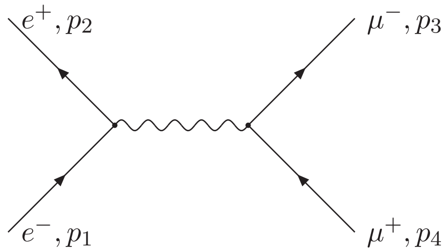
Introduction to QCD

FS 10, Series 1

Due date: 08.03.2010, 6 pm

Exercise 1

a) Consider the process of $e^+e^- \to \mu^+\mu^-$, depicted in the diagram below.

$M(e^+e^- \to \mu^+\mu^-) =$ 

Using the QED Feynman rules and assuming the electrons and muons to be massless show that

$$M(e^+e^- \to \mu^+\mu^-) = ie^2 \frac{[\bar{v}_1\gamma^\mu u_2]\,[\bar{u}_3\gamma_\mu v_4]}{s^2}$$

where $u_2 = u(p_2, s_2)$ etc and we have introduced the following Mandelstam variables.

$$\begin{aligned} s &= 2p_1.p_2 \\ u &= 2p_1.p_3 \\ t &= 2p_2.p_3 \end{aligned} \tag{1}$$

b) Show that the Amplitude squared $|M(e^+e^- \to \mu^+\mu^-)|^2$, summed over outgoing polarisations and averaged over incoming states yields

$$|M(e^+e^- \to \mu^+\mu^-)|^2 = e^4 \left(\frac{L_e^{\mu\nu} L^{\mathtt{muon}}_{\mu\nu}}{s^2}\right).$$

where we have identified the tensors associated with the electron and muon vertices

$$\begin{aligned} L_e^{\mu\nu} &= \frac{1}{4}\sum_{s_1 s_2} (\bar{v}_1\gamma^\mu u_2 \bar{u}_2\gamma^\nu v_1) \\ L^{\mu\nu}_{\mathtt{muon}} &= \sum_{s_3 s_4} (\bar{u}_3\gamma^\mu v_4 \bar{v}_4\gamma^\nu u_3)\,. \end{aligned} \tag{2}$$

Show that

$$
\begin{aligned}
L_e^{\mu\nu} &= (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - p_1.p_2 g^{\mu\nu}) \\
L_{\texttt{muon}}^{\mu\nu} &= 4\,(p_3^\mu p_4^\nu + p_3^\nu p_4^\mu - p_3.p_4 g^{\mu\nu})
\end{aligned} \tag{3}
$$

and use these results to show that

$$|M(e^+e^- \to \mu^+\mu^-)|^2 = 2e^4\left(\frac{u^2+t^2}{s^2}\right).$$

Exercise 2 The differential cross section for a $2 \to n$ particle scattering process can be defined as

$$d\sigma_n = \frac{1}{2E_1 2E_2|\mathbf{v}_1 - \mathbf{v}_2|}\left(\prod_{i=3}^{n}\int\frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}\right)\delta(p_1+p_2-\sum_{i=3}^{n}p_i)(2\pi)^4|M_{2\to n}|^2.$$

where $E_i = \sqrt{\mathbf{p}_i^2 + m_i^2}$.

a) For the case of $2 \to 2$ scattering where both final state particles are massless show that

$$d\sigma_2 = \frac{1}{2s}\frac{d\cos\theta}{16\pi}|M_{2\to 2}|^2$$

where θ shall denote the angle between $\mathbf{p}_1$ and $\mathbf{p}_3$ in the center of mass frame of p_1 and p_2.
Hint: s is the total center of mass energy.
Hint: You can integrate out the ϕ-angle.

b) Show that for $e^+e^- \to \mu^+\mu^-$ (Exercise 1) the differential cross section then becomes

$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha^2\pi}{2s}(1+\cos^2\theta)$$

where $\alpha = e^2/4\pi$ and that the total cross section is thus

$$\sigma = \frac{4\pi\alpha^2}{3s}.$$

c) *optional*
Download the original data sets from `http://hepdata.cedar.ac.uk/view/p4969` and produce your own plot.

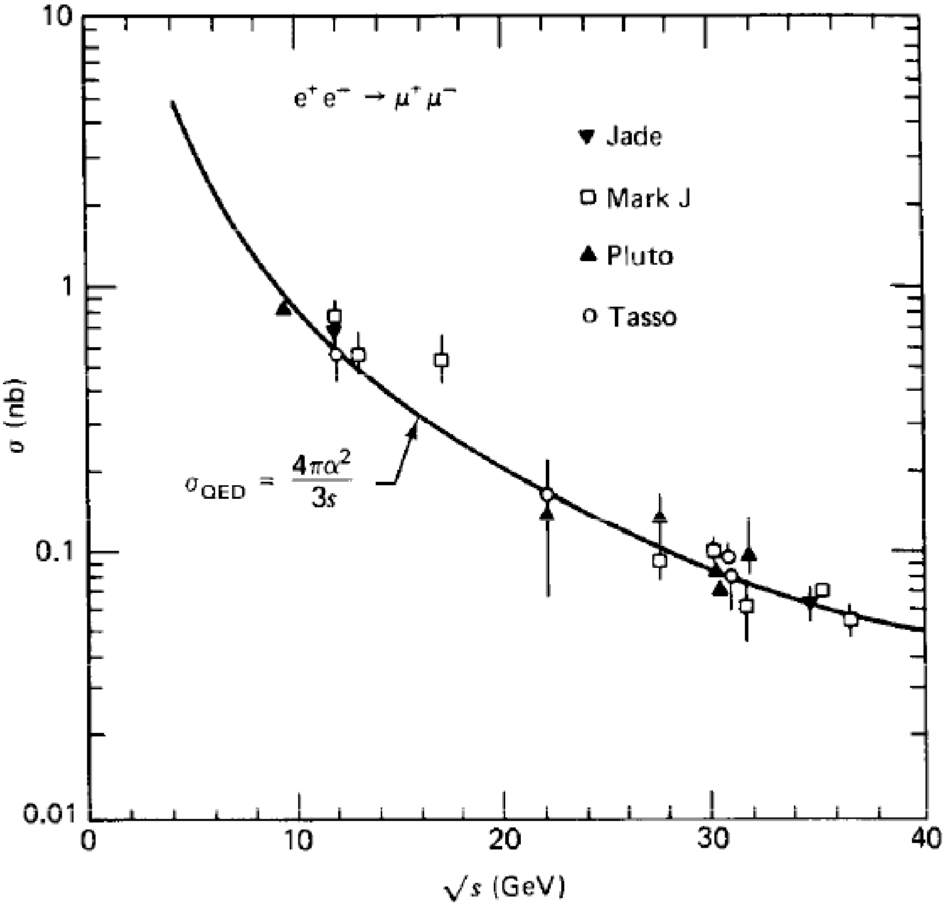


Fig. 6.6 The total cross section for $e^-e^+ \rightarrow \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.