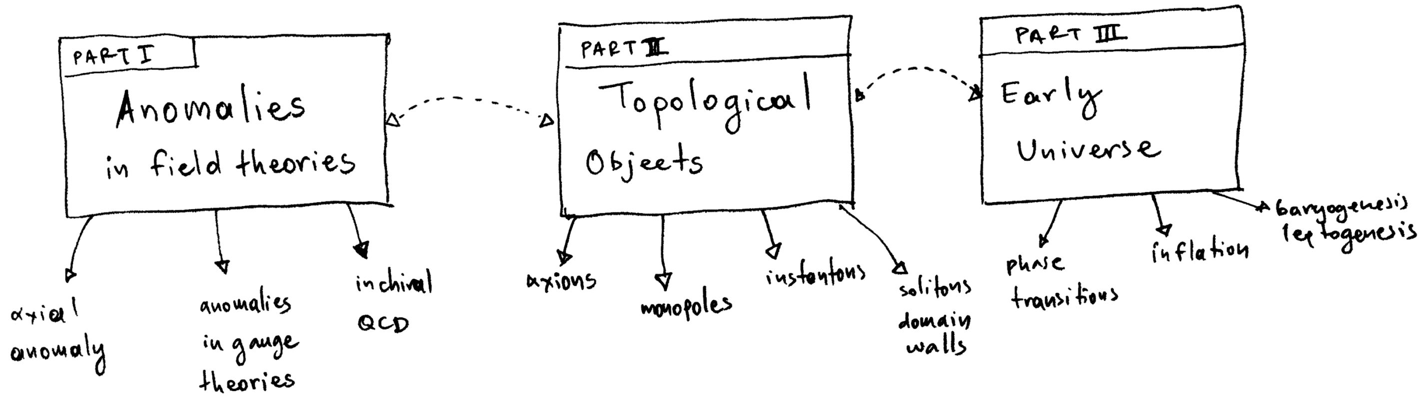
Advanced Field Theory

The course covers topics in three main directions



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metric $\quad g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$

Anomalies

QFT computations are plagued with infinities. Infinite integrals like $\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 (k+p)^2} =$  appear and need to be regulated. But the regulator might violate symmetries of the Lagrangian, and higher order perturbative contributions might break the symmetry even after the regulator is removed.

In the case of global symmetries, such violations (anomalies) may be physical. In the case of local (gauge) symmetries they make the theory ill-defined. The cancelation of all anomalies is, in these cases, a consistency condition for the theory.

Chiral Symmetry

Let's begin with a Lagrangian $\mathcal{L} = \bar{\psi} \not{D} \psi$ where ψ is a Dirac spinor and $\not{D} = (\partial_\mu + i e A_\mu)\gamma^\mu$

1. The transformation $\psi \to e^{i\theta}\psi$ is a symmetry of the Lagrangian.

$$\left.\begin{array}{l} \psi \to e^{i\theta}\psi \\ \bar{\psi} \to \bar{\psi} e^{-i\theta} \end{array}\right\} \quad \bar{\psi}\not{D}\psi \to \bar{\psi}\not{D}\psi$$

Since, infinitesimally, $\psi \to (1+i\theta)\psi \Rightarrow \delta\psi = \psi$, the conserved (Nöther) current is $j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\delta\psi = \underbrace{i\bar{\psi}\gamma^\mu\psi}_{\text{vector current}}$

2. Define $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ $\quad (\gamma_5^\dagger = \gamma_5 \quad \{\gamma_5, \gamma^\mu\} = 0)$

The transformation $\psi \to e^{i\alpha\gamma_5}\psi$ is a symmetry:

$$\bar{\psi}\not{D}\psi \longrightarrow \bar{\psi} e^{i\gamma_5 \alpha} \not{D} e^{i\gamma_5\alpha}\psi = \bar{\psi}\not{D} e^{-i\gamma_5\alpha} e^{i\gamma_5\alpha}\psi = \bar{\psi}\not{D}\psi \quad \boxed{\checkmark}$$

This is the "chiral" symmetry, and theories that respect it are called "chiral" theories.

We could define $\omega_{L/R} = \frac{1 \pm \gamma_5}{2}$ and then $\omega_L + \omega_R = 1$, $\psi = \psi_L + \psi_R$ with $\psi_{L/R} = \omega_{L/R}\cdot\psi$, the "left" and "right" chirality spinors.

The conserved current is now $\underbrace{\bar{\psi}\gamma^\mu\gamma_5\psi}_{\text{axial current}}$

■ Note, however that a mass term in the Lagrangian would $\underline{\underline{\text{not}}}$ be invariant

$$m\bar{\psi}\psi \rightarrow m\bar{\psi}e^{i\gamma_5 a}e^{i\gamma_5 a}\psi \neq m\bar{\psi}\psi$$

3) Let's consider a Lagrangian with two quark flavors, "up" and "down" $\mathcal{L} = \bar{\psi}_u \not{\partial} \psi_u + \bar{\psi}_d \not{\partial} \psi_d + m_u\bar{\psi}_u\psi_u + m_d\bar{\psi}_d\psi_d$

If $m_u, m_d \rightarrow 0$ then the theory is chiral. Now, m_u, m_d are very small and we can start by setting $m_u = m_d = 0$.

Then, we can put ψ_u and ψ_d in one spinor $\psi = \begin{pmatrix}\psi_u \\ \psi_d\end{pmatrix}_{1\times 8}$ and write

$$\mathcal{L} = \bar{\psi}\begin{pmatrix}\not{\partial} & 0 \\ 0 & \not{\partial}\end{pmatrix}\psi$$

The Lagrangian is now invariant under

$$\psi \rightarrow e^{i(\vec{\theta}\cdot\vec{t} + \gamma_5\vec{\theta}_A\cdot\vec{t})}\psi$$ where $\vec{\theta}, \vec{\theta}_A$ are the trs. paramet

and $\vec{t} = \frac{\vec{\sigma}}{2}$, σ^i being the Pauli matrices.

Then $\bar{\psi}\begin{pmatrix}\not{\partial} & 0 \\ 0 & \not{\partial}\end{pmatrix}\psi \rightarrow \bar{\psi}e^{-i(\vec{\theta}\vec{t} - \gamma_5\vec{\theta}_A\vec{t})}\begin{pmatrix}\not{\partial} & 0 \\ 0 & \not{\partial}\end{pmatrix}e^{i(\vec{\theta}\vec{t} + \gamma_5\vec{\theta}_A\vec{t})}\psi$

$$= \bar{\psi}\begin{pmatrix}\not{\partial} & 0 \\ 0 & \not{\partial}\end{pmatrix}e^{-i(\vec{\theta}\vec{t} - \gamma_5\vec{\theta}_A\vec{t})}e^{i(\vec{\theta}\vec{t} + \gamma_5\vec{\theta}_A\vec{t})}\psi = \bar{\psi}\begin{pmatrix}\not{\partial} & 0 \\ 0 & \not{\partial}\end{pmatrix}\psi \; \boxed{\checkmark}$$

Setting $\left.\begin{array}{l}\vec{t}_L = \frac{1}{2}(1+\gamma_5)\vec{t} \\ \vec{t}_R = \frac{1}{2}(1-\gamma_5)\vec{t}\end{array}\right\} \Rightarrow \begin{array}{ll}[t_{Li}, t_{Lj}] = i\,\varepsilon_{ijk}t_{Lk} & \longrightarrow SU(2)_L \\ [t_{Ri}, t_{Rj}] = i\,\varepsilon_{ijk}t_{Rk} & \longrightarrow SU(2)_R. \\ [t_{Li}, t_{Rj}] = 0 & \end{array}$

So $\mathcal{L}$ is invariant under $SU(2)_L \times SU(2)_R$!

The corresponding conserved currents are

$$\underbrace{\bar{\psi}\gamma^\mu \vec{t}\,\psi}_{\text{vector current}} \qquad \text{and} \qquad \underbrace{\bar{\psi}\gamma^\mu\gamma_5 \vec{t}\,\psi}_{\text{axial current}}$$

In real life, this symmetry, if exact, would imply that for each hadron (composed of quarks) there is another hadron with the same quantum numbers and opposite parity. This is not the case, so we are forced to assume that the symmetry is "spontaneously broken".

According to the Goldstone theorem, for every generator of a broken symmetry, the theory should have a massless ("Goldstone") boson. Here we assume that the isospin subgroup $\left(\bar{\psi}\gamma^\mu \vec{t}\,\psi\right)$ is conserved, so we expect three massless bosons.

However, in reality, the masses $m_u, m_d \neq 0$, so the original symmetry was approximate. Hence the ("pseudo-Goldstone") bosons are light but massive. There are exactly three such hadrons, consisting of $q\bar{q}$ pairs, that are much lighter than all other hadrons, the pions: π^0, $\pi^\pm$

Assuming approximate symmetry breaking gives satisfactory predictions for

(1) their mass see lecture 3

(2) the fact that their masses are equal

(3) the decay rate to $\pi^+ \longrightarrow \mu^+ \nu_\mu$

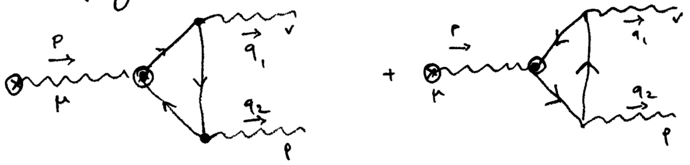
But in the decay $\pi^0 \to \gamma\gamma$

(a) assuming no constraints ~~from~~ from chiral symmetry $\Gamma = 4.4 \cdot 10^{16}\, s^{-1}$

(b) assuming constraints $\Gamma = 1.9\ 10^{13}\, s^{-1}$

$\to$ measurement $\Gamma = 1.19 \cdot 10^{16}\, s^{-1}$

Something anomalous is invalidating the chiral symmetry! The pion, being the Goldstone boson of the broken chiral symmetry is related to the axial current $\bar{\psi}\gamma^\mu\gamma_5\psi$. We would like to investigate whether the current is, in general, conserved in the presence of an electromagnetic field (i.e. in a process with photons), in a chiral theory. Classical considerations (Nöther theorem, eqs of motion) imply it should be conserved. The easiest set-up is the diagrams



$= M_{\mu\nu\rho}(p, q_1, q_2) \cdot p^\mu$

If the current is conserved, then the Ward identity will hold $M_{\mu\nu\rho} \cdot p^\mu = 0$. If $M_{\mu\nu\rho} \cdot p^\mu \neq 0$, however, the axial current is not conserved, and its divergence produces an anomaly.

In order to compute the two one-loop triangles, we will need some extra theoretical tools related to dimensional regularization

Dimensional Regularization