LECTURE 12

Baryogenesis and sphalerons

Baryon and Lepton conservation are accidental symmetries of the Standard Model classical Lagrangian, and remain symmetries to higher orders in perturbation theory. However, only gauge symmetries are really fundamental symmetries of a model. We will see now that both baryon and lepton numbers are not conserved within the SM if one takes into account non-perturbative effects. Under normal circumstances such effects are extremely suppressed and unobservable, but they open the door for various baryogenesis scenaria in Early Universe conditions.

Baryon and lepton numbers

Let's define more precisely the baryon and lepton numbers as charges

$$Q_B = \int d^3x \; J_B^0 \qquad\qquad Q_L = \int d^3x \; J_L^0$$

$$J_B^\mu = \frac{1}{3} \left[\bar{q} \gamma^\mu q + \bar{u}_R \gamma^\mu u_R + \bar{d}_R \gamma^\mu d_R \right] \qquad\qquad J_L^\mu = \bar{l} \gamma^\mu l + \bar{e}_R \gamma^\mu e_R$$

We will work in a simplified SM with one family of quarks and leptons

$$q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad u_R, \quad d_R \qquad\qquad l = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad e_R$$

Note: there is no right-handed neutrino in this simplified model.

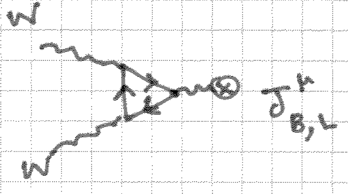
Furthermore, we'll set $\theta_W = 0$ which means

(i) $U(1)$ decouples from the SM and we forget about it, so the SM has now gauge group $SU(3) \times SU(2)$

(ii) The masses of $W^\pm$ and W^3 are all equal to $m_W = \frac{gv}{2}$

Now, in any Feymann graph (and vertex) Q_B and Q_L are conserved.

However, the presense of the Ws that couple only to left handed particles q_L and l, induces the possibility of an axial anomaly for J_B^μ, J_L^μ through triangle diagrams



Note that the theory is consistent, in the sense that all <u>gauge</u> currents are free of anomalies (thanks to the cancelation induced by $tr\left(t_a [t_b, t_c]\right)$ and the fact that we have a family of quarks <u>and</u> leptons).

But the baryon and lepton currents need not be conserved, and indeed

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{g^2}{16\pi^2} F^i_{\mu\nu} \tilde{F}^{\mu\nu;i}$$

where $F^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\, \varepsilon^{ijk} W^j_\mu W^k_\nu$

This is very reminiscent of instanton contributions. In lecture 7 we defined the Chern-Simons current $G_\mu = \varepsilon_{\mu ijk}\left[A_i^a F_{jk}^a - \frac{1}{3} f^{abc} A_j^b A_i^a A_k^c\right]$

Now we have $A_i^a = W_\mu^j$ and $f^{abc} = \varepsilon^{ijk}$ for $su(2)$. We saw then that $\partial_\mu G^\mu = F^i_{\mu\nu} \tilde{F}^{\mu\nu;i}$

Hence $\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{g^2}{16\pi^2} \partial_\mu G^\mu$

* (for n_f families of quarks $\partial_\mu J_B^\mu = n_f \frac{g^2}{16\pi^2} \partial_\mu G^\mu$

$$\Delta Q_B = \Delta Q_L = \frac{g^2}{16\pi^2} \Delta G \qquad \text{with} \qquad G = \int d^3x\, G^0$$

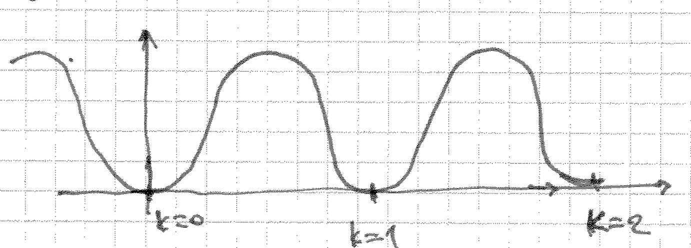
and $\Delta Q = Q(t=\infty) - Q(t=-\infty)$

The expression $K = \frac{g^2}{16\pi^2} \int d^3x\, G^0 = \frac{g^2}{16\pi^2} G$ was the topological charge (the winding number) of the instanton solution.

We now see that for every perturbative solution of the W fields, the topological charge doesn't change (winding number zero) and hence the baryon and lepton numbers are conserved. For non-perturbative configurations, with a non-zero winding number, baryon and lepton numbers are violated!

Instanton mediation

Let's remember that instanton contributions interpolate between equivalen but topologically disconnected vacua of the Yang-Mills field. Looking schematically at the Yang-Mills vacuum structure we have



so an instanton of winding number 1 interpolates between the $k=0$ and $k=1$ vacuum configuration. This is a tunneling effect: to achieve this the instanton needs to depart from the vacuum at some point in time between $t=-\infty$ and $t=+\infty$. The price is the instanton action suppression factor: $S[A] = \frac{8\pi^2}{g^2}|\nu|$, so the path integral ~~becomes~~ gets contribu of the form $e^{-8\pi^2|\nu|/g^2}$

This instanton is violating Q_B and Q_L by a factor of 1, so it mediates a transition like $qqq \to \bar{l} + \gamma$ (or π^0) (!)

where $Q_B(-\infty) = 1$, $Q_L(-\infty) = 0$ $\longrightarrow$ $Q_B(+\infty) = 0$, $Q_L(+\infty) = 1$ $\left\{ \begin{array}{l} \Delta Q_B = -1 \\ \Delta Q_L = -1 \end{array} \right.$

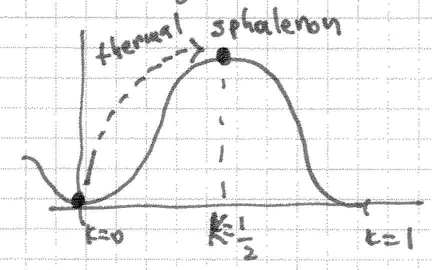
But because g^2 is small, the probability for such a tunneling is $\Gamma_{Q_B} \approx e^{-8\pi^2|\nu|/g^2} \sim 10^{-170}$

Sphalerons

Could thermal effects waive the suppression factor from the instanton contribution? In the Early Universe high temperature environment thermal effects could cause fluctuations to the top of the potential.

The top of the potential is also an extremum of the action of the Yang-Mills field. Such a field configuration is a static solution of the classical equations. It is unstable, and doesn't correspond to an integer winding number. It is not hard to find such solutions, and they end up having half-integer winding numbers. Such solutions are called sphalerons.

~~The height of the~~ The picture now is that thermal fluctuations in high temperature can move the system from winding number $k=0$ to $k=1/2$ from where it falls to $k=1$ because the sphaleron solution is unstable. The net effect is $\Delta K = 1$, which results to baryon and lepton number violation.



The height of the barrier depends on the mass of the sphaleron

$$M_{sph} = 2\sqrt{2}\,\pi^2 \frac{v}{g}$$

This means that in the Early Universe, before the electroweak phase transition, $v=0$, and $M_{sph}=0$, which means that the barrier dissappears and baryon and lepton numbers are violated all the time!

Baryogenesis alternatives

We saw that sphalerons allow for some optimism that non-perturbative effects can explain baryogenesis within the e/w Standard Model. Unfortunately baryon and lepton violating reactions mediated by sphalerons (i) take place at a time of nearly perfect thermal equilibrium* and (ii) are further suppressed by not being a source of CP violation themselves, and hence rely on the small SM CP violation to satisfy the second Sakharov condition.

However, they can transform a lepton asymmetry from some other mechanism to Baryon asymmetry! Hence there have been various efforts to derive Baryogenesis from Leptogenesis.

Note that while non-perturbative effects violate B and L, they preserve B−L. So there has to be a source of L violation from BSM physics to induce leptogenesis, and, through sphalerons, baryogenesis.

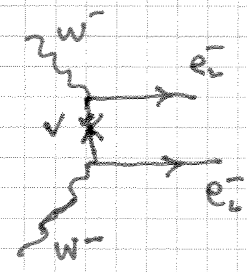
But, if the B−L assymmetry is generated by GUTs, it is completely washed out by the time of e/w phase transition, so it's too late for the sphalerons to create $\not{B}$ from $\not{L}$.

Other idea: Leptogenesis through neutrinos! Neutrinos have masses. It can be that a new Dirac mass term has to be added to the SM Lagrangian.

(*) A way out of equilibrium can be imagined if the e/w phase transition is a first order one: then bubble nucleation is expected (the Higgs field acquires a vev locally within bubbles that expand in space). On the surface of these bubbles there is no equilibrium and sphaleron processes can produce enhanced baryon asymmetry. This has to be turned off (the sphaleron wash-out condition) before equilibrium is restored!

But neutrina are the only fermions without charge. They can be their own antiparticles, i.e. they can be Majorana particles.

This would allow a process like



which obviously violates lepton number (by 2).

Experimental verification for something like this would be an observation of neutrinoless double beta decay. Hasn't been observed (yet?) but many experiments on the way.