ corresponding to $\cos\theta \to -1$ k parallel to $\bar{q}$

$y \to 1$ corresponding to $\cos\theta \to 1$ k parallel to q

The y integration gives a collinear divergent contribution as $\varepsilon \to 0$

$$\frac{\hat{\sigma}_{q\bar{q}\to Zg}}{\sigma_0} = \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon} \left(-\frac{2}{\varepsilon}\right)\left[(1-z)^{1-2\varepsilon} z^{\varepsilon} + 2z^{1+\varepsilon}(1-z)^{-1-2\varepsilon}\right]$$

Using now the identity on distributions

$$\frac{1}{(1-z)^{1-\varepsilon}} = -\frac{1}{\varepsilon}\delta(1-z) + \sum_{n=0}^{+\infty} \frac{1}{n!}\varepsilon^n \frac{\ln^n(1-z)}{(1-z)_+}$$

we set

$$\frac{\hat{\sigma}_{q\bar{q}\to Zg}}{\sigma_0} = \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left\{ 2\cdot\left[\frac{1}{\varepsilon^2}\delta(1-z) + \right.\right.$$

$$\left.\left. + \frac{1+z^2}{(1-z)_+}\left(-\frac{1}{\varepsilon} + 4\ln(1-z) - 2\ln z\right)\right]\right\}$$

This has to be put together with virtual contributions. One starts from one loop corrections to the vertex

$$\gamma_\mu \frac{\alpha_s}{4\pi} C_F \left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon} \frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \pi^2\right] = \gamma_\mu F(M^2)$$

and computes

$$\frac{\hat{\sigma}_{q\bar{q}\to Z}}{\sigma_0} = \delta(1-z)\left[1 + 2\,\mathrm{Re}\left(F(Q^2)\right)\right] =$$

$$= \delta(1-z)\left\{1 + \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 + \frac{2}{3}\pi^2\right]\right\}$$

Note that $1/\varepsilon^2$ poles cancel between real and virtual contributions while $1/\varepsilon$ do not cancel

Putting everything together we obtain the NLO corrections to

$$\frac{\hat{\sigma}^{(1)}_{q\bar{q}\to z}}{\sigma_0} = \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left[+2\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right)\left(-\frac{1}{\varepsilon}\right) + \right.$$

$$\left. + 2\,\frac{1+z^2}{(1-z)_+}\left(4\ln(1-z) - 2\ln z\right) + \left(\frac{2}{3}\pi^2 - 8\right)\delta(1-z)\right]$$

We further expand everything up to the required order in ε

$$\left(\frac{4\pi\mu^2}{M^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} = 1 + \varepsilon \ln\frac{4\pi\mu^2}{M^2} - \varepsilon\gamma_E$$

we obtain the final answer

$$\frac{\hat{\sigma}^{(1)}_{q\bar{q}\to z}}{\sigma_0} = \frac{\alpha_s}{2\pi} = \left[2 P^{(0)}_{qq}(z)\left(-\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi + \ln\frac{M^2}{\mu^2}\right) + D_q(z)\right]$$

$$D_q(z) = 2 P^{(0)}_{qq}(z) + C_F\left(\frac{2}{3}\pi^2 - 8\right)\delta(1-z)$$

Analogously we obtain a result for $\hat{\sigma}_{qg\to z}$

$$\frac{\hat{\sigma}^{(1)}_{qg\to z}}{\sigma_0} = \frac{\alpha_s}{2\pi}\left[P_{qg}\left(-\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi + \ln\frac{M^2}{\mu^2}\right) + D_g(z)\right]$$

$$D_g(z) = T_R\left[(z^2 + (1-z)^2)\ln\frac{(1-z)^2}{z} - \frac{3}{2}z^2 + z + \frac{3}{2}\right]$$

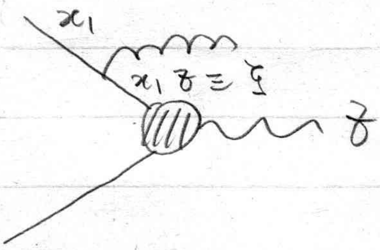
We observe that collinear singularities are the same as for the DIS case. We can therefore see whether collinear singularities can be absorbed in a redefinition of the pdfs

We consider then the hadronic cross section for the production of a Z boson

$$\sigma_{AB\to Z} = \sigma_0 \sum_{q\bar{q}} (V_q^2 + A_q^2)\,\tau \int dx_1\, dx_2\, dz \; \delta(x_1 x_2 z - \tau)$$

$$\Big\{ f_{q/A}(x_1) f_{\bar{q}/B}(x_2) F_q(z) + \big(f_{q/A}(x_1) f_{g/B}(x_2) + f_{g/A}(x_1) f_{q/B}(x_2)\big) F_g(z) \Big\}$$

We now consider hadron A and define $\xi = x_1 z$



ξ is the momentum fraction that enters the hard scattering

Consider now only the collinear singular term

$$\int dx_1 \int d\xi\, dz \; \delta(x_2 \xi - \tau)\; \delta(\xi - x_1 z) \times$$

$$\Big\{ f_{q/A}(x_1)\Big(\delta(1-z) + \Big(-\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi\Big) P_{qq}^{(0)}(z)\Big) + f_{g/A}(x_1)\Big[\Big(-\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi\Big) \frac{\alpha_s}{2\pi} P_{qg}^{(0)}(z)\Big]\Big\} =$$

$$= \int d\xi\, \delta(x_2 \xi - \tau) \Big\{ f_{q/A}(\xi) + \Big(-\frac{1}{\varepsilon} + \gamma_E - \ln 4\pi\Big) \times$$

$$\times \int_\xi^1 \frac{dz}{z} \Big[P_{qq}^{(0)}(z) f_{q/A}\Big(\frac{\xi}{z}\Big) + P_{qg}^{(0)}(z) f_{g/A}\Big(\frac{\xi}{z}\Big)\Big] =$$

$$= \int d\xi\, \delta(x_2 \xi - \tau) \cdot f_{q/A}(\xi, \mu^2)$$

If we wish to reabsorb also the term $\ln \frac{M^2}{\mu^2}$ we have to set $\mu^2 = M^2$.