Introduction to QCD

FS 10, Series 8

Due date: 28.04.2010, 1 pm

Exercise 1 To analyse the hadronic tensor it is convenient to introduce two light like vectors p and n, such that $n.p = 1$. Any 4-vector k can then be written in terms of p,n and a spacelike 2 dimensional transverse vector $k_T = (0, \mathbf{k}_T, 0)$, such that $k^\mu = ap^\mu + bn^\mu + k_T^\mu$ and $p^2 = n^2 = n.k_T = p.k_T = 0$.

a) Show that $p = (P, 0, 0, P), n = (\frac{1}{2P}, 0, 0, -\frac{1}{2P})$ constitute such a representation and that it is possible to parameterise an arbitrary 4 vector k as

$$k^\mu = \xi p^\mu + \frac{\mathbf{k}_T^2 + k^2}{2\xi} n^\mu + k_T^\mu.$$

b) Identify p as the momentum of the incoming proton in the limit where $\nu = p.q >> M^2$. Justify that then we can write

$$q^\mu = \nu n^\mu + q_T^\mu.$$

c) Now we use this to parameterise the phasespace corresponding to a parton radiating off a gluon (with momentum r) before scattering with the photon (momentum q) in DIS:

$$d\Phi = \frac{d^4 r}{(2\pi)^3} \frac{d^4 l}{(2\pi)^3} (2\pi)^4 \delta(r^2) \delta(l^2) \delta^4(q + p - r - l).$$

Integrate out one of the momenta and then set $k = p - r$ to get

$$d\Phi = \frac{1}{4\pi^2} d^4 k \delta((p-k)^2) \delta((p+q)^2).$$

d) Let $k^2 = -|k^2|$ and show that

$$\begin{aligned} d^4 k &= \frac{d\xi}{2\xi} d^2 \mathbf{k}_T d|k^2| \\ (p-k)^2 &= \frac{1}{\xi}((1-\xi)|k^2| - \mathbf{k}_T^2) \\ (q+k)^2 &= 2\nu \left(\xi - x - \frac{|k^2| + 2\mathbf{q}_T.\mathbf{k}_T}{2\nu} \right) \end{aligned} \qquad (1)$$

where $x = \frac{-q^2}{2\nu}$.Hence derive

$$d\Phi = \frac{1}{16\nu\pi^2} d\xi d|k^2| d\mathbf{k}_T^2 d\theta \delta((1-\xi)|k^2| - \mathbf{k}_T^2) \delta\left(\xi - x - \frac{|k^2| + 2\mathbf{q}_T.\mathbf{k}_T}{2\nu} \right).$$

Exercise 2 In Deep Inelastic Scattering the Hadronic tensor $W^{\alpha\beta}$ parameterises the non-perturbative effects of the emission of partons out of the proton.

a) Use the fact that the electromagentic current is conserved ($q.W = 0$) in order to deduce that

$$W^{\alpha\beta}(p,q) = \left(-g^{\alpha\beta} + \frac{q^\alpha q^\beta}{q^2}\right) W_1(x,Q^2) + \left(p^\alpha + \frac{q^\alpha}{2x}\right)\left(p^\beta + \frac{q^\beta}{2x}\right) W_2(x,Q^2).$$

b) Further show that

$$\begin{aligned} W^\alpha_\alpha &= \left(p^2 + \frac{\nu}{x} + \frac{q^2}{4x^2}\right) W_2 - 3W_1 \\ p_\alpha p_\beta W^{\alpha\beta} &= W_2 \left(\frac{\nu}{2x} + p^2\right)^2 - W_1 \left(\frac{\nu}{2x} + p^2\right). \end{aligned} \tag{2}$$

Let $F_1 = W_1$, $F_2 = \nu W_2$ and $F_L = F_2 - 2xF_1$, show that in the Bjorken limit

$$\frac{4x^2}{\nu} p_\alpha p_\beta W^{\alpha\beta} = F_L.$$