Introduction to QCD

FS 10, Series 10

Due date: 12.05.2010, 1 pm

Exercise 1 Consider the collision of two hadrons. In the center of mass frame the hadrons have 4-momenta $P_1 = \frac{\sqrt{s}}{2}(1,0,0,1)$ and $P_2 = \frac{\sqrt{s}}{2}(1,0,0,-1)$, where $\sqrt{s}$ shall denote the total center of mass energy. We assume that in the collision the partons emitted from the hadrons have 4-momenta $p_1 = x_1 P_1$ and $p_2 = x_2 P_2$. The hadronic cross section σ is then given as a convolution of the partonic cross section $\hat{\sigma}$ over the parton distribution functions $f_i(x)$,i.e.

$$\sigma = \sum_{ij} \int_0^1 dx_1 dx_2 f_i(x_1) f_j(x_2) \hat{\sigma}_{ij}. \tag{1}$$

a) Show that the rapidity of the partonic center of mass system is given by $y = \frac{1}{2}\log\left(\frac{x_1}{x_2}\right)$.

b) Show that eqt. (1) can be written as

$$\sigma = \sum_{ij} \frac{1}{s} \int d\hat{s} dy f_i(x_1) f_j(x_2) \hat{\sigma}_{ij}$$

where $\hat{s} = s x_1 x_2$.

Exercise 2 The partonic cross section for the process $q\bar{q} \to l^+ l^-$ via a virtual photon is given by

$$\hat{\sigma}(q\bar{q} \to l^+ l^-) = \sigma_0 \frac{Q_q^2}{N} \quad , \quad \sigma_0 = \frac{4\pi\alpha^2}{3\hat{s}}. \tag{2}$$

a) At leading order the invariant Mass of the lepton pair is $M^2 = \hat{s}$. Show that

$$\frac{d^2\sigma}{dM^2 dy} = \frac{\sigma_0}{Ns}\left[\sum_q Q_q^2 \left\{f_q(x_1) f_{\bar{q}}(x_2) + f_q(x_2) f_{\bar{q}}(x_1)\right\}\right].$$

b) Plot your result using one of the available pdfsets for $y = 0$ (the central rapidity bin) and $\sqrt{s} = 1800 GeV$. Links to various pdf sets are available on *http://durpdg.dur.ac.uk/hepdata/pdf.html*.
Compare your result to the attached plot.

c) *(optional)* Repeat the analysis for hadrons to Z and W. Plot rapidity distributions.