Introduction to QCD

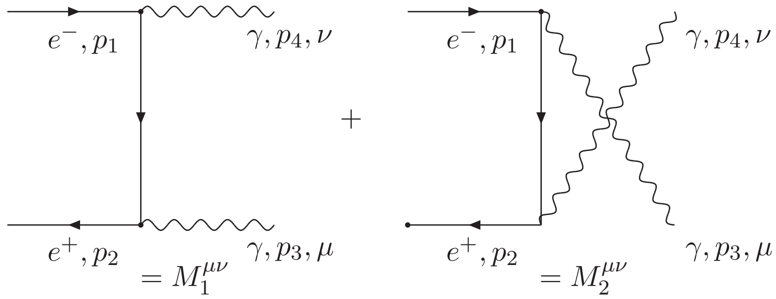
FS 10, Series 3

Due date: 17.03.2010, 1 pm

Exercise 1

a) Consider electron positron annihilation into photons in QED, i.e. $e^+e^- \to \gamma\gamma$. At leading order the Feynman diagrams contributing to this process are

$$M^{\mu\nu}_{e^+e^-\to\gamma\gamma} =$$



Show that

$$\begin{aligned} M_1^{\mu\nu} &= (-ie^2)\frac{\bar{v}_2\gamma^\mu(\not{p}_3 - \not{p}_2)\gamma^\nu u_1}{(p_2-p_3)^2} \\ M_2^{\mu\nu} &= (-ie^2)\frac{\bar{v}_2\gamma^\nu(\not{p}_1 - \not{p}_3)\gamma^\mu u_1}{(p_1-p_3)^2} \end{aligned} \tag{1}$$

b) Verify that the QED Ward identity

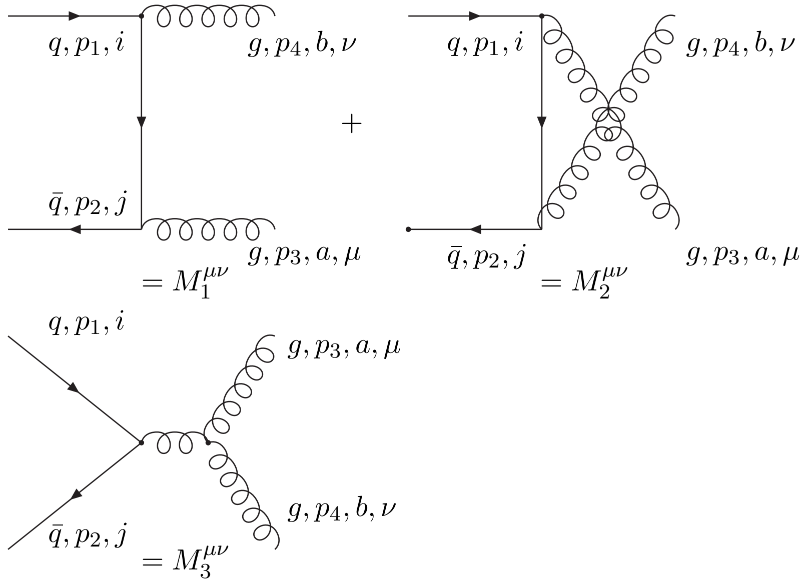
$$(p_3)_\mu M^{\mu\nu}_{e^+e^-\to\gamma\gamma} = 0 = (p_4)_\nu M^{\mu\nu}_{e^+e^-\to\gamma\gamma}$$

is fulfilled. Conclude that QED amplitudes are purely transverse.

Exercise 2

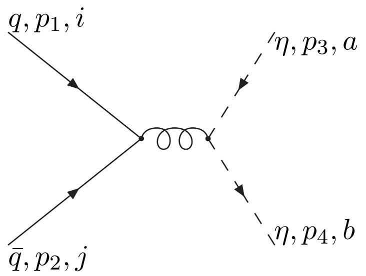
a) Now consider the process $q\bar{q} \to gg$ in QCD. At leading order the following diagrams contribute

$M^{\mu\nu}_{q\bar{q}\to gg} =$



and

$M_{Ghost} =$



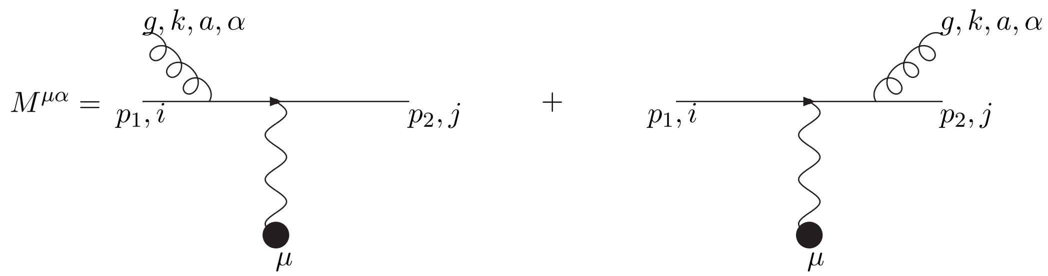
Show that the above diagrams yield

$$
\begin{aligned}
M_1^{\mu\nu} &= (-ig^2)(T^aT^b)_{ji}\frac{\bar{v}_2\gamma^\mu(\not{p}_3-\not{p}_2)\gamma^\nu u_1}{(p_2-p_3)^2} \\
M_2^{\mu\nu} &= (-ig^2)(T^bT^a)_{ji}\frac{\bar{v}_2\gamma^\nu(\not{p}_1-\not{p}_3)\gamma^\mu u_1}{(p_1-p_3)^2} \\
M_3^{\mu\nu} &= (-ig^2)[T^a,T^b]_{ji}\frac{-2g^{\mu\nu}\bar{v}_2\not{p}_3 u_1-(p_3+2p_4)^\mu\bar{v}_2\gamma^\nu u_1+(p_4+2p_3)^\nu\bar{v}_2\gamma^\mu u_1}{(p_1+p_2)^2} \\
M_{Ghost} &= (-ig^2)[T^a,T^b]_{ji}\frac{\bar{v}_2\not{p}_3 u_1}{(p_1+p_2)^2}.
\end{aligned}
\tag{2}
$$

b) Confirm that the QCD Ward identity is fulfilled

$$(p_3)_\mu M^{\mu\nu}_{q\bar{q}\to gg} = p_4^\nu M_{Ghost}.$$

Exercise 3 Consider soft radiative QCD corrections to a quark scattering off an electromagnetic current.



a) Show that (in the soft limit) the Matrix element becomes

$$M_{ij}^{\mu\alpha a} = (-ie\bar{u}_2\gamma^\mu u_1)\, J_{ij}^{\alpha a}(1 + O(k))$$

where

$$J_{ij}^{\alpha a} = -g(T^a)_{ji}\left[\frac{p_2^\alpha}{p_2.k} - \frac{p_1^\alpha}{p_1.k}\right].$$

Hence prove that $J_{ij}^{\alpha a}k_\alpha = 0$ to show that that the current J is conserved.

b) Compute the square of the matrixelement, i.e.

$$|M^{\mu\nu}|^2 = \sum_\lambda \epsilon_\alpha(\lambda)(\epsilon^*)_\beta(\lambda)M_{ij}^{\mu\alpha a}(M^*)_{ij}^{\nu\beta a}.$$

and show that the Matrix element square factorises into a soft (eikonal) part times the squared amplitude of the underlying leading order process ($|M_0^{\mu\nu}|^2$) as follows

$$|M^{\mu\nu}|^2 = -g^2C_F\left(\frac{p_1^2}{p_1.k} - \frac{2p_1.p_2}{p_1.kp_2.k} + \frac{p_2^2}{p_2.k}\right)|M_0^{\mu\nu}|^2.$$