Phase transitions in the Early Universe, part II

We have seen that in the Early Universe, thermal effects can change the effective potential of scalar fields. In a simple Higgs model with a $U(1)$ gauge field

$$\mathcal{L} = D_\mu \phi D^\mu \phi^* - m^2 \varphi^* \varphi - \frac{\lambda}{4} (\varphi^* \varphi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

the effective potential, at LO is just

$$V_{eff}^{LO} = \frac{m^2}{2} \phi_c^2 + \frac{\lambda}{16} \phi_c^4 \qquad [\ \phi_c \text{ is the shift of } \varphi$$

where m is the bare mass in the Lagrangian. A condition for symmetry breaking is that the VEV is non-zero, which means $m^2 < 0$.

At NLO and finite temperature, the effective potential gets corrections

$$V_{eff} = V_{eff}^{LO} + V_{eff}^{NLO} + V_{eff}^{T}$$

where the last term describes the finite temperature effects.

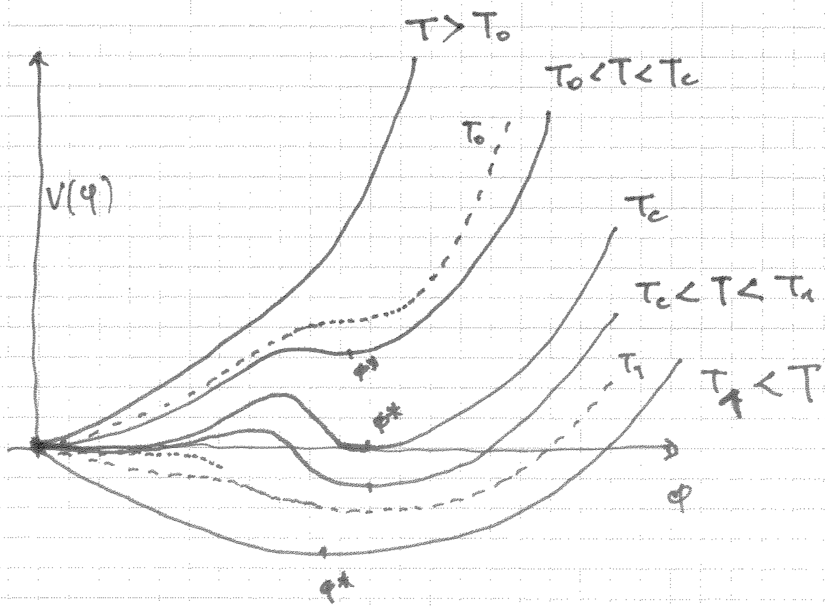
V_{eff}^{NLO} is a correction to the coefficient of ϕ^4, i.e. a correction to λ, whose magnitude depends on λ^2 and e^4. If λ is small and e^4 is much smaller than λ, i.e. if the gauge interaction is very weak compared to the φ^4 interaction, the correction V_{eff}^{NLO} is small compared to V_{eff}^{LO}, and we can ignore it. Then the effective potential is

$$V_{eff} = \frac{1}{2} m^2(T) \phi_c^2 - \frac{CT}{3} \phi_c^3 + \frac{\lambda}{16} \varphi_c^4$$

where a φ^2 term from V_{eff}^T contributes to the mass,

$$m^2(T) = m^2 + \frac{(\lambda + 3e^2)}{12} T^2$$

For large enough T, i.e if $T > T_0 = \frac{-12 m^2}{\lambda + 3e^2}$, the effective mass becomes positive. Then the ~~minimum~~ extremum at $\phi_c =$ is a minimum. If $m^2(T)\cdot\lambda < C^2T^2$, there are two more extrema, a maximum and a minimum. Let's denote T_1 the temperature for which $m^2(T)\cdot\lambda = C^2T^2$. Then if $T_0 < T < T_1$ there are two local minima. There is a temperature T_c for which ~~those~~ the local minimum at $\phi_c = 0$ is degenerate with the one at ϕ_c^*. ~~Below Tc~~ Above T_c we are at the "symmetric phase", where no spontaneous symmetry breaki takes place, the gauge boson is massless, Higgs has ~~no~~ zero vaccuum expectation valu Below T_c we are at the "asymmetric phase", where the gauge boson is massive, and the Higgs has an non-zero vev.



The VEV here changes discontinuously from $\phi_c = 0$ to $\phi_c = \phi_c^*(T_c)$ once the temperature crosses T_c. This is a first order transition, defined by the fact that the minimum of the potential (the equivalent of the "free energy" of Ehrenfest) changes discontinuously. The transition needs to happen through a tunneling effect, and there is a time period, around T_c, when the system is in a degenerate vacuum situation, with the two phases co-existing.

Note: if $C=0$ (or very small) then $T_1 = T_c = T_0$ and the phase transition is continuous, i.e. of the second order!

Phase transitions in e/w theory

The e/w sector of the SM has an $SU(2)\times U(1)_Y$ gauge group structure with a Lagrangian (excluding QCD terms)

$$\mathcal{L}_{SM} = -\frac{1}{4} W^{\alpha}_{\mu\nu} W^{\alpha\mu\nu} - \frac{1}{4} B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger (D^\mu H) - m^2 H^\dagger H - \lambda \left(H^\dagger H\right)^2$$
$$+ \bar{\psi}_L D_\mu \gamma^\mu \psi_L + \bar{\psi}_R D_\mu \gamma^\mu \psi_R + y\left(\bar{\psi}_L H\right)\psi_R + y\bar{\psi}_R\left(H^\dagger \psi_L\right)$$

with the particle content

W^i_μ	$i=1,2,3$	SU(2) adjoint
B_μ		SU(2) singlet
$\psi_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$		SU(2) doublet
$\psi_R = u_R, d_R$		SU(2) singlet
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$		SU(2) doublet

and $D_\mu = \partial_\mu + ig\dfrac{\sigma^i W^i_\mu}{2} + i\dfrac{g'}{2} B_\mu$

The $SU(2)\times U(1)$ is broken by the Higgs vev $\langle H\rangle = \begin{pmatrix} 0 \\ \frac{\varphi_c}{\sqrt{2}} \end{pmatrix}$.

We want to study the phase transition associated with the symmetry breaking.

Looking at the $(D_\mu H)^\dagger (D^\mu H)$ term, setting $H = \langle H\rangle = \begin{pmatrix} 0 \\ \varphi_c/\sqrt{2} \end{pmatrix}$ and selecting the terms quadradic at the fields W^i_μ, B_μ, we get

$$|D_\mu \langle H\rangle|^2 = \left| \left(i g \frac{\sigma_i}{2} W_\mu^i + i \frac{g'}{2} B_\mu\right) \begin{pmatrix}0\\ \varphi_c/\sqrt{2}\end{pmatrix}\right|^2$$

$$= \left| i\frac{g}{2}\begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1+iW_\mu^2 & -W_\mu^3\end{pmatrix}\begin{pmatrix}0\\ \varphi_c/\sqrt{2}\end{pmatrix} + i\frac{g'}{2}B_\mu \begin{pmatrix}0\\ \varphi_c/\sqrt{2}\end{pmatrix}\right|^2$$

$$= \left|\frac{\varphi_c}{2\sqrt{2}}\begin{pmatrix} g(W_\mu^1 - iW_\mu^2)\\ -gW_\mu^3 + g'B_\mu\end{pmatrix}\right|^2$$

$$= \frac{\varphi_c^2 g^2}{8} W_\mu^- W_\mu^+ + \frac{\varphi_c^2}{8}\left(g'B_\mu - gW_\mu^3\right)^2$$

We see that a linear combination of B_μ and W_μ^3 acquires mass and the orthogonal linear combination remains massless. Let's define

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$$

$$W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu$$

The mass term becomes $\frac{\varphi_c^2}{8}\left[(g'\cos\theta_W - g\sin\theta_W) A_\mu - (g'\sin\theta_W + g\cos\theta_W)\right.$

$$= \frac{\varphi_c^2}{8}\left(g'\sin\theta_W + g\cos\theta_W\right)^2 Z_\mu Z^\mu$$

if we impose $\quad g'\cos\theta_W = g\sin\theta_W$ (where* $g\sin\theta_W = e$)

So $\quad M_W^2 = \frac{\varphi_c^2 g^2}{4} \qquad M_Z^2 = \frac{g^2\varphi_c^2}{4\cos^2\theta_W}$

→ The effective potential for the Higgs field at LO is

$$V_{eff}^{LO} = \frac{m^2}{2}\langle H^\dagger H\rangle + \lambda\langle (H^\dagger H)^2\rangle = \frac{m^2\varphi_c^2}{2} + \lambda\varphi_c^4$$

and SSB requires $m^2 < 0$. At NLO, the effective

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* as can be seen from the $\bar\Psi D_\mu \gamma^\mu \Psi$ vertex containing $g\sin\theta_W \bar\Psi \not{A} \Psi$

potential gets one-loop corrections and temperature-dependent corrections

$$V_{eff} = \frac{1}{2} m^2(T)\varphi_c^2 - \frac{CT}{3}\phi_c^3 + \frac{\lambda}{4}\phi_c^4 + B\phi_c^4\left(\log\frac{\phi_c^2}{\mu^2} - \frac{25}{6}\right)$$

$$- \left(N_B + \frac{7}{8}N_F\right)\frac{\pi^2 T^4}{90}$$

where $m^2(T) = m^2 + \left(\frac{\lambda}{2} + \frac{e^2(1+2\cos^3\theta_W)}{4\sin^2(2\theta_W)} + \sum_f \frac{y_f^2}{12}\right)T^2$

and $C = \frac{3e^3(1+2\cos^3\theta_W)}{4\pi\sin^3 2\theta_W} + f(\lambda)$

$$B = \frac{3}{4}\left(\frac{e^2}{4\pi}\right)^2 \frac{1+2\cos^4\theta_W}{\sin^4 2\theta_W} - \frac{1}{64\pi^2}\sum_f y_f^2$$

Note that the quark masses are $m_q = y_q \cdot \frac{\varphi_c}{\sqrt{2}}$. We can determine what φ_c is in the zero temperature limit, since $\frac{m_W^2}{m_Z^2} = \cos^2\theta_W$, and $m_W^2 = \frac{g^2}{4}\varphi_c^2 = \frac{e^2}{4\sin^2\theta_W}\varphi_c^2$. Knowing φ_c (≈ 263 GeV) we can find $y_q = \frac{m_q}{\varphi_c\cdot\sqrt{2}}$. Plugging all numbers known into the formula for B shows that $\boxed{B<0}$

A dirty way to see that $\phi_c \neq 0$ at the low temperature limit:

$V_{eff}(T=0)$ has a minimum at $\left.\frac{dV(\phi)}{d\phi}\right|_{\phi=v} = 0$, which relates v with μ: $\log\mu = \log v + \frac{m^2}{8Bv^2} + \frac{\lambda}{8B} - \frac{11}{6}$ (*)

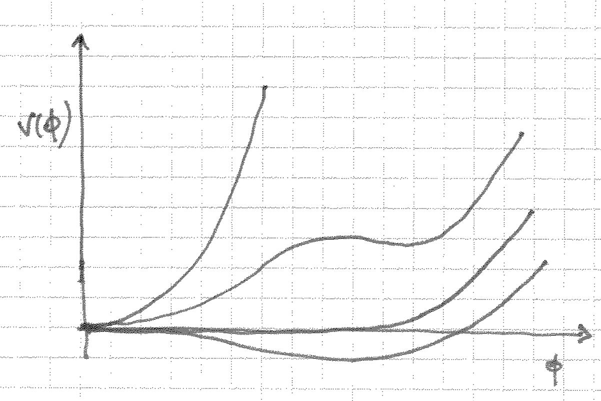
We can then substitute $\log\mu$ in $V_{eff}(T=0)$. Setting $\mu = v$ in (*) we get $m^2 = v^2\left(\frac{44}{3}B - \lambda\right)$, so we can trade m^2 for v^2 and arrive at

$$V_{eff}(T=0) = \frac{Bv^2}{2} a \phi^2 - \frac{B}{4}(a+2)\phi^4 + B\phi^4 \log \phi^2/v^2 \quad , \quad a = \frac{14}{3}$$

Now $\phi = 0$ is an extremum, but $\left.\frac{d^2 V_{eff}(T=0)}{d\phi^2}\right|_{\phi=0} = Bv^2 a$

and with $B<0$ this is never positive. So the effective potenti[al] at zero temperature has a minimum different than zero.

For very high T the effective potential can have only one minimum $\phi = 0$. There is a critical temperature T_c where the new minimu[m] becomes degenerate with the minimum at zero. Given that, also C is parametrically small, the e/w phase transition as described here can be considered a second order transition, i.e. one where the vacuum exp. val[ue] for the Higgs field changes continuously from 0 to v



The critical temperature, according to the above discussion, is of the order of $v \approx 263$ GeV.

Caveat: the above discussion is in reality much more complicated, since one should treat properly the renormalization of all couplings and masses. This involves solving systems of coupled RGE equations for g_1, g_2 and λ, as well as for the masses, including the mass of the Higgs itself that is plagued by quadradic divergences.

Baryon number, baryon asymmetry

Why is there more matter than anti-matter in the Universe?

Possible answers:

(a) There isn't! By some physical process in the Early Universe regions where matter dominated were seperated from regions where anti-matter dominated. This would mean there are anti-matter galaxies. But in the boundary regions we should be observing extremely strong e/m radiation, which we don't! Also the intergalactic medium that permeates everything would produce by anihilation strong light emission. $\boxtimes$

(b) Because the Universe started off with more matter. Possible but puzzling and unsatisfactory

(c) Because some physical process created a small surplus of matter vs. antimatter, and everything else anihilated to photons.

The matter vs. antimatter issue can be quantified by defining the <u>baryon number</u> $B = \frac{1}{3}(n_q - n_{\bar{q}})$, where every quark has $+1/3$ and every anti-quark has $-\frac{1}{3}$. Hence baryons have $B=1$, anti-baryons $B=-1$, mesons $B=0$ and leptons also $B=0$.

We can also define the <u>lepton number</u> $L = n_\ell - n_{\bar{\ell}}$ where all leptons have $L=1$ and all anti-leptons $L=-1$.

Within the SM, both baryon and lepton numbers are conserved, as can be seen by inspecting all SM interactions.

In the early universe, before nucleons were formed, there must have been an imbalance between quarks and anti-quarks, so that after anihilation some quarks survived to make nucleons. We actually need to satisf three conditions (the "Sakharov conditions")

1. Some physical process should violate baryon symmetry
2. C and CP symmetries should be violated, otherwise the process that produces more quarks than anti-quarks will have an inverse with which it will be in equilibrium
3. The universe needs to be out of equilibrium

→ C and CP are violated in weak interactions. Also, the universe can be out of equilibrium as it cools down either because interactions freeze out or because of phase transitions. And finally one can imagine that the SM is only an effective low energy theory, and in higher energies there is some GUT where quarks and leptons are in the same multiplet.

→ B violation means L violation, so that the net charge of the universe is kept zero (otherwise electrostatic expansion would be overwhelming)

→ B and L are often called accidental symmetries: they are symmetries of the SM but, if the SM is an effective theory valid up to a cutoff M, then there are also higher-order (non-renormalizable) operators, suppressed by powers of $\frac{E}{M}$, that break B and L conservation!

→ Any GUT theory that puts quarks and leptons in the same multiple has to explain why the proton is stable (or unstable with such a low decay rate)