Relativistic Kinematics

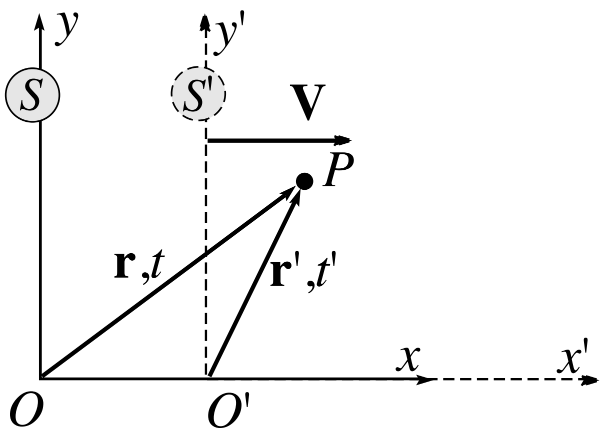
Relativistic Kinematics

Just a quick recall of the basic ideas and the most useful formulas we will need.

Galilean relativity = physics in all inertial system is the same
Special relativity = Galilean relativity + constant speed of light

Lorentz transformations.
Consider two systems (inertial frames). One is moving at a constant velocity **v** w.r.t to the other along the axis **x** (it can be any direction)

Synchronise the clocks in the two frameworks when x = x' = 0 t = t' = 0.
How is the "event" (t,x,y,z) in S seen in the S' frame ?
(t,x,y,z) is called a four-vector



$$x' = \gamma(x - \beta ct)$$
$$y' = y$$
$$z' = z$$
$$ct' = \gamma(ct - \beta x)$$

<— Lorentz Transformations

$$\Lambda^\mu_\nu = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix}$$

where $\boldsymbol{\beta} \equiv \frac{\mathbf{V}}{c}$ $\quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$

Because v < c —> β < 1 and γ > 1

Q: What is the opposite transformation ?
(instead S—>S' go S' —>S)

Lorentz Transformations

Q: What is the opposite transformation ? (instead S—>S' go S' —>S)

Just invert the previous system of equations
Can you see intuitively why without the math ?

$$
\begin{aligned}
x' &= \gamma(x - \beta ct) \\
y' &= y \\
z' &= z \\
ct' &= \gamma(ct - \beta x)
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
x &= \gamma(x' + \beta ct') \\
y &= y' \\
z &= z' \\
ct &= \gamma(ct' + \beta x')
\end{aligned}
$$

Relativistic Kinematics

Consequences of the Lorentz transformations:

- Relativity of simultaneity:
  events simultaneous in one frame are not simultaneous in the other

- Lorentz space contraction
  moving objects are shortened by γ $L = L' / \gamma$

- Lorentz time dilation
  moving clocks run slower by γ $T = \gamma T'$

- Modified velocity addition
  a particle is moving at speed u' wrt S', it's speed w.r.t S is $u = \frac{u' + v}{1 + (u'v/c^2)}$

  —> for v<<c it goes back to the normal addition
  —> if one of the two velocities is c, the sum is c
  (can't go faster than c or c is the same in all inertial frames)

Q: Can you prove this relation ?

Relativistic Kinematics

Relativity of simultaneity

● Relativity of simultaneity

$$\Delta t = 0 \Rightarrow t_A = t_B$$

$$t'_A = \gamma (c t_A - \beta x_A)$$

$$t'_B = \gamma (c t_B - \beta x_B)$$

$$\Delta t' = t'_A - t'_B = \gamma \left(c \cancel{(t_A - t_B)} - \beta (x_A - x_B) \right)$$

Relativistic sum of velocitites

● Velocity addition: u' in S', what is u in S?

$$u = \frac{\Delta x}{\Delta t} = \frac{\cancel{\gamma}(\Delta x' + v \Delta t')}{\cancel{\gamma}(\Delta t' + \frac{v}{c^2} \Delta x')} =$$ divide all by $\Delta t'$

$$= \frac{\Delta x'/\Delta t' + v}{1 + \frac{v}{c^2} \Delta x'/\Delta t'} =$$

$$= \frac{u' + v}{1 + \frac{v \cdot u'}{c^2}}$$

Relativistic Kinematics

We will work mainly with one four-vector:
(Energy, momentum) = (E,**p**) = (E/c, px, py, pz)

The spatial component is:

$$\vec{p} = \gamma m v = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

The time component is:

$$p^0 = \gamma m c = E/c$$

The relativistic energy is:

$$E = \gamma m c^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

Q: Can you explain why muons (lifetime=2 10^{-6} sec) make it to earth ?

Lorentz Transformations

Q: Can you explain why muons (lifetime=2 10^{-6} sec) make it to earth ?

$$\tau_\mu = 2.197\ 10^{-6}\text{s},\ m_\mu = 105.658\ MeV/c^2$$

The muon lifetime is $\tau = 2.197\ 10^{-6}$s; $c\tau = 658.654$ m = L'.
To travel L= 10 km it needs a γ factor of:

$$L = \gamma L' \Rightarrow \frac{L}{L'} = \gamma$$

$$\gamma = \frac{10000 \quad m}{658.654 \quad m} = 15.182$$

Knowing its mass $m_\mu = 105.658\ MeV/c^2$ we can compute the energy of the muon:

$$E_\mu = \gamma m_\mu = 15.182 \times 105.658 MeV/c^2 = 1.6\ GeV$$

Relativistic Kinematics

We will work mainly with one four-vector:
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Four vector multiplications:

$$a^\mu b_\mu = a_\mu b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

$$p^\mu = \left(\frac{E}{c}, p_x, p_y, p_z \right) \qquad p_\mu p^\mu = \frac{E^2}{c^2} - \mathbf{p}^2 = m^2 c^2$$

Massless particles

Massless particles are nonsense in classical non relativistic mechanics.
It the mass is zero, the momentum is zero, the energy is zero.

Massless particles makes their appearance with special relativity.
Handwaving argument: look at the energy

$$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

when m=0 —> E =0. But if at the same time v = c, we get something mathematically interesting 0 / 0. A massless particle can exist if it moves at the speed of light (photons)

Special relativity has little to say about the photon energy: just E = p
The energy of a photon is given by QM: E = hν

Natural Units

Quantities computed in QFT have typically a bunch of ħ and c in their expression.
We can redefine our units to get rid of them

$$c = \hbar = 1$$

In these units mass (m), momentum (mc) and energy (mc^2) have the same units.
As energy, mass, momentum units we will typically use GeV: 10^9 eV

[mass]=[momentum]=[energy] = GeV

Using the values of c and ħ in another set of units (here GeV-m-s) we get all the conversion factors:

$\hbar = 6.6\ 10^{-25}$ GeV sec $\Rightarrow 1 = 6.6\ 10^{-25}$ GeV s

1 GeV = $1/6.6\ 10^{25}\ s^{-1}$

1 s = $1/6.6\ 10^{25}\ GeV^{-1}$ [time] = GeV^{-1}

$c = 3\ 10^8$ m/s $\Rightarrow 1 = 3\ 10^8$ m/s

1 s = $3\ 10^8$ m [length] = GeV^{-1}

With these you can do all combinations:

1 fm ~ 5 GeV^{-1}

1 mb = 10^{-27} cm ~ 2.6 GeV^{-2} [area] = GeV^{-2}

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