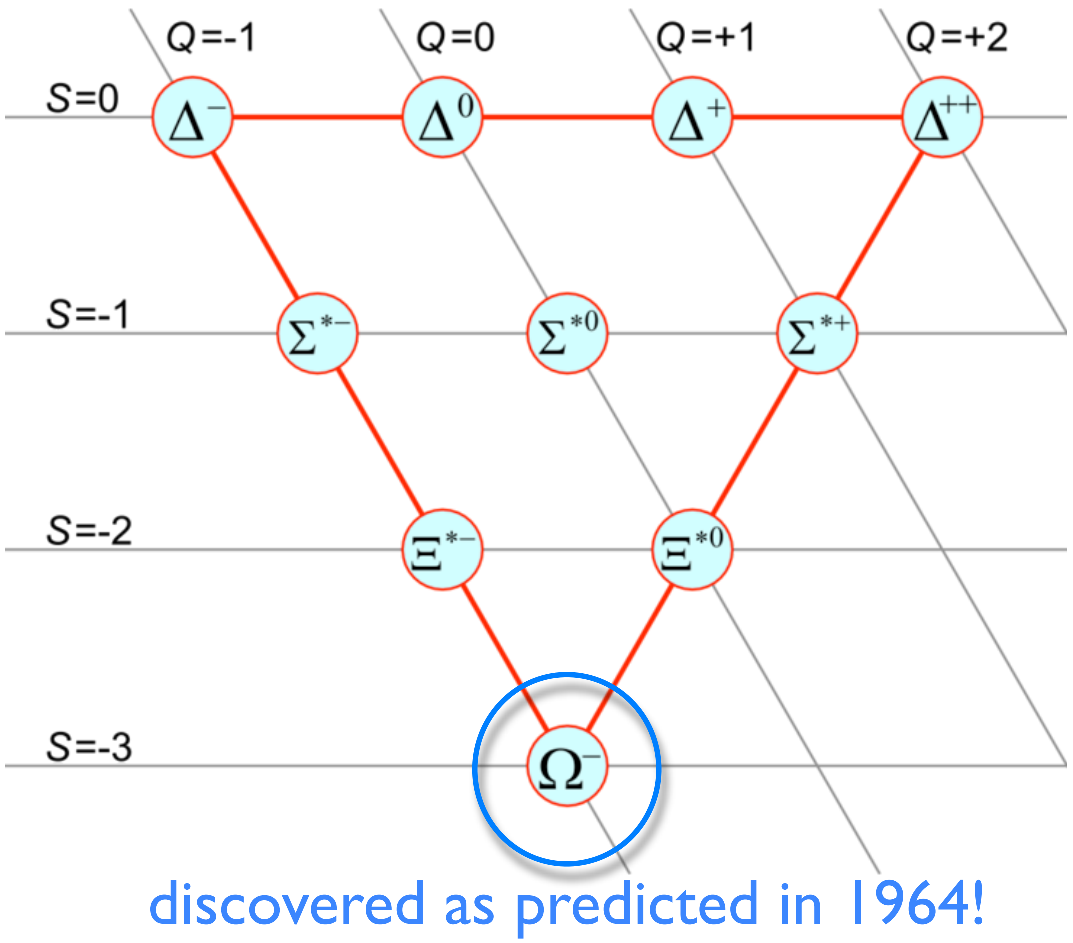
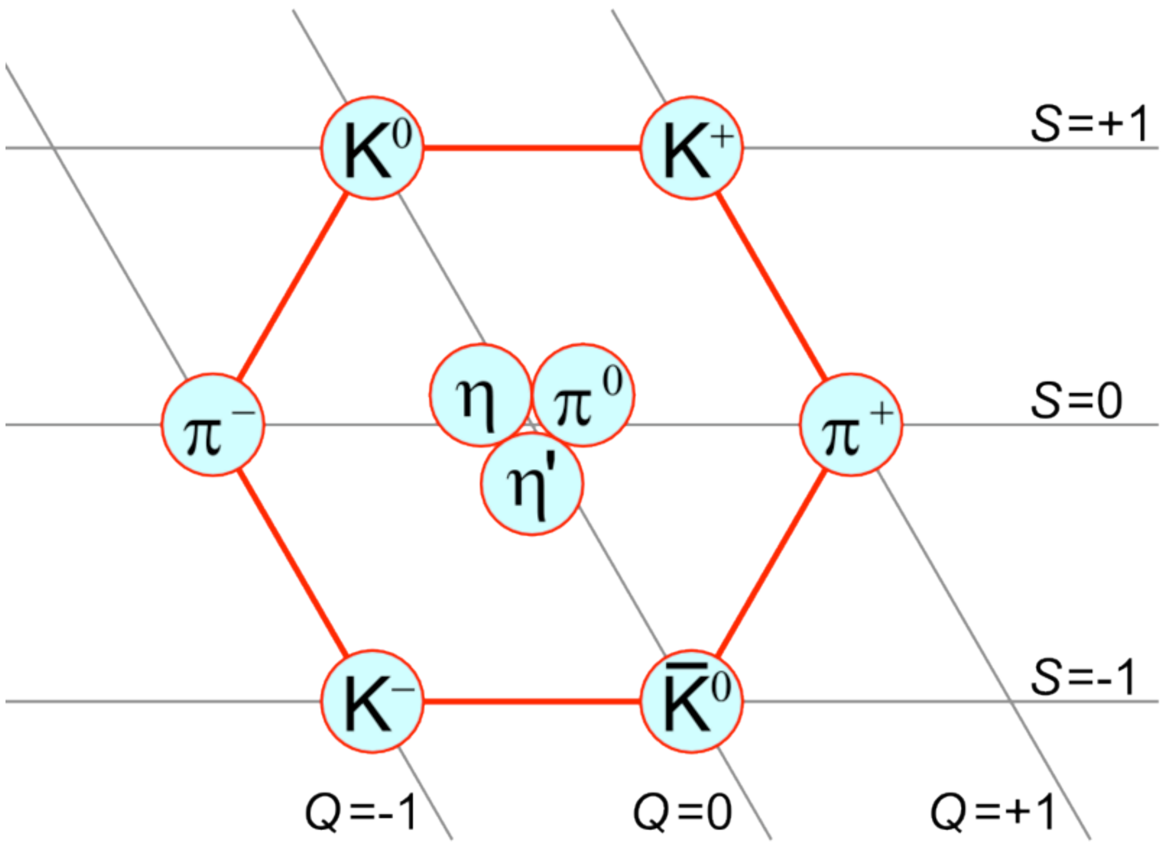
Introduction to QCD

lecture 1: Introduction to color, quarks and gluons

Quarks in flavour SU(3)

The eightfold way (1961)





- Why do Hadrons (baryons and mesons) fit the pattern ?

Quarks in flavour SU(3)

1964: Gell-Mann and Zweig propose quarks

u

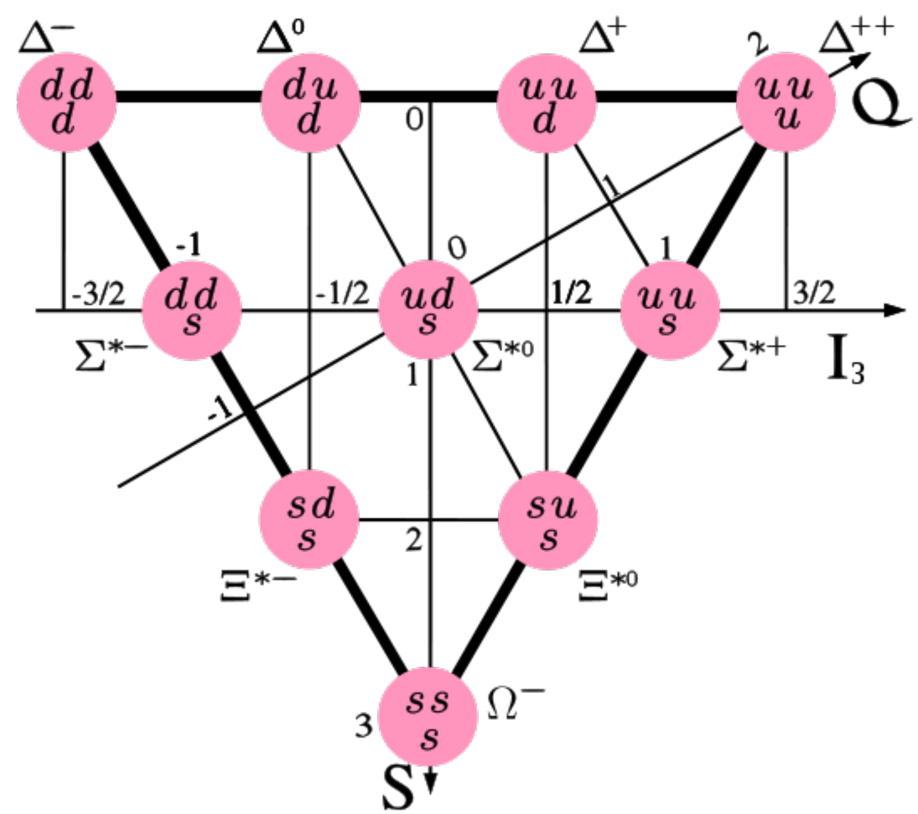
$m_u \approx 3 - 9MeV$

d

$m_d \approx 1 - 5MeV$

s

$m_s \approx 75 - 170MeV$



"Explains" the pattern

Fractional charge!

No free quarks to be seen!

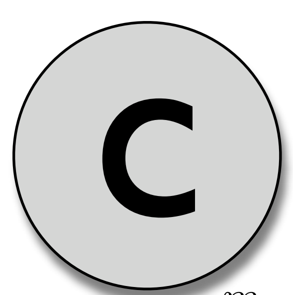
More quarks

e	μ
ν_e	ν_μ
u	?
d	s

Bjorken and Glashow proposed a fourth quark to fit the pattern.

GIM mechanism (1970)

1971: J/Ψ discovery at Brookhaven and SLAC



$m_c \approx 1.1 - 1.3 GeV$

$$J/\Psi = (c\bar{c})$$

More quarks

- 1975-1976 naked charm
- 1975: tau discovered at SLAC
- 1977: $\Upsilon = (b\bar{b})$ discovered at Fermilab (E288)
- 1980: $\Lambda_b^0 = (udb)$ naked beauty
- 1995: top quark identified at Tevatron

$m_b \approx 4.0 - 4.4 GeV$

b

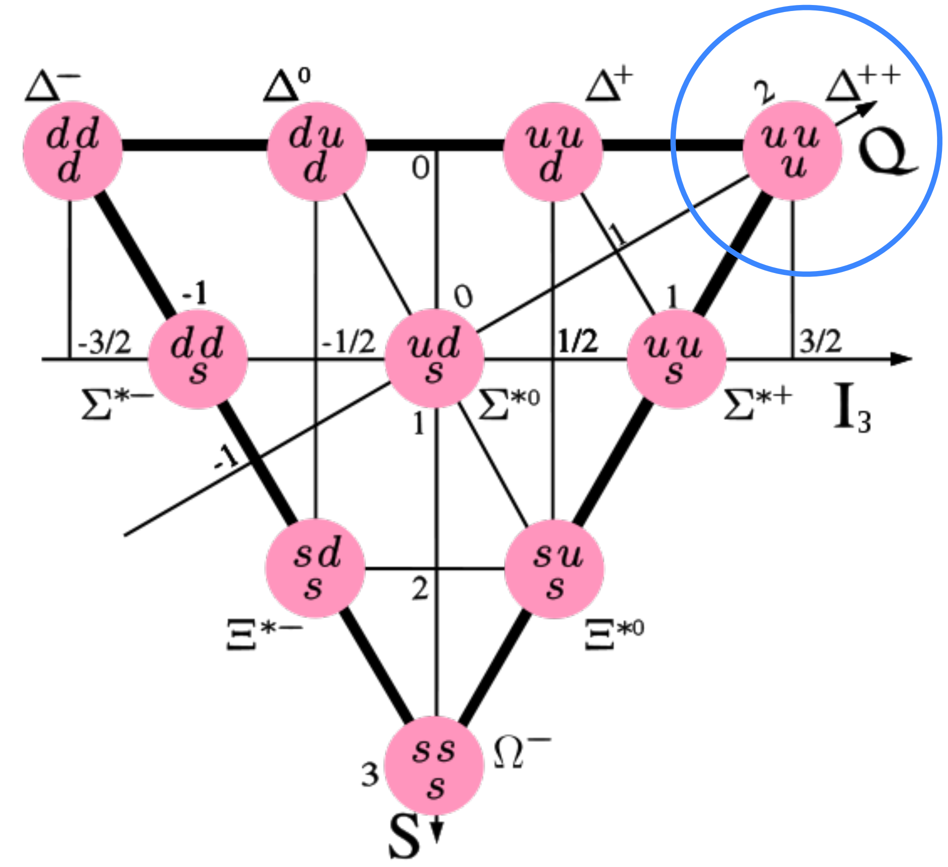
t

$m_t \approx 171 GeV$

The spin-statistics issue

Δ^{++} is a spin 3/2 particle with 3 “identical” up quarks !

St. Pauli’s exclusion principle endangered!

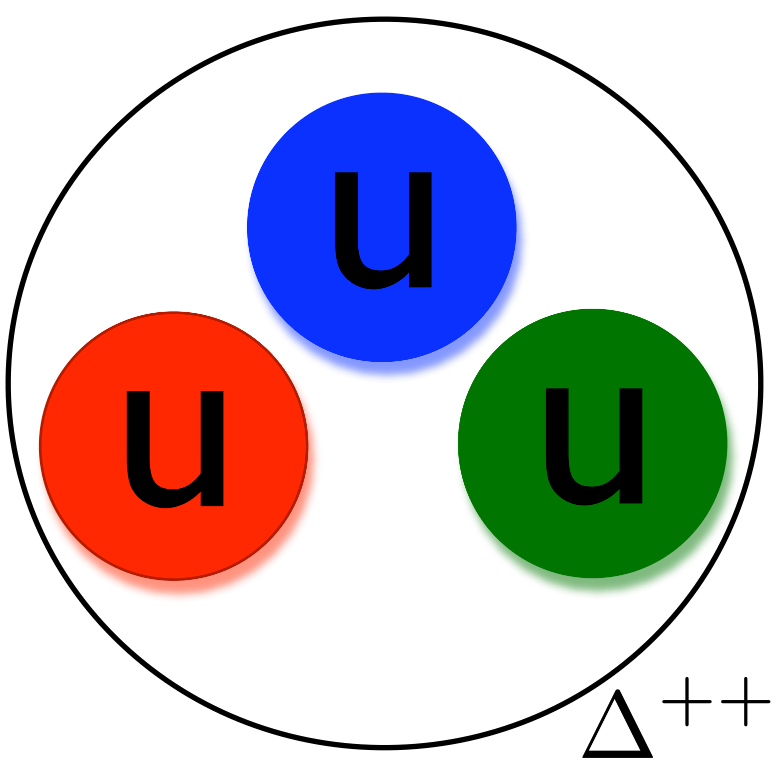


Color SU(3)

Greenberg proposes a new degree of freedom:

Color

There are now 3 kinds of up quarks

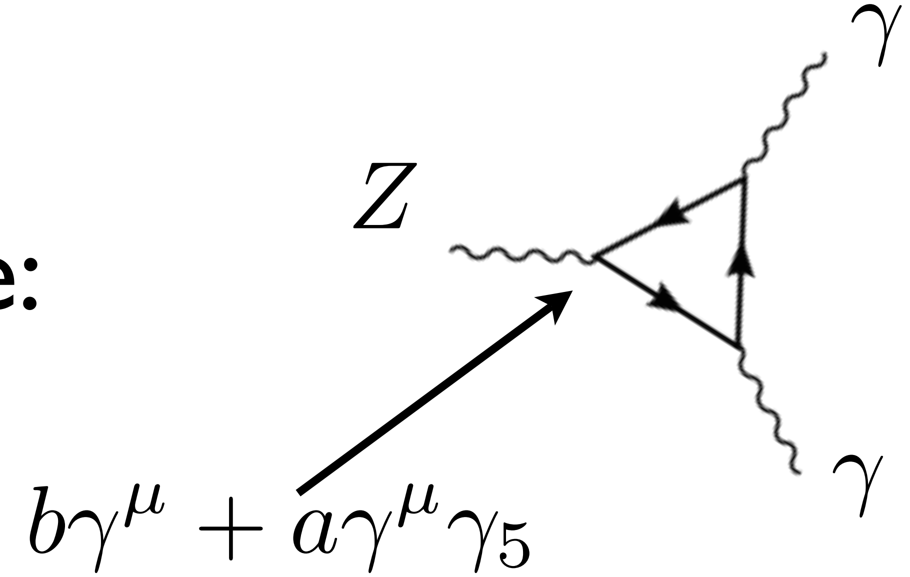


Why 3?

Adler-Bell-Jackiw anomaly

Loop diagrams introduce violation of symmetries of the Lagrangian (in this case the chiral symmetry)

example:



The anomaly has to cancel when summing over fermions.

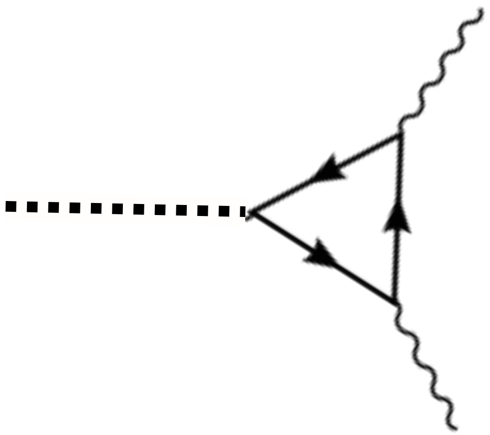
$$a_e = -1$$
$$a_{up} = 1$$
$$a_{down} = -1$$

$$\sum_f e_f^2 a_f = \frac{1}{2}\left(-1 + N_c(\frac{4}{9} - \frac{1}{9})\right)$$

$N_c = 3 \rightarrow$ anomaly cancelation

pion decay

$\pi^0 = (q\bar{q})$

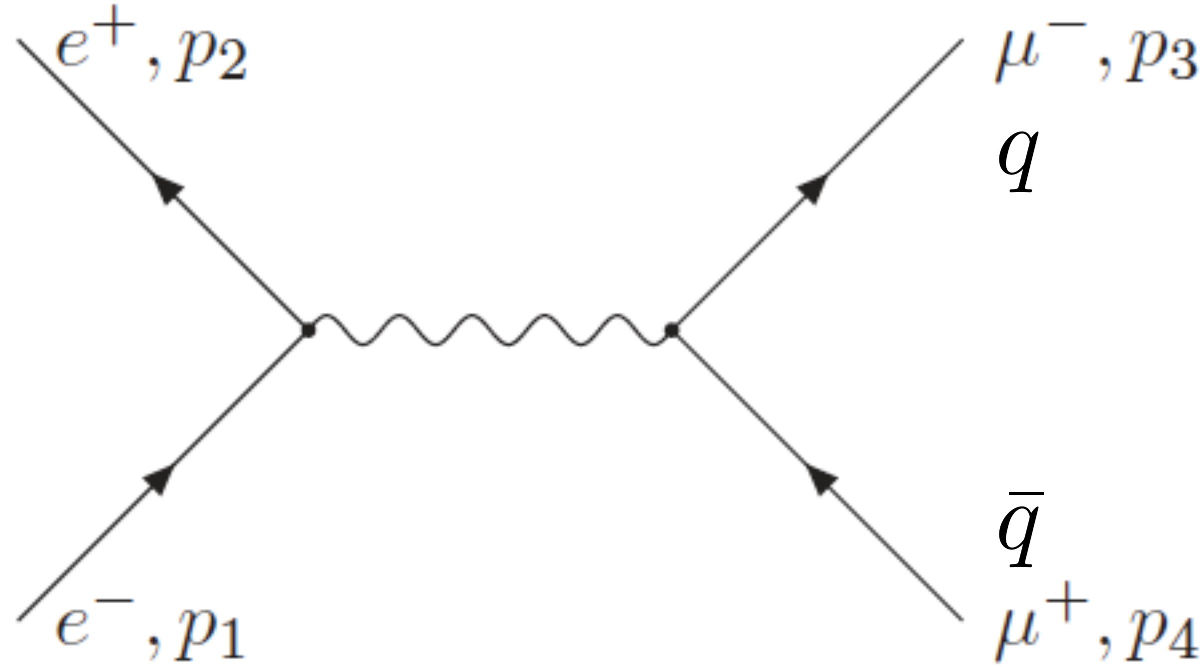


$$\Gamma(\pi^0 \to \gamma\gamma) = N_c^2 (e_u^2 - e_d^2)^2 \frac{a_{em}^2 m_\pi^3}{64\pi^3} \frac{1}{f_\pi^2} = 7.63 eV (\frac{N_c^2}{3})$$

Experimental value: $7.84 \pm 0.56 eV$

$N_c = 3 \to$ pion decay ok.

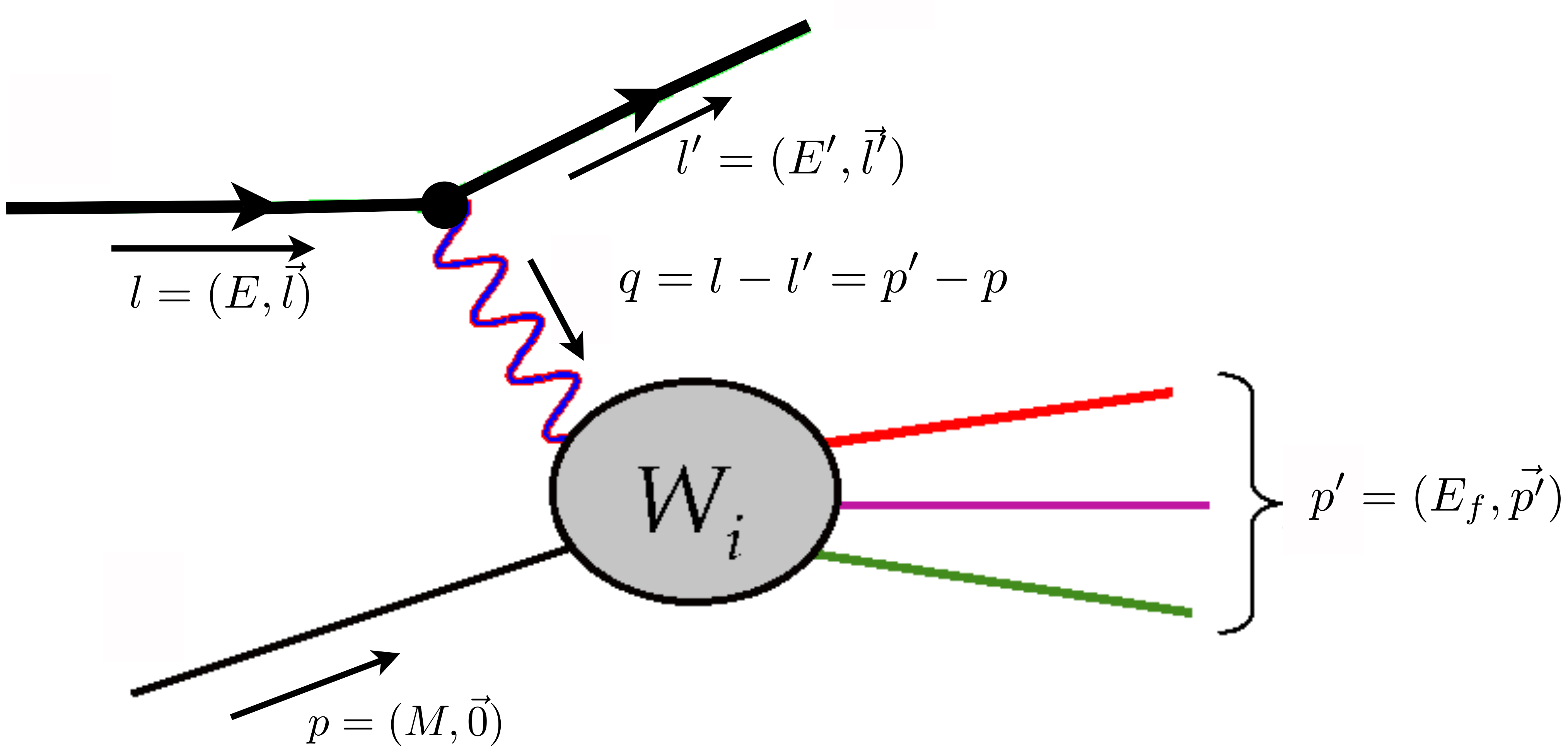
Hadron production



$$\frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = N_c \sum e_q^2 = Nc\frac{11}{9}$$

You can therefore measure the number of colors. Experiment yields $N_c \approx 3.2$

DIS introduction

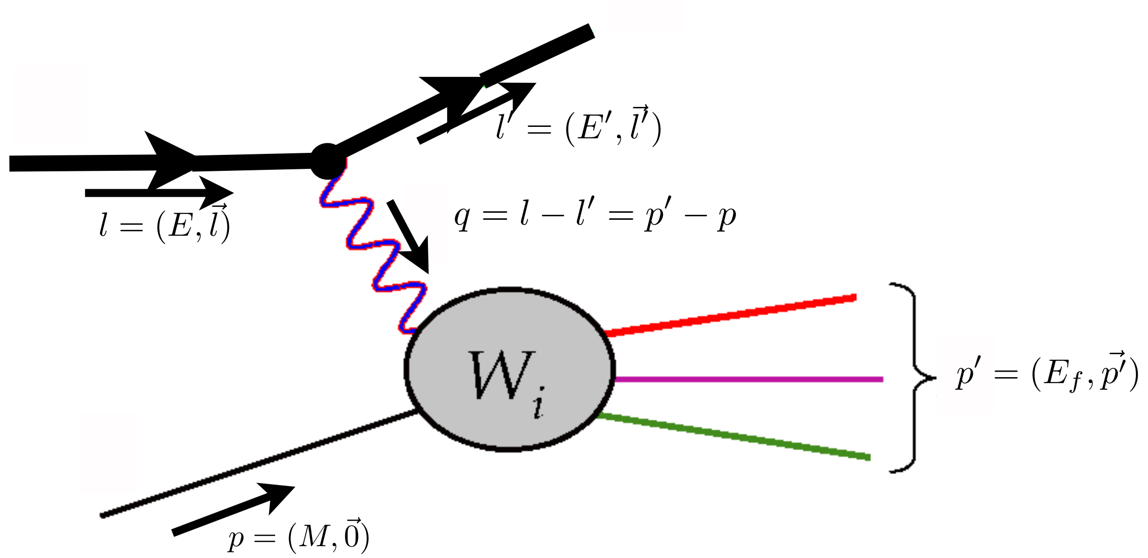


$$\nu = E - E'$$

$$Q^2 = -q^2 = -(p - p')^2 = -M^2 - p'^2 + 2M(M + \nu)$$

DIS introduction

In the elastic scattering case



$$p'^2 = M^2 \rightarrow$$

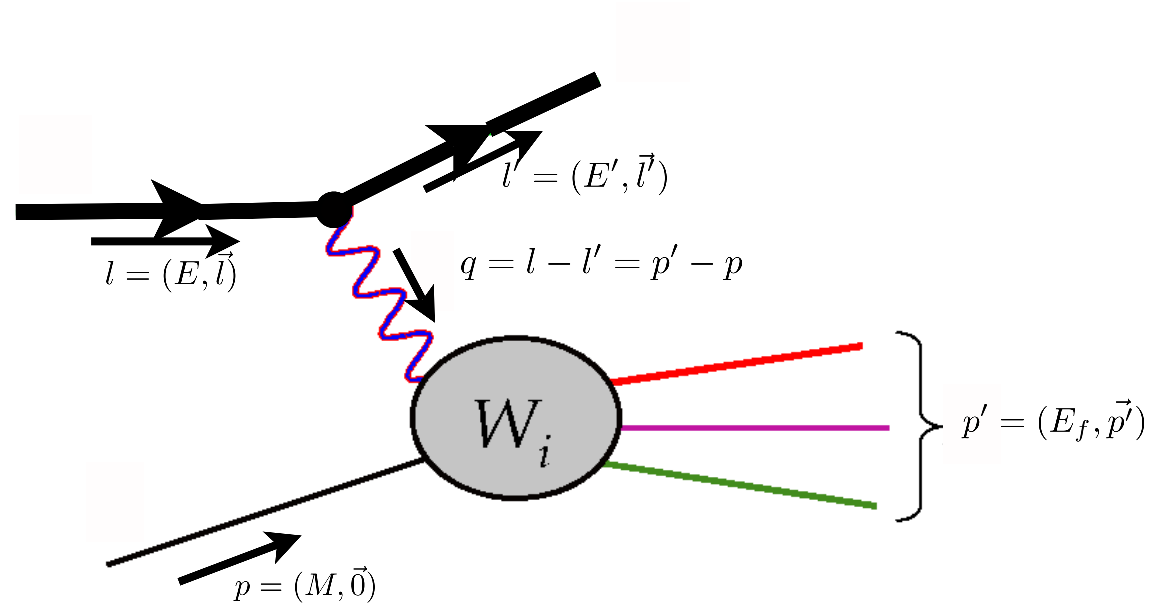
$$Q^2 = -q^2 = -(p - p')^2 = -M^2 - p'^2 + 2M(M + \nu) = 2M\nu$$

So $$x_B = \frac{Q^2}{2M\nu}$$ deviation from elastic scattering

"Bjorken - x"

DIS introduction

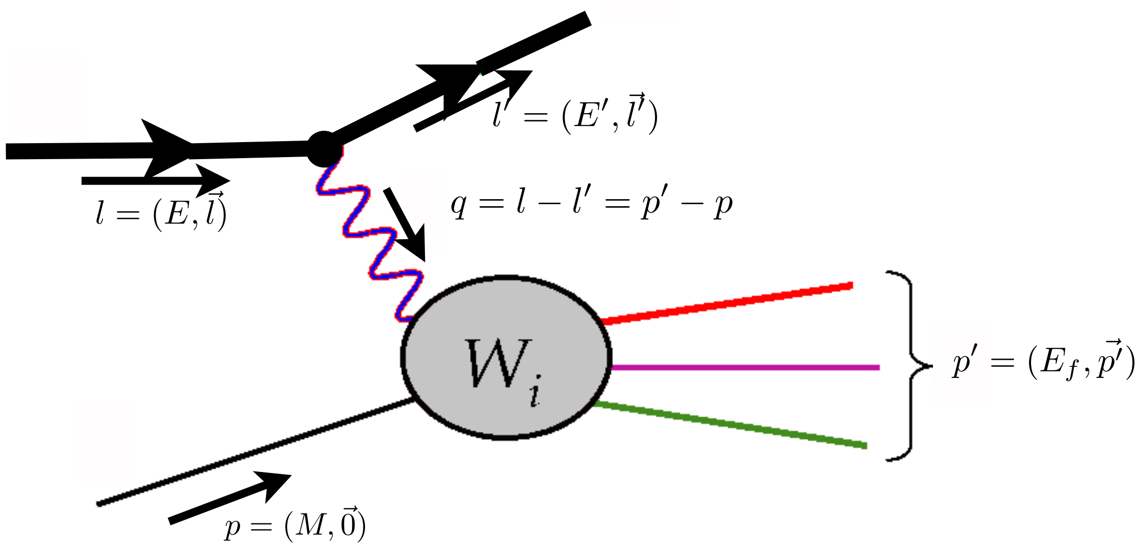
Assuming elastic scattering with a point-like proton (of spin 1/2)



$$\frac{d\sigma}{dQ^2} = \frac{4\pi a^2}{Q^4} e_q^2 \frac{E}{E'} \left(cos^2(\theta/2) + \frac{Q^2}{2M^2} sin^2(\theta/2) \right)$$

DIS introduction

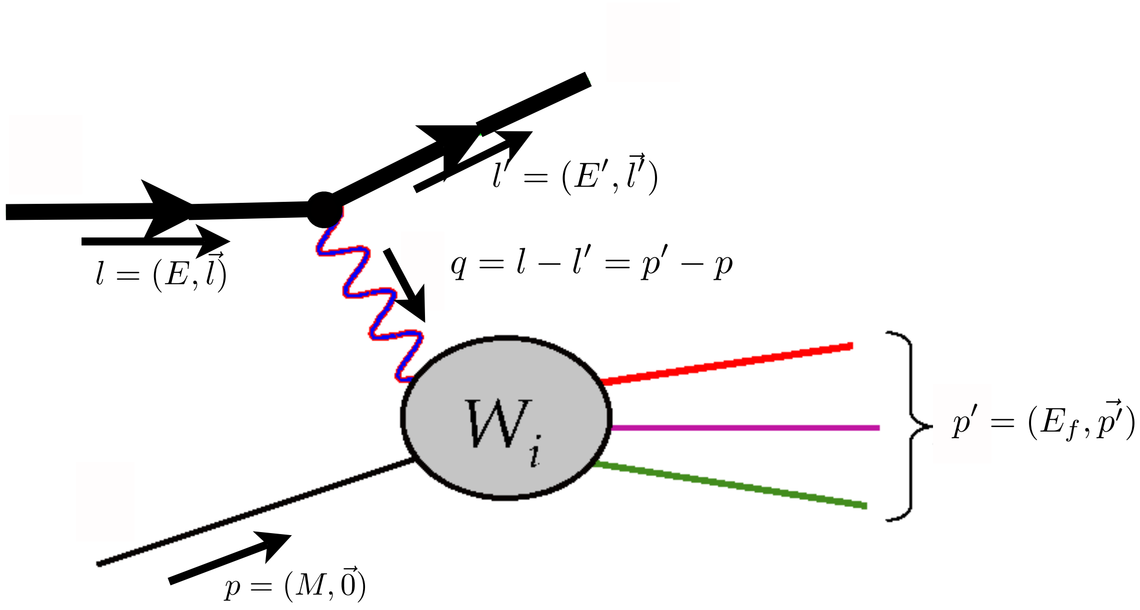
Assuming elastic scattering
with a point-like proton
(of spin 1/2)



$$\frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi a^2}{Q^4} e_q^2 \frac{E}{E'} \left(cos^2(\theta/2) + \frac{Q^2}{2M^2} sin^2(\theta/2) \right) \delta(\nu - \frac{Q^2}{2M})$$

DIS introduction

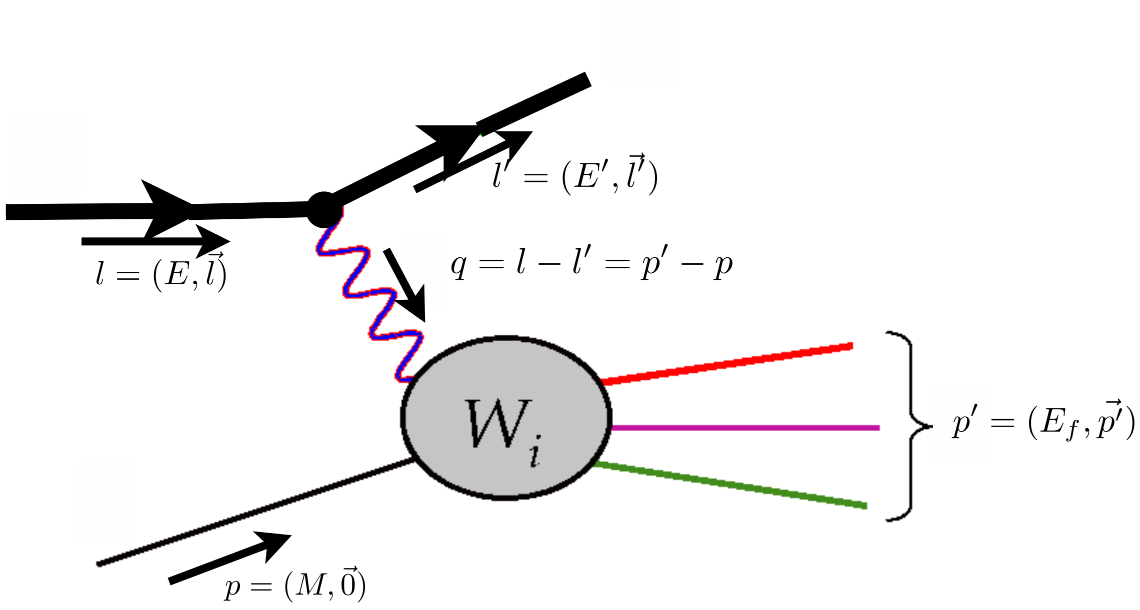
Assuming elastic scattering with a point-like proton (of spin 1/2)



$$\frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi a^2}{Q^4} e_q^2 \frac{E}{E'} \left(cos^2(\theta/2) + \frac{Q^2}{2M^2} sin^2(\theta/2) \right) \delta(\nu - \frac{Q^2}{2M})$$

DIS introduction

Assuming elastic scattering with a point-like proton (of spin 1/2)



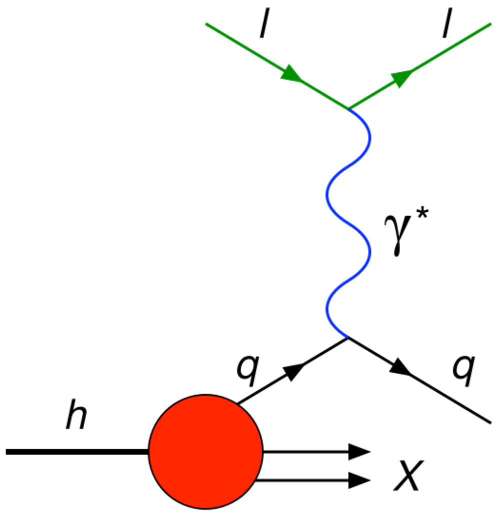
$$\frac{d\sigma}{dQ^2 d\nu} = \frac{4\pi a^2}{Q^4}\frac{E}{E'}\left(W_2(Q^2,\nu)cos^2(\theta/2) + 2W_1(Q^2,\nu)sin^2(\theta/2)\right)$$

$$W_2(Q^2,\nu) = e_q^2\delta(\nu - \frac{Q^2}{2M}) \qquad W_1(Q^2,\nu) = e_q^2\frac{Q^2}{4M^2}\delta(\nu - \frac{Q^2}{2M})$$

Structure functions

DIS introduction

If you assume elastic scattering with a constituent carrying a fraction of the proton momentum

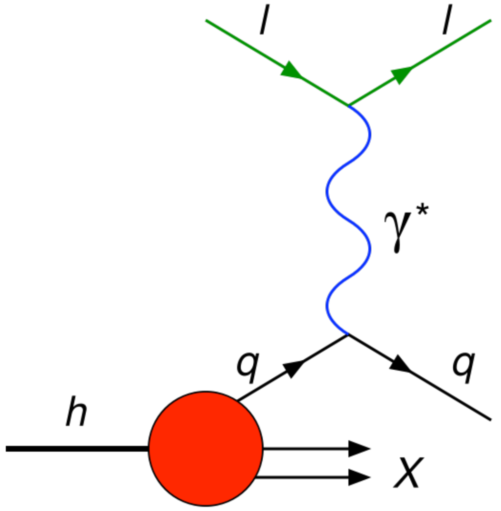


$$W_1(Q^2,\nu) = \sum_i \int dx f(x_i) e_i^2 \frac{Q^2}{4x_i M^2} \delta(\nu - \frac{Q^2}{2Mx_i}) = \sum_i e_i^2 f_i(x_B) \frac{1}{2M}$$

$$W_2(Q^2,\nu) = \sum_i \int dx f(x_i) e_i^2 \delta(\nu - \frac{Q^2}{2Mx_i}) = \sum_i e_i^2 f_i(x_B) \frac{x_B}{\nu}$$

DIS introduction

If you assume elastic scattering with a constituent carrying a fraction of the proton momentum



$$F_1(x) = MW_1(Q^2, \nu) = \frac{1}{2}\sum_i e_i^2 f_i(x)$$

$$F_2(x) = \nu W_1(Q^2, \nu) = \sum_i e_i^2 x f_i(x)$$

Structure functions redefined!

Bjorken scaling

So, assuming that there are constituents of spin 1/2 and that the scattering is elastic on them, the structure functions should only depend on Bjorken-x (not on Q^2 or ν independently)

Callan-Gross relation

Moreover one expects that

$$F_2(x) = 2xF_1(x)$$

$$F_1(x) = MW_1(Q^2,\nu) = \frac{1}{2}\sum_i e_i^2 f_i(x)$$

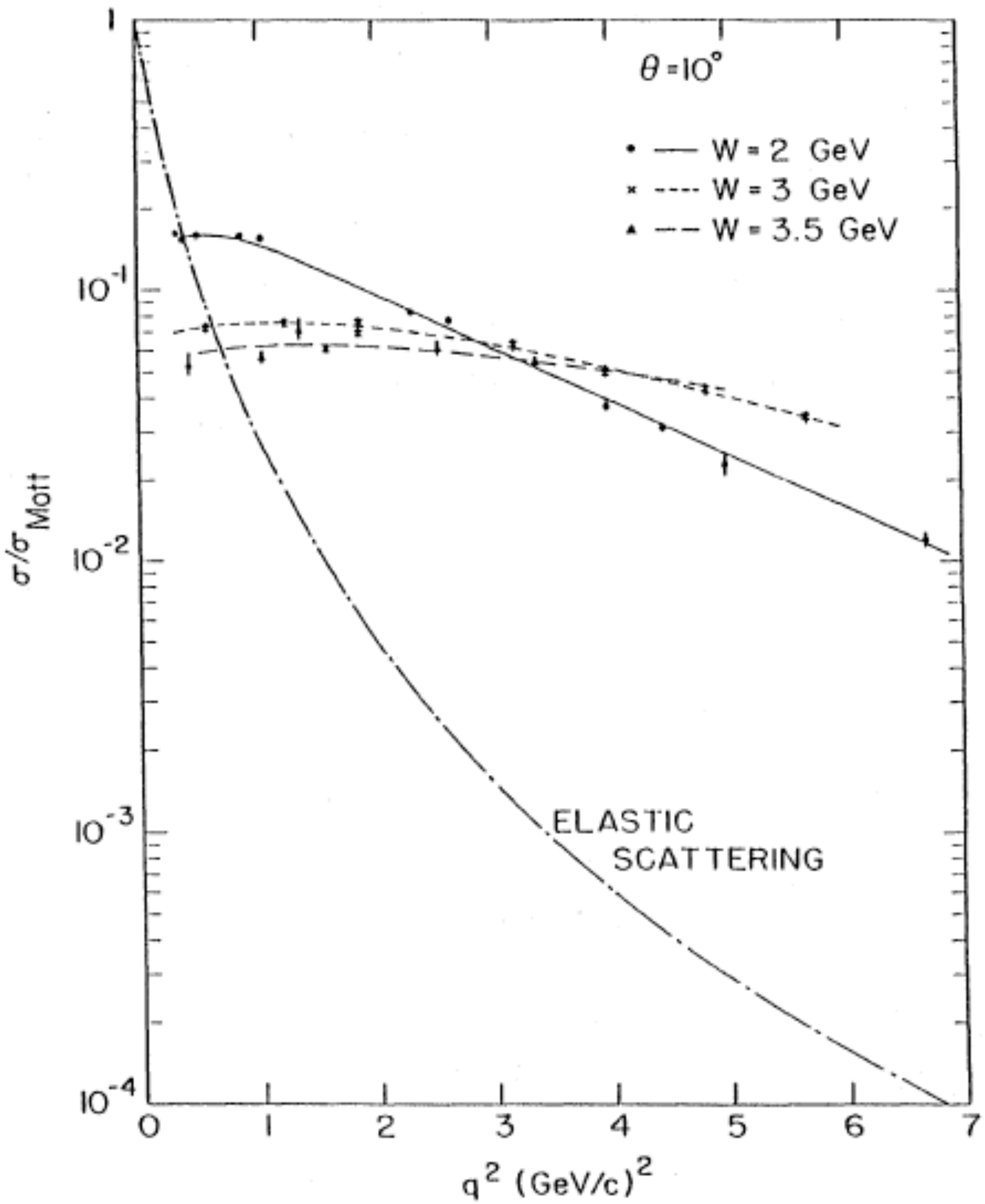
$$F_2(x) = \nu W_1(Q^2,\nu) = \sum_i e_i^2 x f_i(x)$$

DIS SLAC-MIT experiment

They actually expected rapidly falling structure functions as predicted by the uniform charge distribution assumption (Hofstadter, 1956)

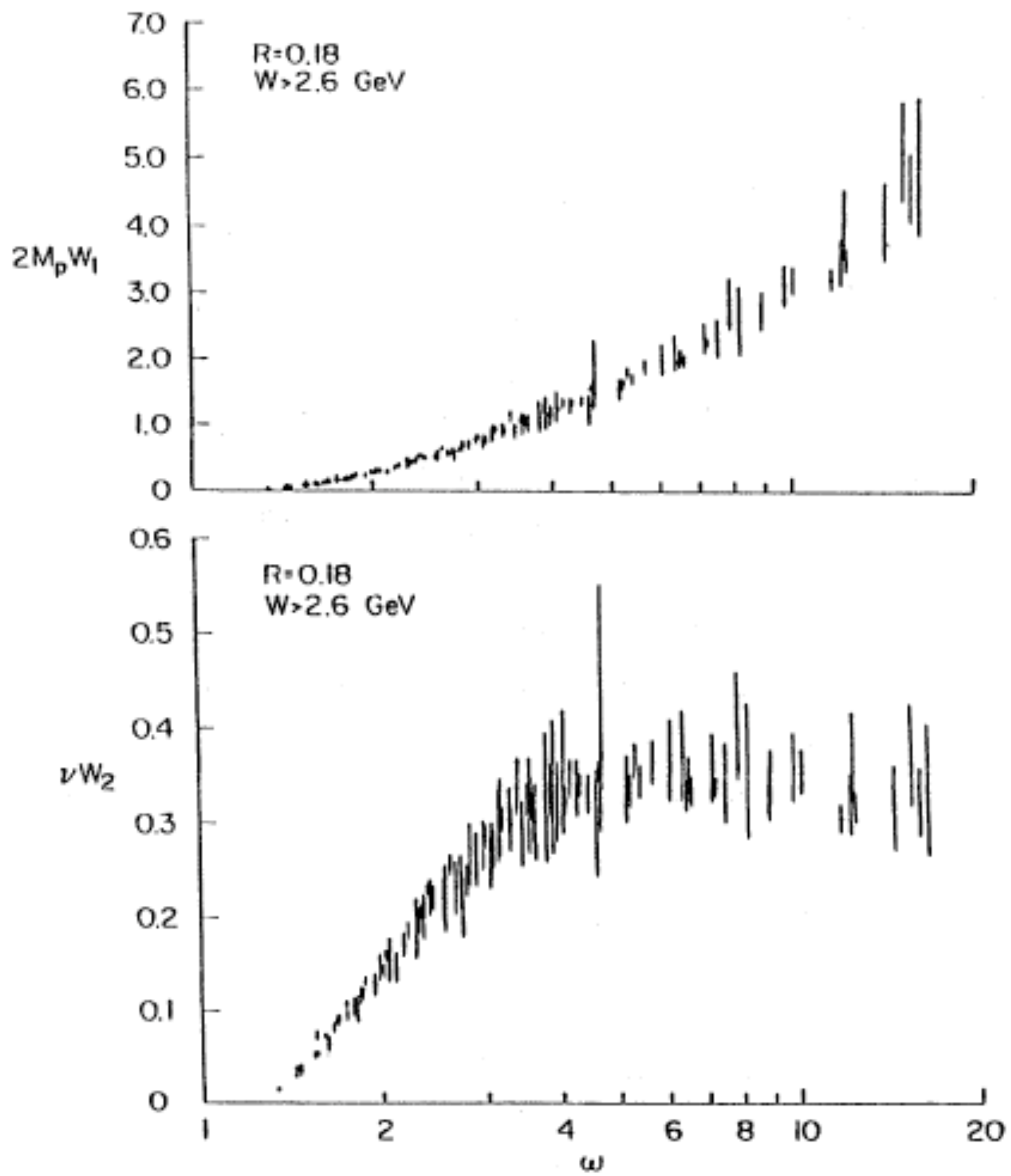
MIT-SLAC experiment

They found (a) a much milder behavior of the structure function related part of the cross section



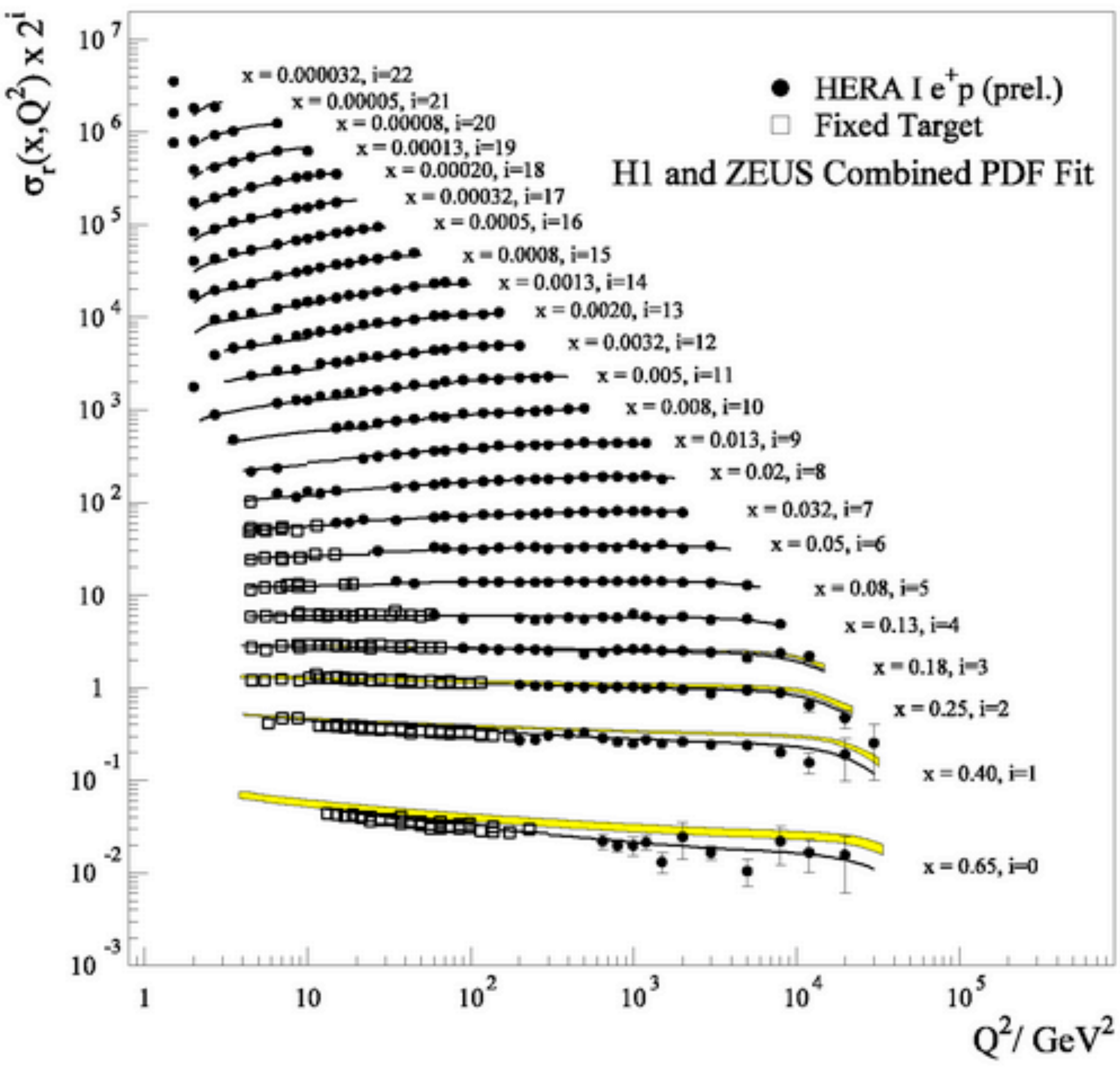
MIT-SLAC experiment

...and (b) that both structure functions obey Bjorken scaling (they only depend on Bjorken-x)



Friedman's nobel lecture, RevModPhys.63.615

Modern DIS data



DIS with neutrina (charges of quarks)

$$F_2^{eP}(x) = \sum_i e_i^2 x f(x) = x \left(\frac{4}{9}(u(x) + \bar{u}(x)) + \frac{1}{9}(d(x) + \bar{d}(x)) \right)$$

$$F_2^{eN}(x) = \sum_i e_i^2 x f(x) = x \left(\frac{4}{9}(d(x) + \bar{d}(x)) + \frac{1}{9}(u(x) + \bar{u}(x)) \right)$$

$$F_2^{eCa}(x) = x \frac{5}{18} \left(d(x) + \bar{d}(x) + u(x) + \bar{u}(x) \right)$$

$$F_2^{\nu_\mu Ca}(x) = x \left(d(x) + \bar{d}(x) + u(x) + \bar{u}(x) \right)$$

} charge measurement

Momentum sum rules
Gluons

$$\frac{18}{5}\int_0^1 dx F_2^{eCa}(x) = \int_0^1 dx(u(x) + d(x) + \bar{u}(x) + \bar{d}(x)) \approx 0.5$$

The structure functions come from experiment.
The sum over all quarks is less than one!
There are other particles inside the proton.
Particles that don't interact
electromagnetically or weakly!

Scaling violations

Bjorken scaling is only approximate - early calculations showed that in any interacting field theory gross corrections appear to all orders in perturbation theory.

“however, a mild violation of scaling would be possible in a special class of theories that are *asymptotically free*- characterized by effective couplings that approach zero as the renormalization scale increases indefinitely. But, there was no known example of such a theory at that time.”

Summary

- Hadrons are composed of quarks
- Quarks are spin 1/2 particles
- They have a color degree of freedom
- The number of different colors is 3
- There is another particle in the hadrons that interacts only strongly