Introduction to QCD

FS 10, Series 11

Due date: 26.05.2010, 1 pm

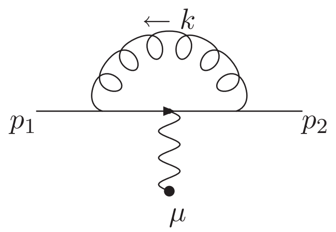
Exercise 1 In **Series 3 Exercise 3** we showed (for a massive quark scattering off an electromagnetic current, although this property is process independent) that in the soft limit the Matrix element squared corresponding to the *real* correction factorises into an *eikonal* factor times the Tree level amplitude itself:

$$|M^{\mu\nu\,soft}_{R}|^2 = -g^2 C_F \left(\frac{p_1^2}{p_1.k} - \frac{2p_1.p_2}{p_1.kp_2.k} + \frac{p_2^2}{p_2.k} \right) |M_0^{\mu\nu}|^2$$

In this exercise You are invited to show that the virtual correction has the same behaviour when the *virtual* gluon becomes soft.

a) First show that

$M_V^\mu =$



$$= -eg^2 C_F \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}_2 \gamma^\alpha (\not{p}_2 + \not{k} + m)\gamma^\mu (\not{p}_1 + \not{k} + m)\gamma_\alpha u_1}{[(p_2+k)^2 - m^2 + i\epsilon][(p_1+k)^2 - m^2 + i\epsilon][k^2 + i\epsilon]}$$

where $u_i = u(p_i)$.

b) Show that in the soft limit ($k \to 0$) the integrand simplifies to

$$M_V^\mu = -eg^2 C_F (4p_1.p_2 \bar{u}_2 \gamma^\mu u_1) \int \frac{d^4k}{(2\pi)^4} \frac{1 + O(k)}{[k^2 + 2p_1.k + i\epsilon][k^2 + 2p_2.k + i\epsilon][k^2 + i\epsilon]}.$$

c) One can use the residue theorem in order to evaluate the integration over k_0 by closing the contour in the lower plane. In principle each of the three propagators should contribute with 1 pole, however only the propagator $[k^2 + i\epsilon]$ contributes to the physical singularities, we therefore neglect the others. Show that thus

$$M_V^{\mu\,soft} = -g^2 C_F \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k_0} \frac{p_1.p_2}{(p_1.k)(p_2.k)} M_0^\mu$$

where $M_0^\mu = -ie\bar{u}_2\gamma^\mu u_1$ is the leading order matrix element.

d) It appears that if our quarks were massless we would have canceled this virtual soft piece with the one found in **Series 3 Exercise 3**. In the massive case however our calculation is not yet complete. In order to comply with the *LSZ* formulism we must make sure that our external quarks are on shell. The *onshell-scheme* demands that we have to renormalise our wavefunctions with $(Z_2^{os})^{1/2}$, where

$$Z_2^{os} = 1 + \frac{d\Sigma(p,m)}{d\not{p}}\Big|_{\not{p} = m}$$

see **Series 4**. Show (by dropping irrelevant terms and picking up the $k^2 \to 0$ residue) that

$$Z_2^{os\ soft}(p) = 1 + g^2 C_F \int \frac{d^3k}{(2\pi)^3}\frac{1}{2k_0}\left[\frac{m^2}{(k.p)^2}\right].$$

*Hint:*Show $\frac{d}{d\not{p}} = 2\not{p}\frac{d}{dp^2}$.

e) Finally show that in the soft limit the virtual contribution indeed factorises in the same way as the real contribution and in particular that both soft pieces cancel. Prove that

$$\left|\sqrt{Z_2^{os}(p_1)}(M_0^\mu + M_V^{\mu\, soft} + O(g^3))\sqrt{Z_2^{os}(p_2)}\right|^2 = |M_0^{\mu\nu}|^2$$

$$+g^2 C_F \int \frac{d^3k}{(2\pi)^3}\frac{1}{2k_0}\left[\frac{m^2}{(k.p_1)^2} - \frac{2p_1.p_2}{(p_1.k)(p_2.k)} + \frac{m^2}{(k.p_2)^2}\right]|M_0^{\mu\nu}|^2 + O(g^3).$$