Advanced Field Theory
Series 8.

Due by: 25 April

Exercise 1. ***Soliton of φ^4-theory in $1+1$ dimensions***

Consider the scalar field theory with Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - V(\varphi), \qquad V(\varphi) = \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 \tag{1}$$

in one spatial dimension ($g_{\mu\nu} = \mathrm{diag}(+1,-1)$) with $m^2 < 0$.

(a) Determine the constant solutions of the equation of motion, shift the potential such that these have vanishing energy.

(b) Find the static solutions of the equations of motion interpolating between two constant solutions (these interpolating solutions are called solitons), use the ansatz $\varphi(x) = a\tanh(bx)$.

(c) Calculate the energy of the static soliton ($\int dx \cosh^{-4}(x) = 4/3$)

(d) Check that the current

$$J^\mu = \varepsilon^{\mu\nu}\partial_\nu\varphi \tag{2}$$

is conserved. What are the possible values of $\int dx\, J^0$ for a solution of the equation of motion?