Due by: 7 March

**Exercise 1.** ***Feynman parameters***

To evaluate loop diagrams one combines propagators with the use of *Feynman parameters*. The basic version is

$$\frac{1}{AB} = \int_0^1 \frac{dx}{[xA + (1-x)B]^2}, \tag{1}$$

but it can be generalised to $n$ propagators

$$\frac{1}{\prod_{i=1}^n A_i} = \Gamma(n) \int_0^1 \left( \prod_{i=1}^n dx_i \right) \delta \left( 1 - \sum_{i=1}^n x_i \right) \left[ \sum_{i=1}^n x_i A_i \right]^{-n}. \tag{2}$$

(a) Prove (1) and (2).

**Exercise 2.** ***Dimensional regularization and loop momenta***

The momentum of loop integrals are taken to be $d$-dimensional. We would like to address this a bit more explicitly by defining projectors into the physical and unphysical dimensions:

$$g_{\parallel}^{\mu\nu} = \begin{cases} g^{\mu\nu} & \mu, \nu \leq 3, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

$$g_{\perp}^{\mu\nu} = \begin{cases} g^{\mu\nu} & \mu, \nu > 3, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

It is quite straightforward to see that $g^{\mu\nu} = g_{\perp}^{\mu\nu} + g_{\parallel}^{\mu\nu}$. Raising and lowering is still performed using $g^{\mu\nu}$.

(a) Infer the value of $g_{\perp}^{\mu\nu} g_{\mu\nu}$ from $g^{\mu\nu} g_{\mu\nu} = d$ and $g_{\parallel}^{\mu\nu} g_{\mu\nu} = 4$.

(b) Using this decomposition and expanding the product, how can the loop integral (10) from last exercise sheet can be decomposed?

**Exercise 3.** ***Triangle topology II***

Redo the computation of the Adler-Bell-Jackiw anomaly

$$q_\mu \left[ A_5^\mu \left( (p_1, \varepsilon_1), (p_2, \varepsilon_2) \right) + A_5^\mu \left( (p_2, \varepsilon_2), (p_1, \varepsilon_1) \right) \right] \tag{5}$$

that you have seen in the lecture. In particular perform the steps that have not being treated explicitly in the lecture.