Due by: 23 May

**Exercise 1.** ***Phase transitions in the electroweak theory***

We will consider the electroweak theory a $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ broken by a Higgs scalar. The Lagragian density of this model has the form

$$\mathcal{L} \supset \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \tag{1}$$

where the gauge part consists of the usual field strength tensors:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \qquad a = 1,2,3 \tag{2}$$

with the $\mathrm{SU}(2)_L$ $W^a_{\mu\nu} \equiv \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\varepsilon^{abc}W^b_\mu W^c_\nu$ and $\mathrm{U}(1)_Y$ $B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$. The Higgs is now an $\mathrm{SU}(2)_L$ doublet,

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \tag{3}$$

with Lagrangian density

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger(D^\mu H) - m^2 H^\dagger H - \lambda(H^\dagger H)^2 \tag{4}$$

and covariant derivative

$$D_\mu H = (\partial_\mu + ig\frac{\sigma^a}{2}W^a_\mu + ig'\frac{1}{2}B_\mu)H. \tag{5}$$

The fermions acquire their mass through their Yukawa couplings to the Higgs field:

$$\mathcal{L}_{\text{Yukawa}} = -\sum_f y_f F_L^T i\sigma^2 H f_R + \text{h.c.} \tag{6}$$

where $y_f$ denotes the Yukawa coupling, $F_L \equiv \begin{pmatrix} u_f \\ d_f \end{pmatrix}_L$ denotes the left-handed doublet, and $f_R$ the right-handed singlet.

In the electroweak theory the symmetry is spontaneously broken, by the Higgs field acquiring a VEV

$$\langle H \rangle = \begin{pmatrix} 0 \\ \varphi_c/\sqrt{2} \end{pmatrix} \tag{7}$$

(a) Compute the masses of the fermions as a function of $\varphi_c$.

(b) Defining the photon, Z and $\mathrm{W}^\pm$ by

$$A_\mu \equiv \cos\theta_W B_\mu + \sin\theta_W W^3_\mu, \qquad Z_\mu \equiv -\sin\theta_W B_\mu + \cos\theta_W W^3_\mu, \qquad W^\pm_\mu \equiv W^1_\mu \pm iW^2_\mu \tag{8}$$

compute their masses as a function of $\varphi_c$.

When fermions are involved the result from the last exercise sheet needs to be adapted as follows:

$$\bar{V}_1^{T=0}(\varphi_c) = B'\varphi_c^4\left[\ln\frac{\varphi_c^2}{v'^2} - \frac{25}{6}\right], \tag{9}$$

$$\bar{V}_1^{T\neq0}(\varphi_c) = -\frac{\pi^2T^4}{90}(n_S + n_V + \frac{7}{8}n_F) + \frac{T^2}{24}\left[t_S + 3t_V + 2t_F\right] - \frac{C'T}{3}, \qquad (T \gg 0) \tag{10}$$

where the the unprimed parameters are the same as previously.

(c) With this result compute the 1-loop effective potential $\bar{V}(\varphi)$ and show that

$$m^2(T) = m^2 + \left( \frac{\lambda}{2} + \frac{e^2(1+2\cos^2\theta_W)}{4\sin^2 2\theta_W} + \sum_f \frac{y_f^2}{12} \right) T^2 \tag{11}$$

(d) Retaining only the gauge boson contribution to the $\varphi_c^3$-term in $\bar{V}(\varphi)$ show that

$$C' \approx \frac{3e^3(1+2\cos^3\theta_W)}{4\pi\sin^3 2\theta_W} \tag{12}$$

Dropping the $\lambda^2$ contributions to the radiative corrections one can prove that

$$B' \approx \frac{3}{4}\left(\frac{e^2}{4\pi}\right)^2 \frac{1+2\cos^4\theta_W}{\sin^4 2\theta_W} - \frac{1}{64\pi^2}\sum_f y_f^2. \tag{13}$$

(e) Find the sign of $B'$ with the values of the parameters in the standard model ($e^2/4\pi \approx 1/137$,...). Write the numerial values and comment on which terms are essentially important and which ones are not.