Passage of particles through matter

Overview

For "interaction of particles with matter" we mean the phenomena we can use to build detectors.

Interactions of particles with matter are qualitatively different depending on:

- type of interaction: electromagnetic or strong; weak is relevant only for neutrinos
- charge of the particle
- mass of the particle
- energy/momentum of the particle
- A,Z of the target (incident particles see the structure of the target matter)

(we will be typically interested in interactions of "high energy" particles (E>MeV))

Because of this we typically group them as:

Heavy particles
(heavy = not electrons)

Electrons

Photons

Electromagnetic Showers

Hadronic Showers

Cerenkov Radiation

Transition Radiation

One remark

The passage of particles through matter is a very complicated process to model. We get an good description only in very idealised situations.

There are zillions of empirical formulas to describe specific cases.
We will provide only the most useful ones and focus on the physics of the interaction processes.

Bethe Bloch

Ionization: heavy particles

Main microscopic processes:

- elastic scattering from nuclei → deflection of particle trajectory
- inelastic scattering from electrons → energy loss of the particle
  (ionization excitation of target electrons)

Main macroscopic effect:
deflection of particle trajectory
energy loss of the particle

We'll focus first on the inelastic scattering.
—> the fraction of energy of the incident particle transferred to the electron in each collision is very small, but the number of collisions is so large that we obtain a macroscopic effect

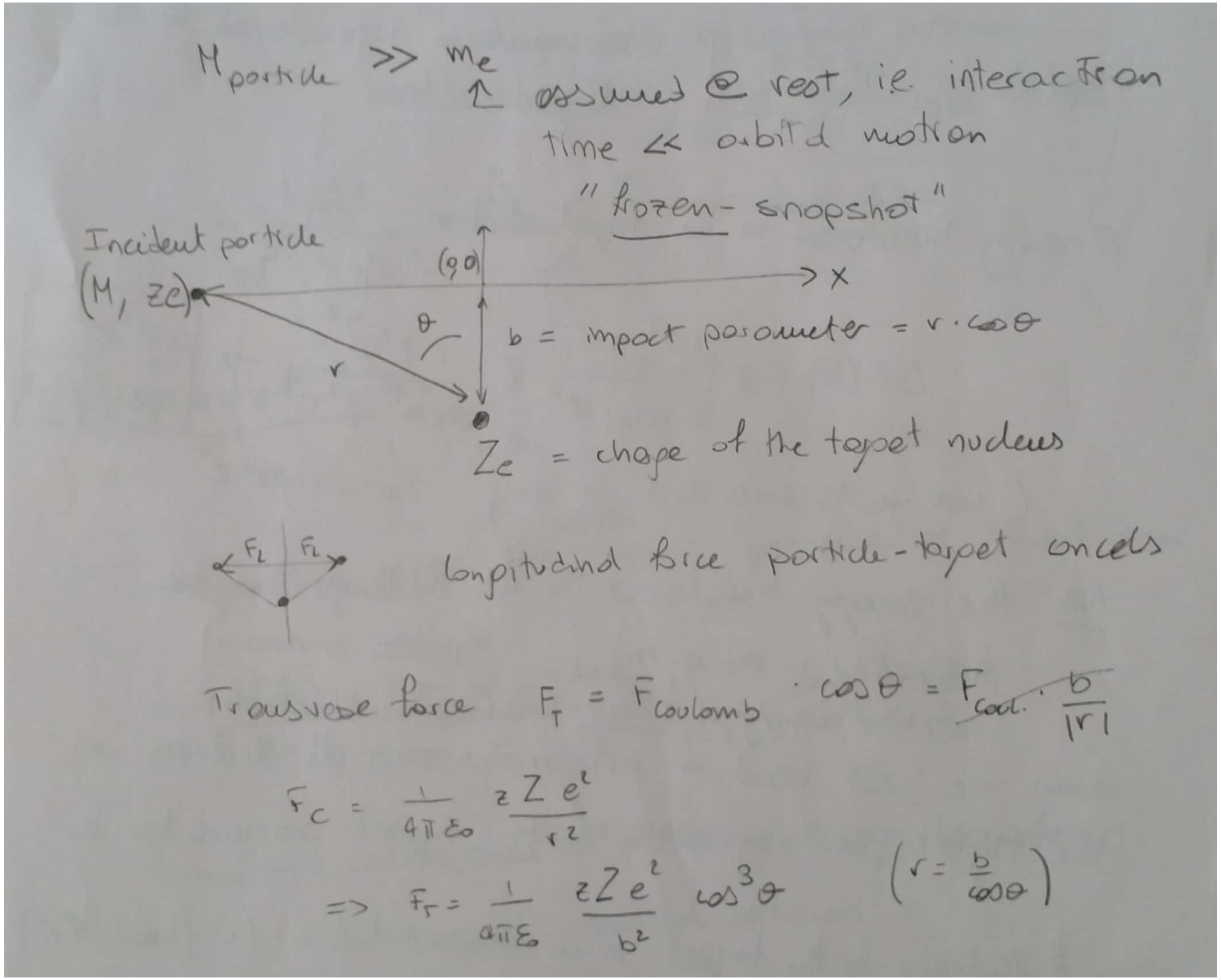
e.g. a 10 MeV proton gets stop by 0.25 mm of copper

We want to get to the value of -dE/dx, i.e. the specific (per unit length) energy loss, also known as stopping power

The first computation of the -dE/dx has been performed using classical electromagnetism by Bohr, then Bethe and Bloch gave the QM formulation

Ionization: heavy particles

Bohr derivation of the -dE/dx: classical derivation
(no QM, no relativistic treatment of the electric field of the incident particle)



$M_{particle} \gg m_e$

↑ assumed @ rest, i.e. interaction time $\ll$ orbital motion

"frozen-snapshot"

b = impact parameter $= r \cdot \cos\theta$

Ze = charge of the target nucleus

longitudinal force particle-target cancels

Transverse force $F_T = F_{Coulomb} \cdot \cos\theta = F_{Coul.} \cdot \frac{b}{|r|}$

$$F_C = \frac{1}{4\pi\varepsilon_0} \frac{zZe^2}{r^2}$$

$$\Rightarrow F_T = \frac{1}{4\pi\varepsilon_0} \frac{zZe^2}{b^2} \cos^3\theta \qquad \left(r = \frac{b}{\cos\theta}\right)$$

Ionization: heavy particles

1

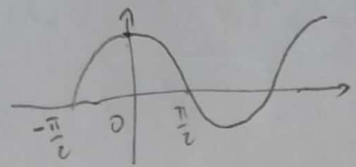
Momentum transfer to the target nucleus

$$\Delta p(b) = \int_{-\infty}^{+\infty} F_T(b)\,dt = \int_{-\infty}^{+\infty} F_T(b)\,\frac{dx}{v} = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{b^2}\int_{-\infty}^{+\infty}\cos^3\theta\,\frac{dx}{v}$$

$$dx = b\,d\tan\theta = \frac{b}{\cos^2\theta}\,d\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{b^{\cancel{2}}}\int_{-\pi/2}^{+\pi/2}\cos^{\cancel{3}}\theta\,\frac{1}{v}\,\frac{\cancel{b}\,d\theta}{\cancel{\cos^2\theta}} = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{bv}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{bv}\underbrace{\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta}_{} = \frac{1}{4\pi\varepsilon_0}\frac{zZe^2}{bv}\left[\sin\theta\right]_{-\pi/2}^{\pi/2} =$$



$$= \frac{zZe^2}{bv}\left[1-(-1)\right]$$

$$\boxed{\Delta p(b) = \frac{1}{4\pi\varepsilon_0}\frac{2zZe^2}{bv}}$$

momentum transferred to the target electron

Ionization: heavy particles

Energy transferred to the target electron $\frac{\Delta p^2}{2 m_e}$

$$\Delta E(b) = 2\frac{z^2 Z^2 e^4}{b^2 m_e \cdot v^2} = \frac{2 z^2 Z^2 e^4}{m_e b^2 \beta^2 c^2} \rightarrow \boxed{\frac{2 z^2 e^4}{m_e b^2 \beta^2 c^2}}$$

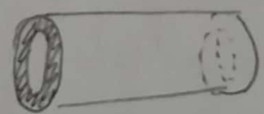
(set units such that $\frac{1}{4\pi\varepsilon_0} = 1$) For a single e^-

<u>NB</u> the energy transferred to the nucleus can be neglected $m_e \ll m_{nucl.}$ $\frac{m_{proton}}{m_e} \sim 4000$

$\Rightarrow$ the energy is <u>MOSTLY</u> transferred to electrons.

Now we just need to integrate over all electrons in the target in any value of the impact parameter b.

<u># electrons in the target</u> in a tube $= 2\pi \cdot b\,db \cdot dx \cdot$ (density of e^-)



$$\# \text{atoms} = \frac{N_A \cdot \rho \cdot V}{A}$$

ρ = density

V = volume

A = mass number

$$\frac{\# \text{electr}}{V} = \frac{N_A \cdot \rho \cdot Z}{A}$$

atom Z electrons

Ionization: heavy particles

$\Rightarrow$ Energy loss by the incident particle

$$-dE(b) = \underbrace{\Delta E(b)}_{} \cdot \frac{\#e}{V} \cdot dV =$$

loss

energy loss by incident particle encountering one (Z=1) electron on its path.

$$= \frac{2 z^2 e^4}{m_e b^2 \beta^2 c^2} \cdot \frac{N_A \rho \cdot Z}{A} \cdot 2\pi\, b\, db\, dx$$

$$-\frac{dE(b)}{dx} = \frac{4\pi z^2 e^4 N_A \cdot \rho}{m_e \beta^2 c^2} \cdot \frac{Z}{A} \cdot \frac{db}{b}$$

Ionization: heavy particles

Integrating over the impact parameter $b_{min} \div b_{max}$

$$-\frac{dE(b)}{dx} = \frac{4\pi z^2 e^4}{m_e} \cdot \frac{Z}{A} N_A \rho \cdot \frac{1}{\beta^2 c^2} \cdot \ln\left(\frac{b_{max}}{b_{min}}\right)$$

b_{max} = max distance to have an effect.
the minimum effect is to ionize/excite the shell electrons $= I$

2

$$I = \Delta E = \frac{2 z^2 e^4}{m_e b^2 \beta^2 c^2}$$

$$\Rightarrow \boxed{b_{max} = \frac{z e^2}{\beta c} \cdot \sqrt{\frac{2}{m_e I}}}$$

b_{min} = min distance gives the maximal energy transfer

max energy transfer

$$\Delta E(b) = \frac{1}{2}\gamma^2 m_e \beta^2 c^2 = \frac{2 z^2 e^4}{m_e b^2 \beta^2 c^2}$$

Exercise: compute the max energy transfer in the "low energy" approximation: $2\gamma m_e \ll M$

3

$$\Rightarrow b_{min}^2 = \frac{2 z^2 e^4}{\frac{1}{2}\gamma^2 m_e^2 \beta^4 c^4}$$

$$\boxed{b_{min} = \frac{2 z e^2}{\gamma m_e \beta^2 c^2}}$$

Ionization: heavy particles

Putting all together

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e \beta^2 c^2}\, \rho N_A \frac{Z}{A} \ln\left(\frac{\cancel{ze^2}}{\cancel{\beta c}} \sqrt{\frac{2}{m_e I}} \cdot \frac{\gamma m_e c^2 \beta^2}{2\, \cancel{ze^2}} \right)$$

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e}\, \rho N_A \frac{Z}{A} \cdot \frac{1}{\beta^2 c^2} \cdot \ln\left(\frac{\gamma m_e \beta c}{\sqrt{2 m_e I}} \right)$$

$$\ln \sqrt{\frac{\gamma^2 m_e^{\cancel{2}} \beta^2 c^2}{2\, \cancel{m_e}\, I}} = \frac{1}{2} \ln \frac{m_e \gamma^2 \beta^2 c^2}{2 I}$$

$$\boxed{-\frac{dE}{dx} = \frac{2\pi z^2 e^4}{m_e}\, \rho N_A \frac{Z}{A} \cdot \frac{1}{\beta^2 c^2} \cdot \ln\left(\frac{m_e \gamma^2 \beta^2 c^2}{2 I} \right)}$$

Bethe Bloch

$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Where:

z	charge number of incident particle	
Z	atomic number of absorber	
A	atomic mass of absorber	g mol^{-1}
K	$4\pi N_A r_e^2 m_e c^2$	0.307 075 MeV mol^{-1} cm^2
I	mean excitation energy	eV (*Nota bene!*)
$\delta(\beta\gamma)$	density effect correction to ionization energy loss	

3

$$W_{\max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

energy transfer to an electron in a single collision — MeV

Bethe Bloch

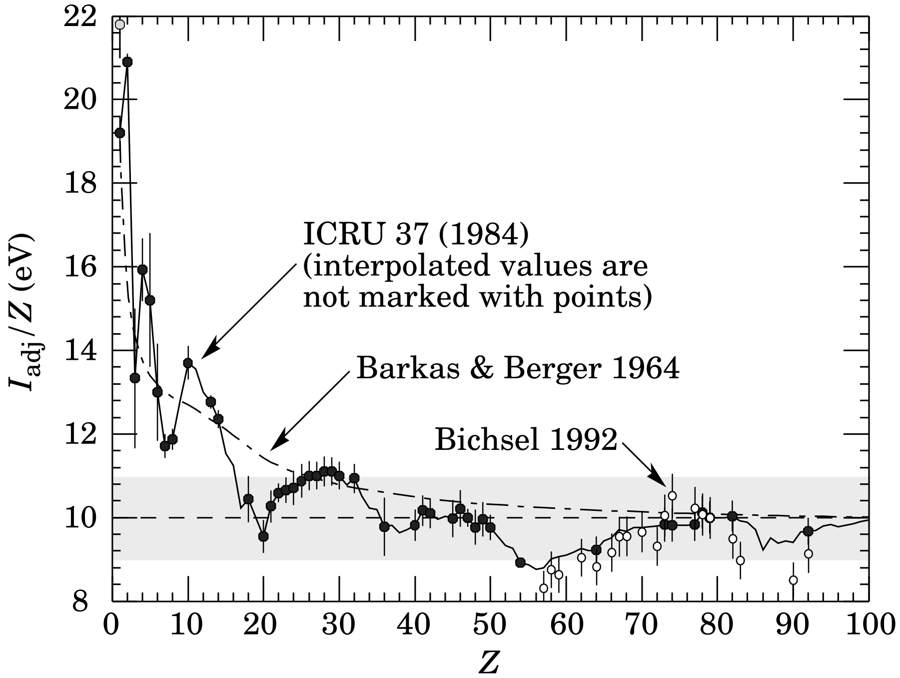
$$\left\langle -\frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

2

I = mean excitation potential = hν_{mean} ; where ν_{mean} is a (logarithmic) weighted average of the orbital frequencies of the target atoms.

"The determination of the mean excitation energy is the principal non-trivial task in the evaluation of the Bethe stopping-power formula"

for this reason we use measured values (fit I in a dE/dx measurement)



A simple approximation is:

$$\frac{I}{Z} = 12 + \frac{7}{Z}\ \text{eV} \qquad Z < 13$$

$$\frac{I}{Z} = 9.76 + 58.8 Z^{-1.19}\ \text{eV} \qquad Z \geq 13$$

Bethe Bloch

$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Density effect correction to ionization energy loss.

The electric field of the incident particle polarize atoms on its way. Because of this, electrons far from the trajectory of the particle are shielded from the electric field and contribute less to the energy loss.

Typically the Sternheimer parametrization is used:

$$\delta(\beta\gamma) = \begin{cases} 2(\ln 10)x - \overline{C} & \text{if } x \geq x_1; \\ 2(\ln 10)x - \overline{C} + a(x_1 - x)^k & \text{if } x_0 \leq x < x_1; \\ 0 & \text{if } x < x_0 \text{ (nonconductors);} \\ \delta_0 10^{2(x - x_0)} & \text{if } x < x_0 \text{ (conductors)} \end{cases}$$

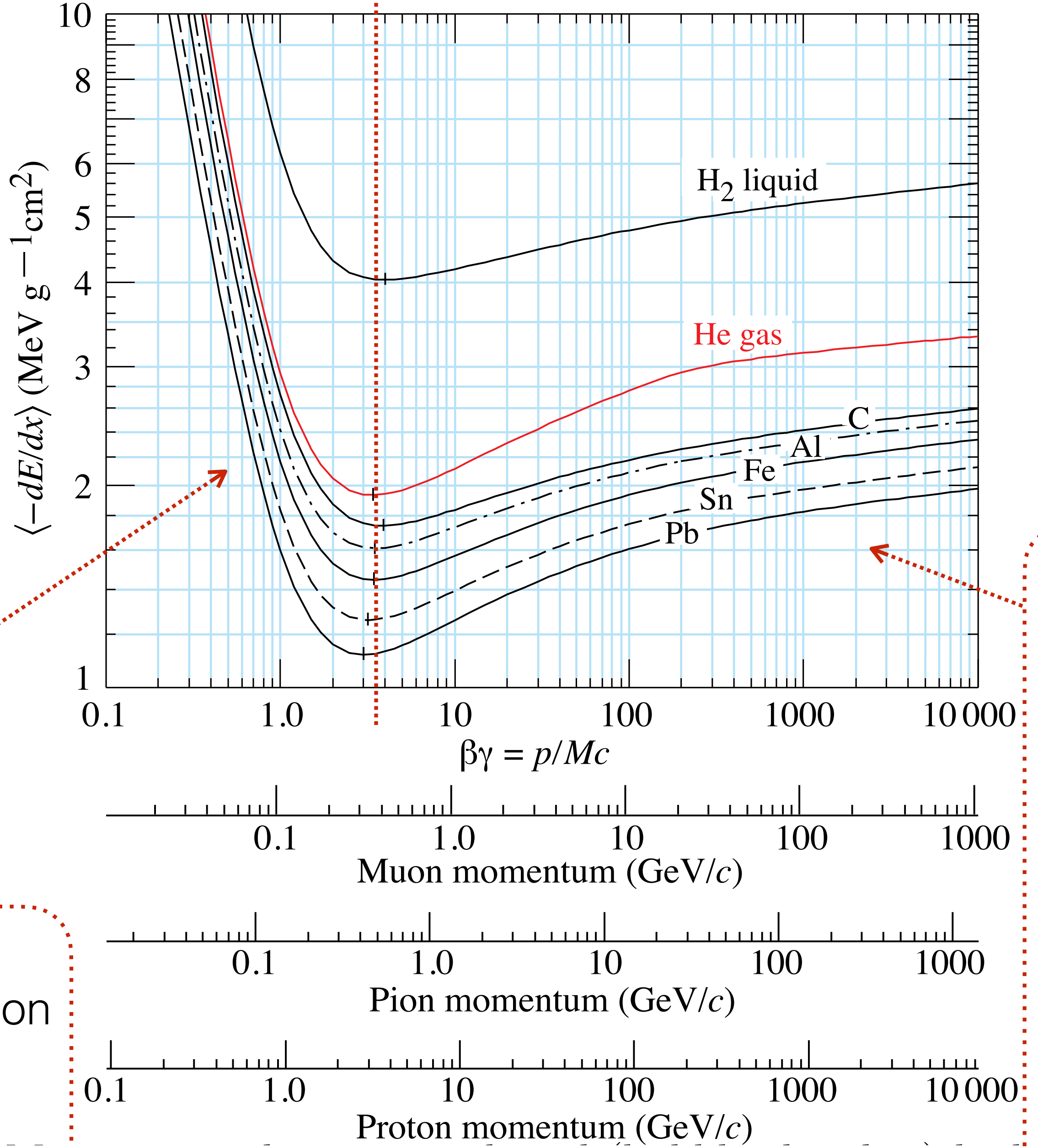
$$x = \log_{10} \eta = \log_{10}(p/Mc)$$

All parameters in the expression are material dependent and are tabulated (see e.g. PDG)

Bethe Bloch

MIP = minimum ionization particle

MeV / mass thickness



$\frac{1}{\beta^2}$ rise

1

The slower the particle the larger the Δp_T

$$\Delta p_T = \int F_T dt = \int F_T \frac{dx}{v}$$

For a given material dE/dx depends only on the speed βγ of the particle (not its momentum or energy)

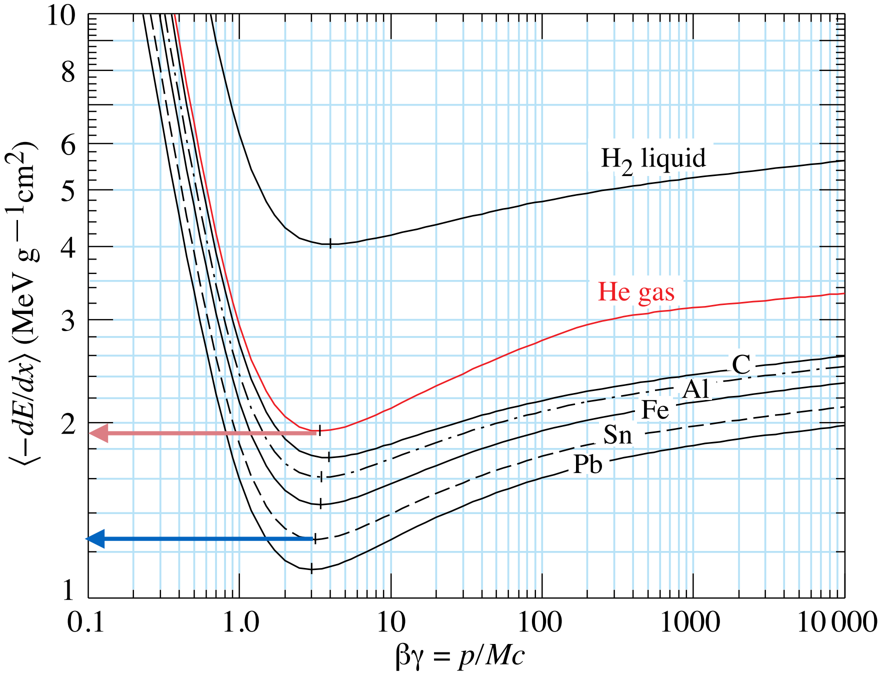
log rise reduced by the density corrections

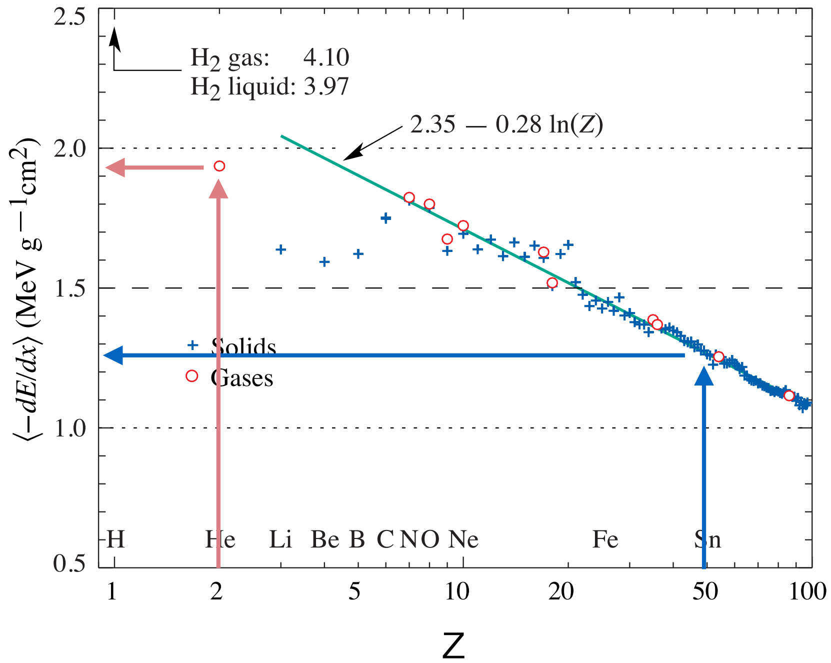
The rise comes from the increase in transversal electric field at high energy Ey' = γ Ey. Charges further away from the particle trajectory are involved

Bethe Bloch

$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

-dE/dx at the minimum ionizing point for different materials

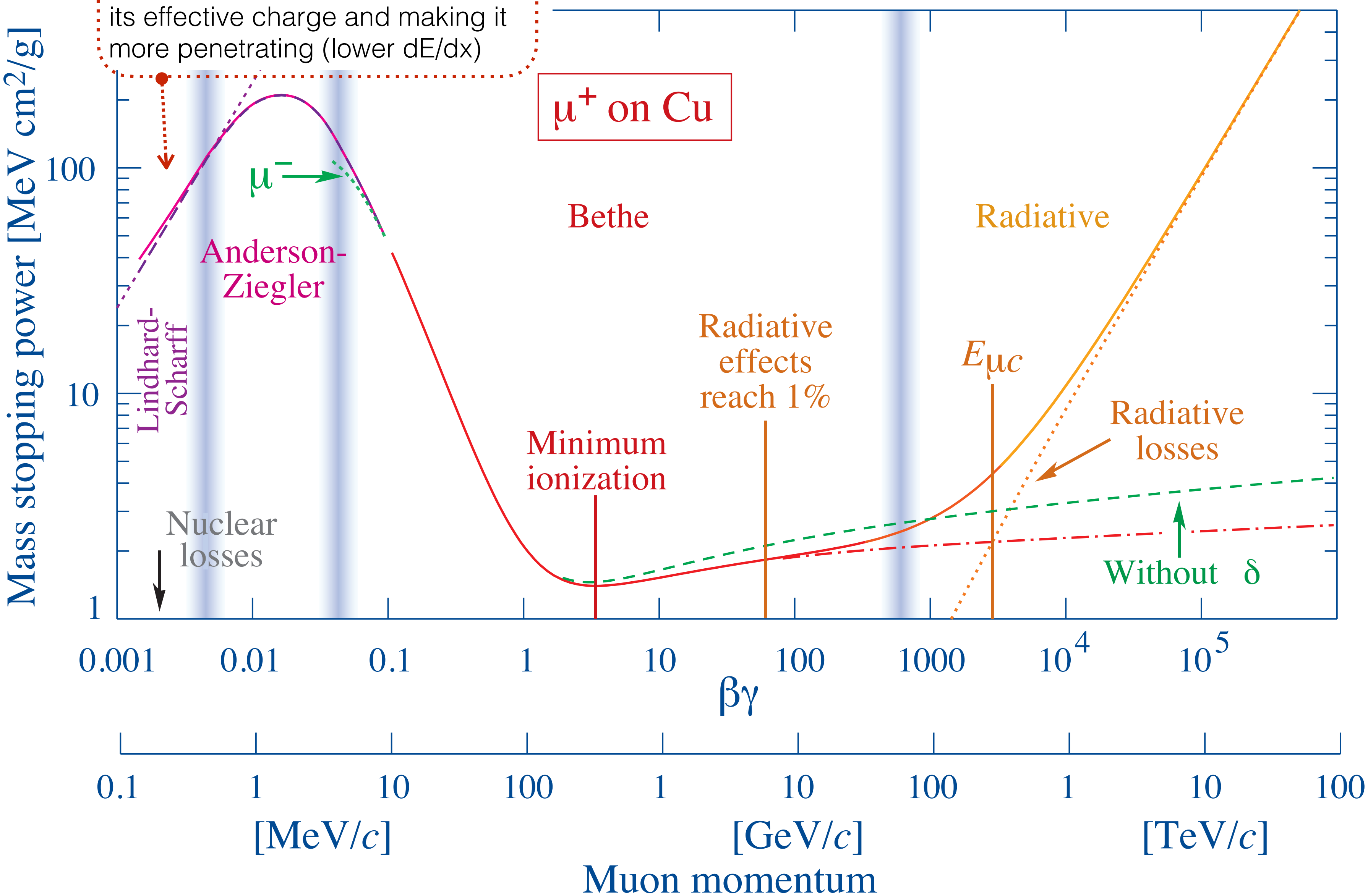




The dependency is not perfectly linear especially at low Z

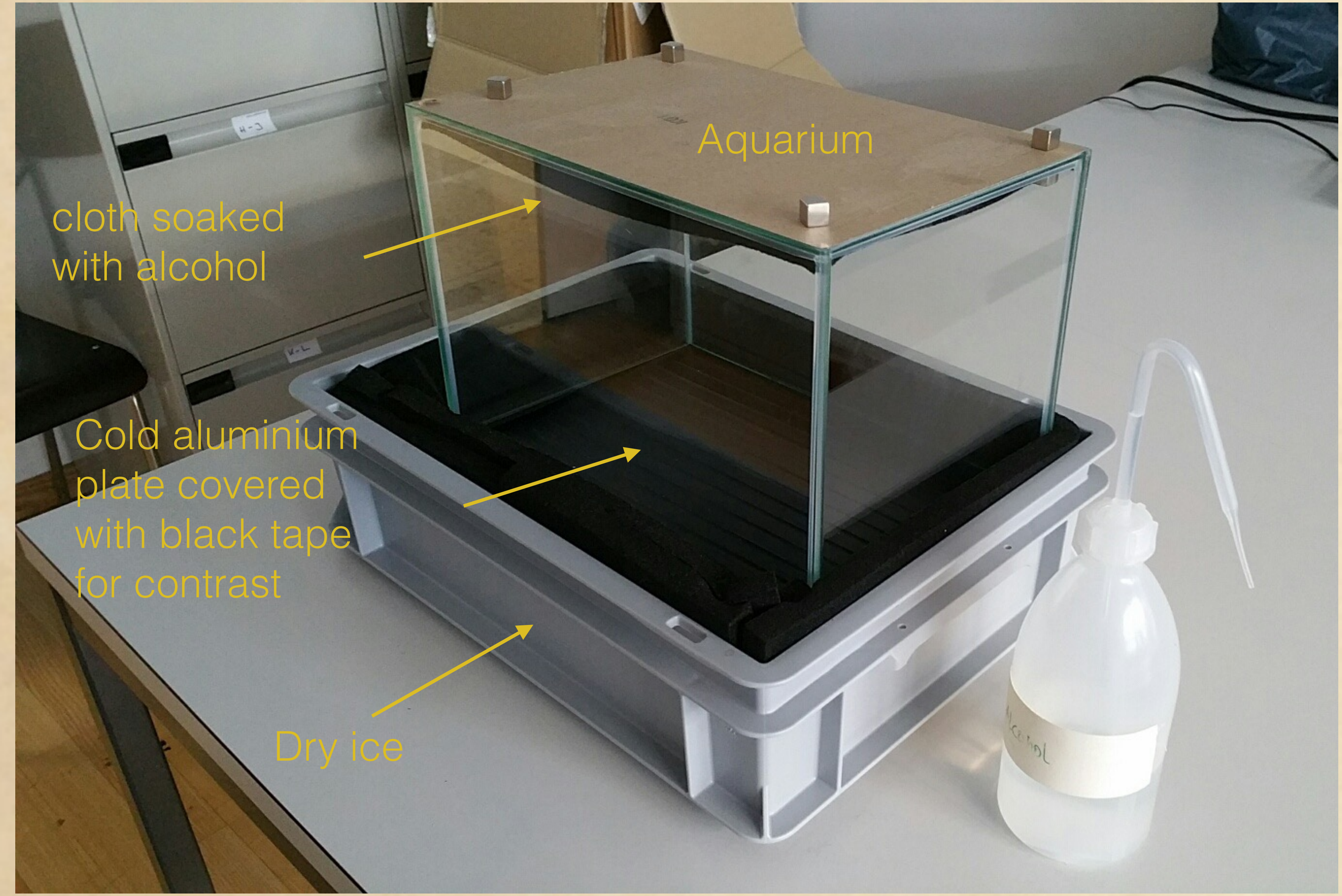
Range of validity of the Bethe Bloch

incident particle tends to pick up electrons part of the time, reducing its effective charge and making it more penetrating (lower dE/dx)



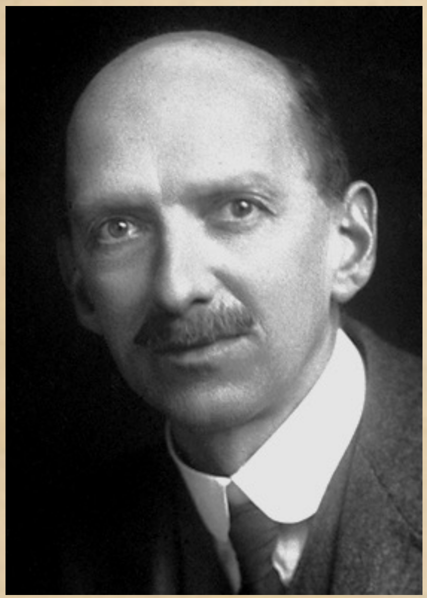
In class demonstration:
Cloud Chamber

Cloud chamber



Cloud chamber

The sealed environment contains a supersaturated vapour of alcohol (in our case ethyl alcohol). On the cold plate at the bottom a thick fog builds.
When a charged particle goes through the vapour it ionizes the alcohol (Bethe-Bloch).
The ionized atoms act as condensation centres around which a trail of small droplets is visible by naked eye. The trails persists for a couple of seconds and, while falling to the bottom by gravity, dissolve in the cloud because of diffusion.



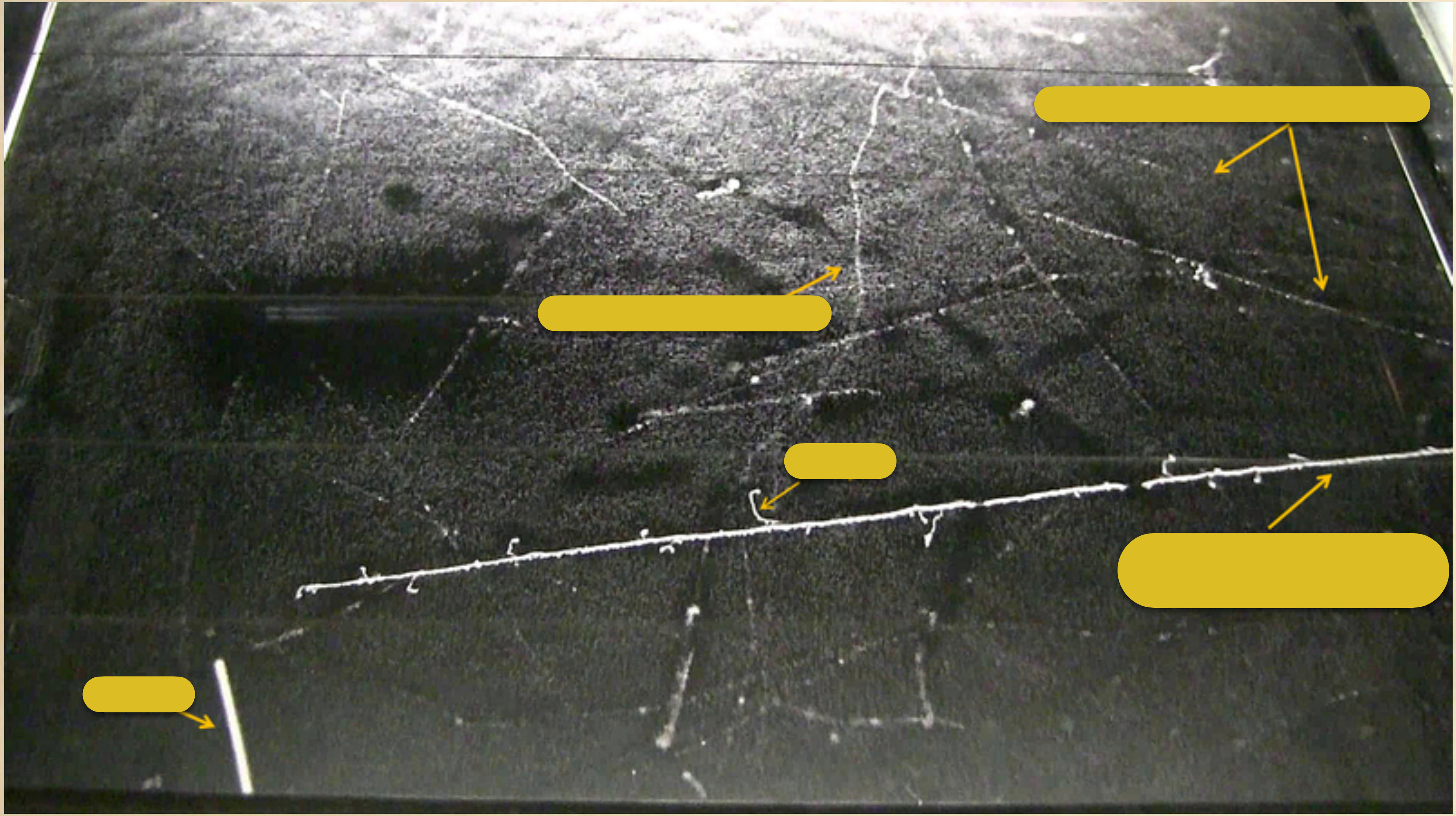
Charles Wilson (1869 –1959)
1927 Nobel prize in Physics
BSc in biology then moved to physics/meteorology and got interested in cloud formation.

Thanks to him it was possible for the first time to see single particles !

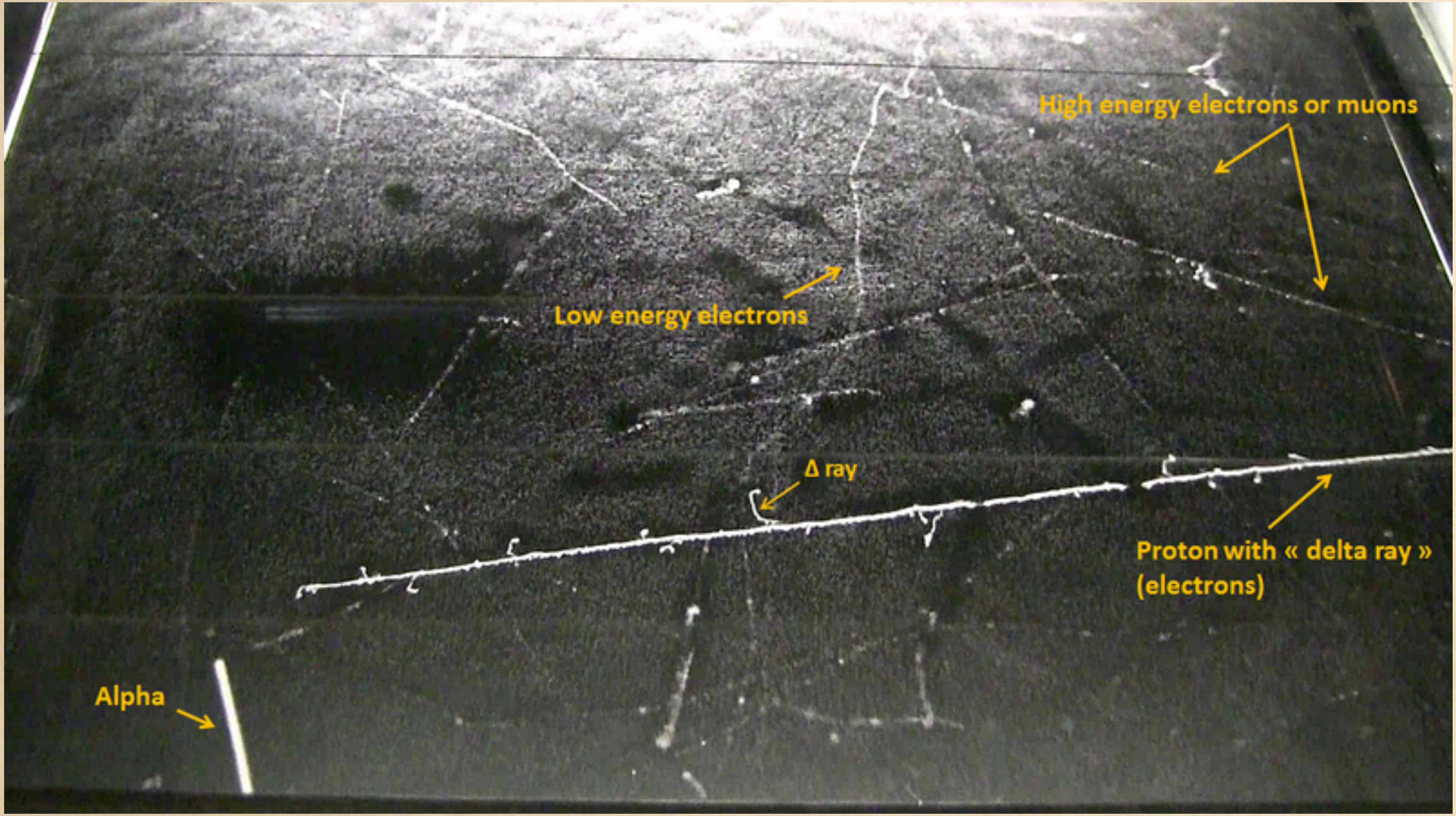
Academic advisors J. J. Thomson
Doctoral students Cecil Frank Powell

https://www.nobelprize.org/nobel_prizes/physics/laureates/1927/wilson-lecture.pdf

What particles could these be?



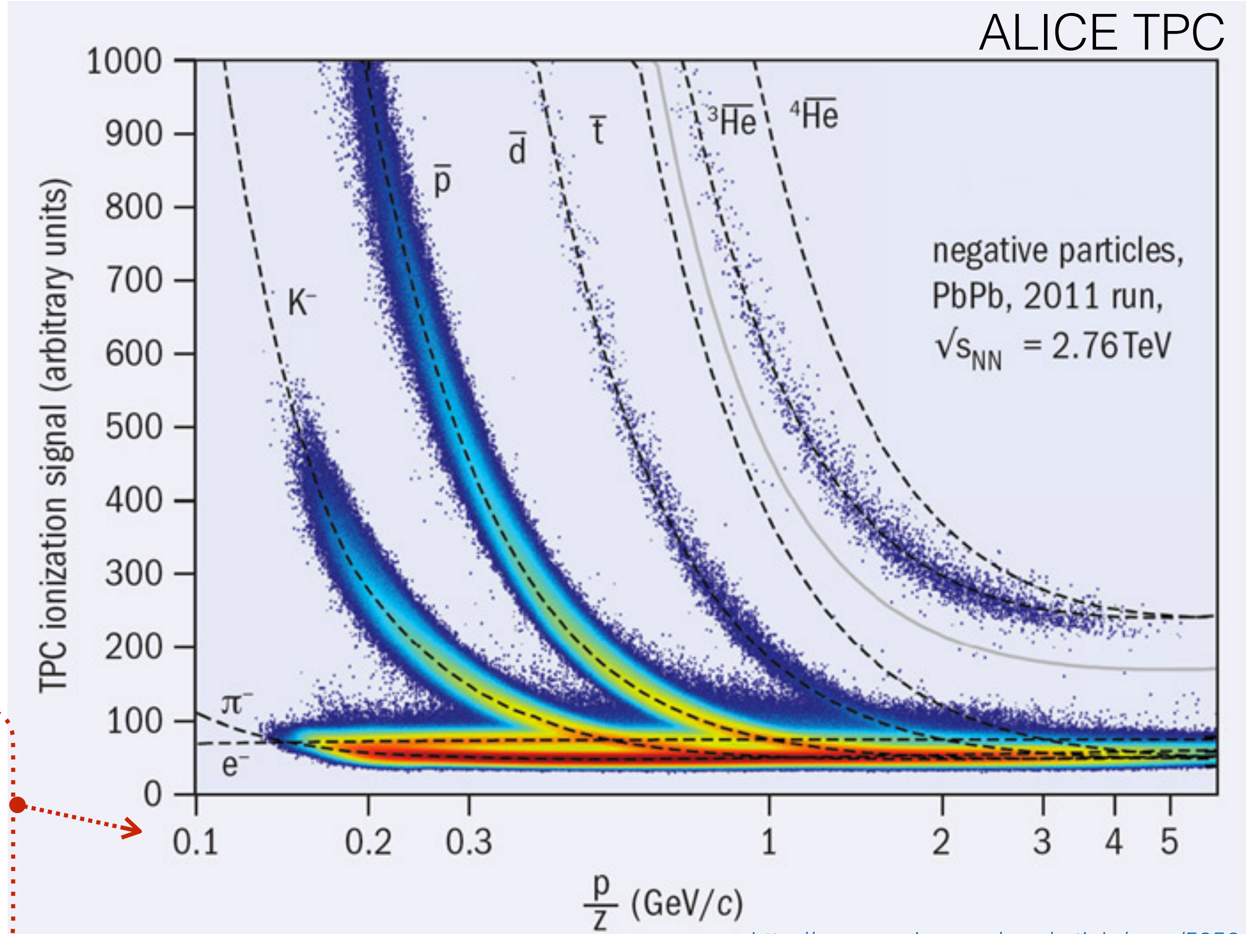
What particles could these be?



Momentum dependence

The stopping power -dE/dx is "universal" as a function of the speed of the particle, it you plot it against the momentum, you bring in a mass dependence

$\beta\gamma = p/m$



order of magnitude of the lowest momenta measured with LHC tracker

http://cerncourier.com/cws/article/cern/50561

PID = Particle identification —> get the mass and charge of a particle

Knowing momentum of a particle and measuring its dE/dx you can infer its mass

This only works at low momenta (velocity) where the $1/\beta^2$ rise strongly depends on the mass.

At high momenta there is no separation

δ-electrons

Delta electrons (or secondary electrons) are electrons from the target knocked out by the incident ionizing particle which have enough energy to further ionize.

$$\frac{d^2N}{dTdx} = \frac{1}{2} K z^2 \frac{Z}{A} \frac{1}{\beta^2} \frac{F(T)}{T^2} \qquad \text{for } I \ll T \ll W_{max}$$

The factor F(T) is about unity to $T \ll W_{max}$ and it depends on the spin of the incident particle $F(T) = 1-\beta^2\, T/T_{max}$ (for spin 0)

The angle of emission is $\cos\theta = (T_e/p_e)(p_{\max}/W_{\max})$
with p_e, T_e momentum, energy of the emitted photon; p_{max} momentum of an electron emitted with the maximum energy trasfer W_{max}

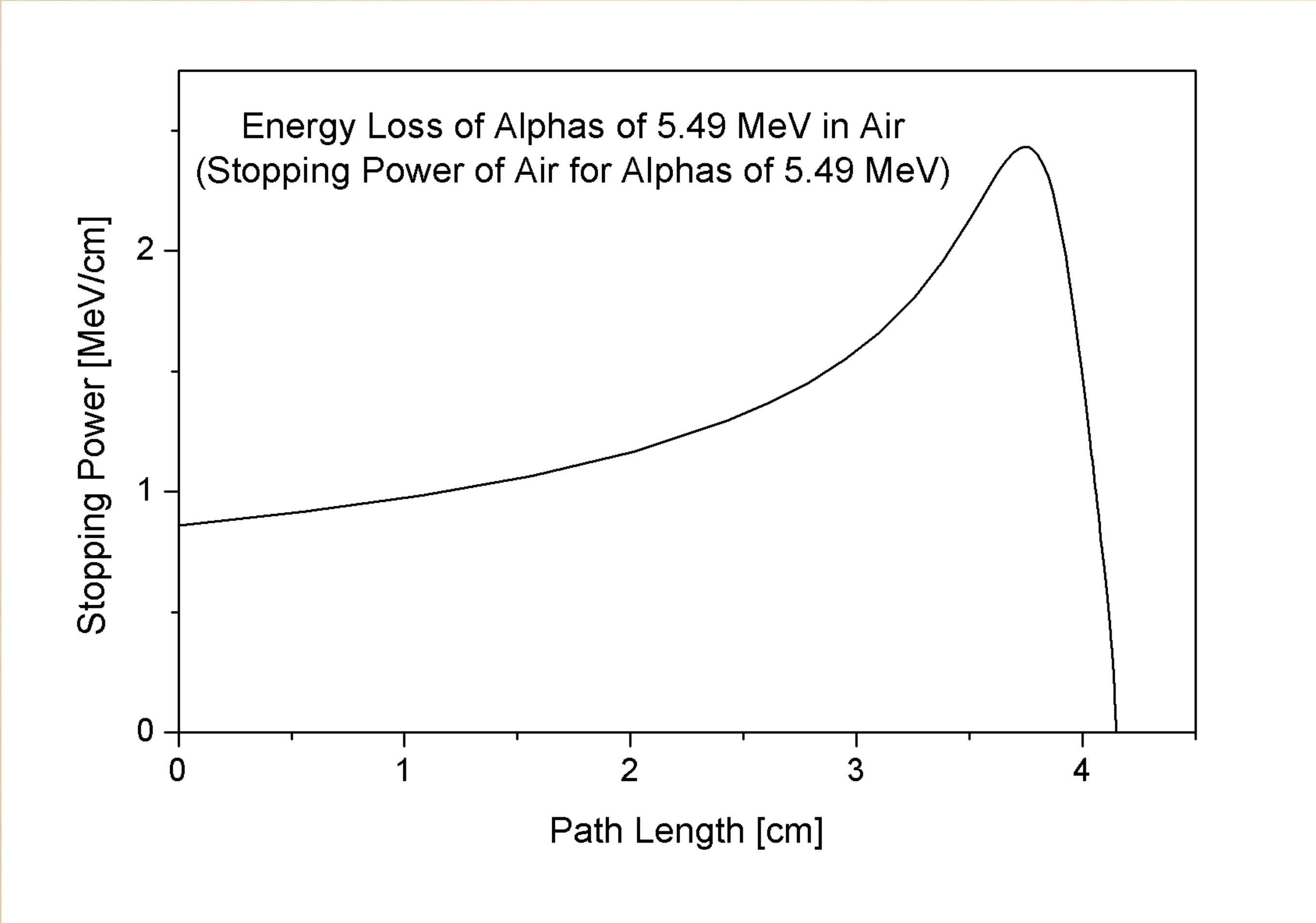
Bragg peak

-dE/dx vs. penetration depth: at the beginning of the path the particle will have its higher energy and sit on the relativistic (log) rise. Then it will slow down and rolls down in -dE/dx to the minimum ionizing point and finally, when very slow enter the $1/\beta^2$ rise.

Q: Can you plot the dE/dx as a function of the depth in matter ?

Bragg peak

Q: Can you plot the dE/dx as a function of the depth in matter ?



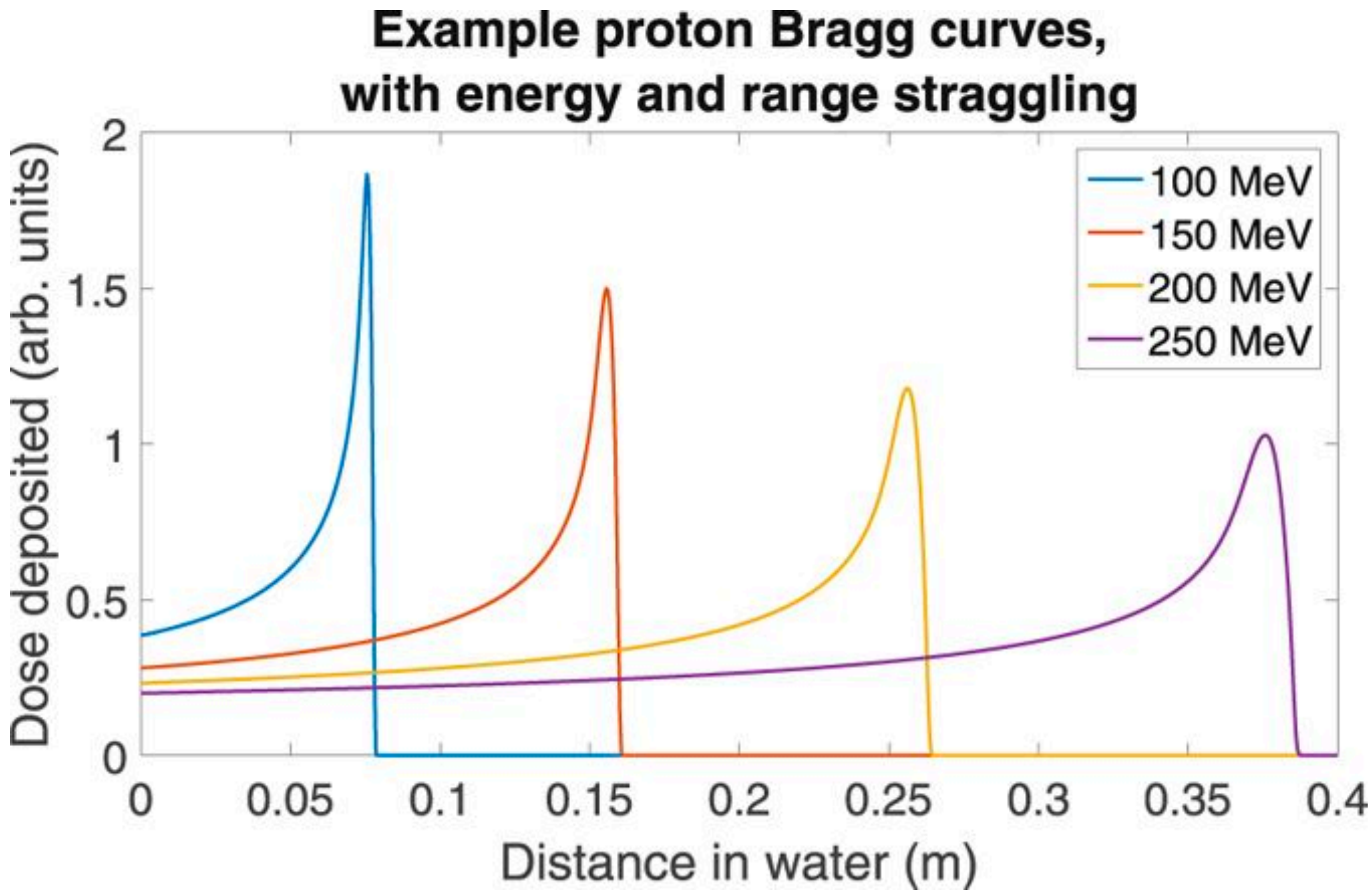
Bragg peak

-dE/dx vs. penetration depth: at the beginning of the path the particle will have its higher energy and sit on the relativistic (log) rise. Then it will slow down it and roll down in -dE/dx to the minimum ionizing point and finally, when very slow enter the $1/\beta^2$ rise.

The $1/\beta^2$ rise you might have noticed with the cloud chamber:
(see movie at min 01:57)

Bragg peak

An important application of this is used in cancer treatment (R. Wilson 1946).



https://www.nature.com/articles/s41598-017-10554-0

The target material for these measurements is water as a proxy for human body

Mixtures and compounds

Tabulated values for the -dE/dx for mixtures and compounds come from direct measurements, but a good approximation can be obtained by the weighted average of the -dE/dx of the elements (Bragg additivity):

$$\frac{1}{\rho}\frac{dE}{dx} = \frac{w_1}{\rho_1}\left(\frac{dE}{dx}\right)_1 + \frac{w_2}{\rho_2}\left(\frac{dE}{dx}\right)_2 + \dots$$

where w_i are the fractions by weight of the elements in the compound

We will see that the weighted average will be used to get to the behaviour of compounds given the values of the single elements for several microscopic quantities.

Range

Q: how would you (ideally) measure the range of a proton in copper ?

Range

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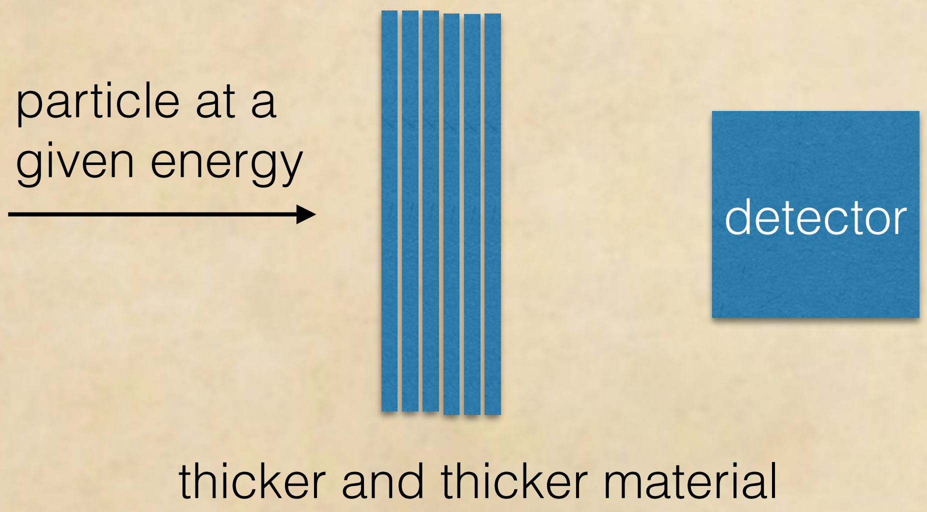


Range

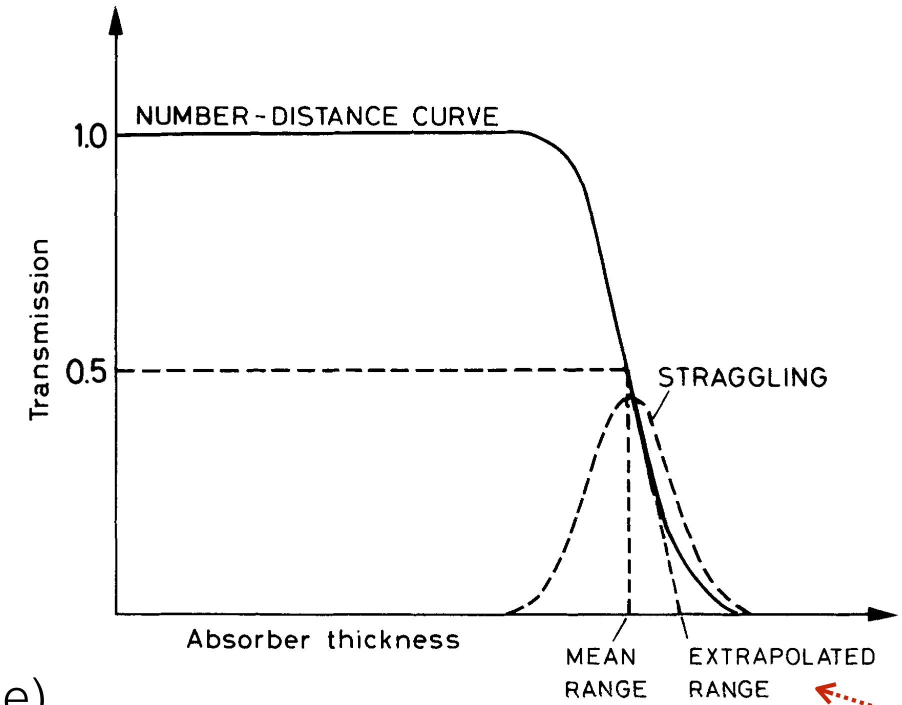
The range can be easily obtained integrating the stopping power curve

$$S(T_0) = \int_0^{T_0} \left(\frac{dE}{dx}\right)^{-1} dE \qquad \text{T = kinetic energy}$$

(ignoring the fact that the particle doesn't go straight, but it bounces around in a zig-zag path)

The smearing observed at the end of the curve, comes from the fact that the energy loss is a statistical process ("range straggling").

The range is typically given as the 50% point, i.e. where 50% of the particles are adsorbed.



Important to get a first estimate of the size of detectors / shielding etc.. (i.e. before running a complete simulation —> see later in the course)

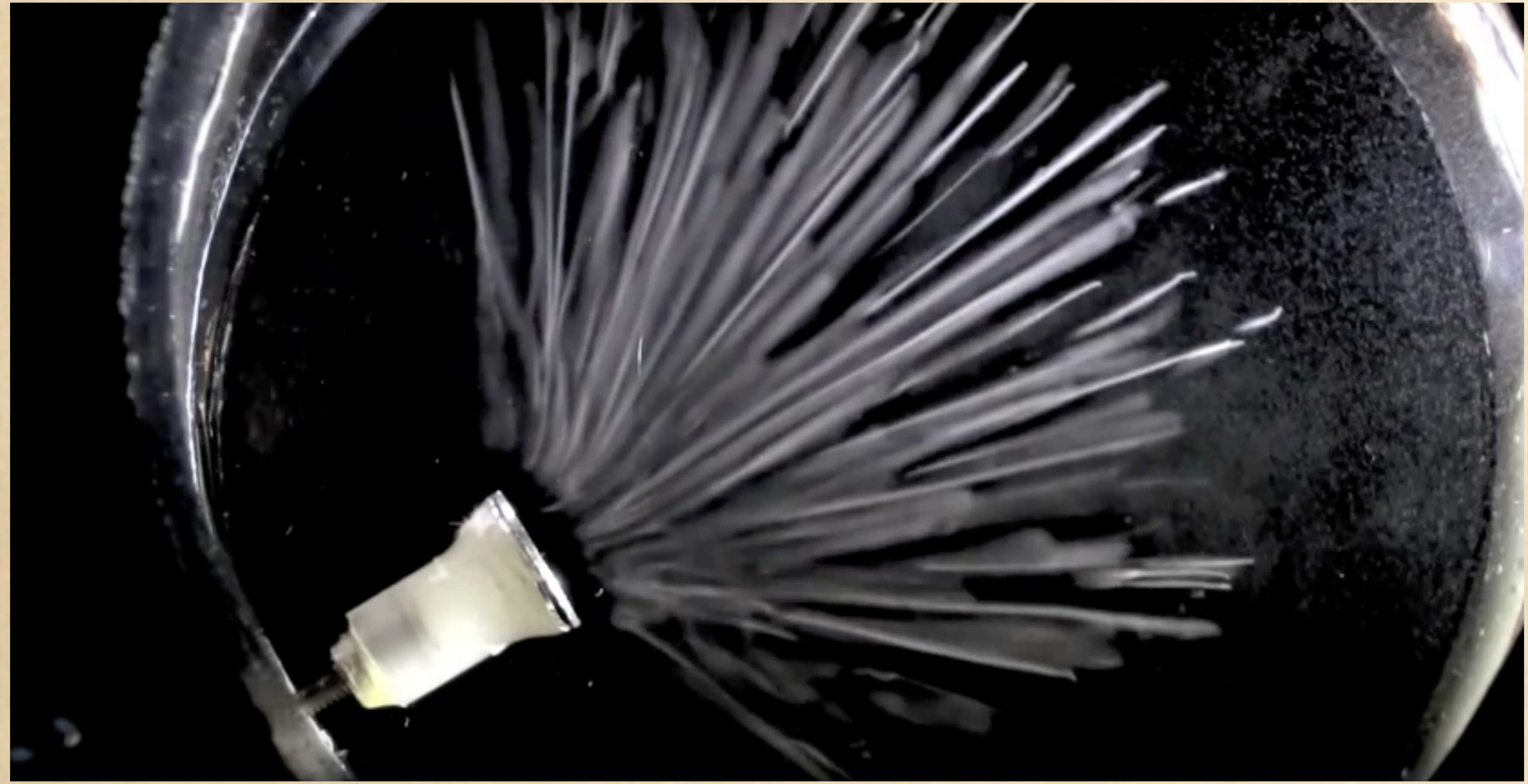
Q: how would tracks from an (~monocromatic) alpha emitter appear in the cloud chamber ?

particles = background level of the detector

Alphas

Q: how would tracks from an (~monocromatic) alpha emitter appear in the cloud chamber ?

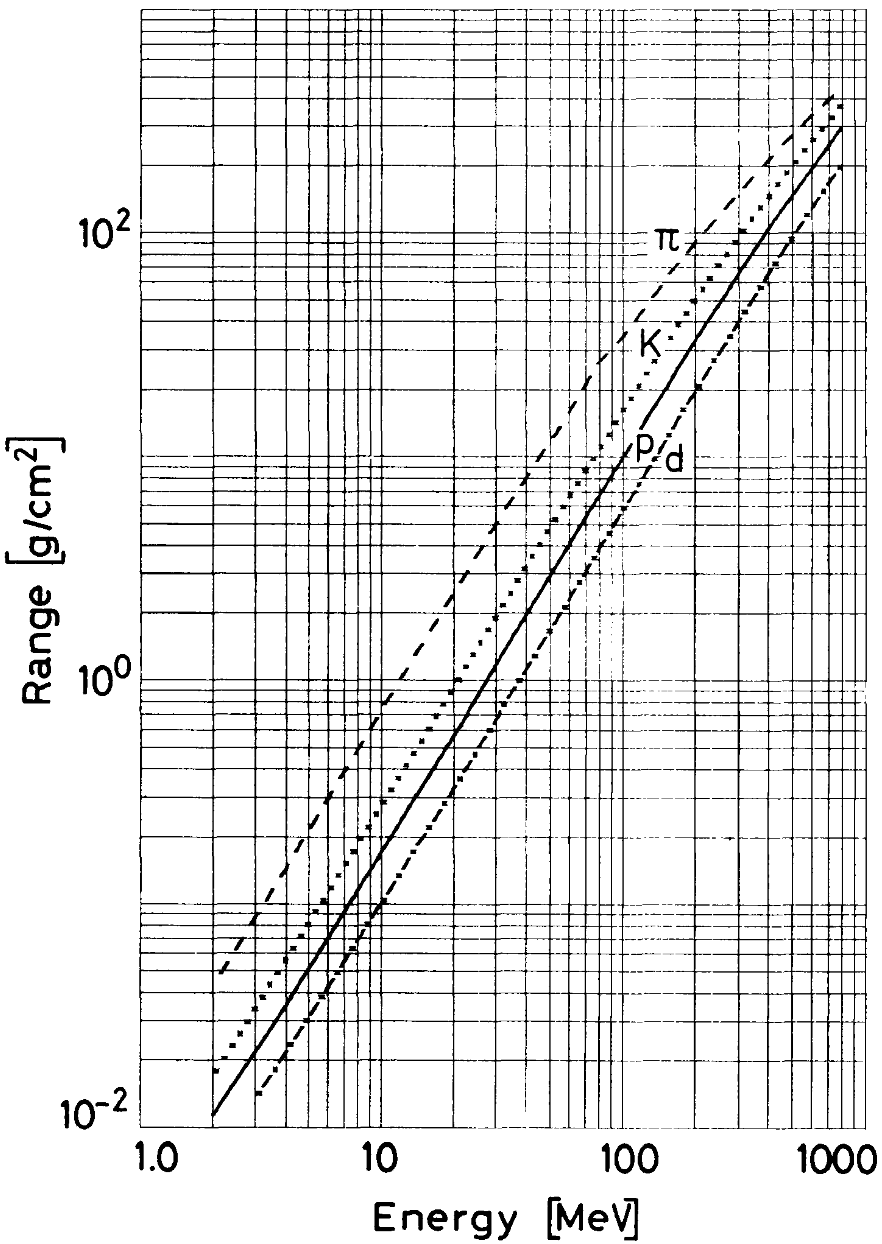
$$^{241}_{95}\mathrm{Am} \xrightarrow{432.2\mathrm{y}} {}^{237}_{93}\mathrm{Np} + {}^{4}_{2}\alpha^{2+} + \gamma\ 59.5409\ \mathrm{keV}$$



Range

Example of calculated range for different heavy particles in Al

Al density = 2.7 g/cm³



Electrons: bremsstrahlung and radiation length

Electrons/Positrons

(Here, unless otherwise specified, I will say electrons to mean both electrons and positrons)

$$\left(\frac{dE}{dx}\right)_{\rm tot} = \left(\frac{dE}{dx}\right)_{\rm rad} + \left(\frac{dE}{dx}\right)_{\rm coll}$$

On top of energy loss by collisions (as heavy particles) electrons also lose energy by bremsstrahlung. We will define "critical energy" the energy where the energy loss by collisions is equal to the energy loss by bremsstrahlung

$$\left(\frac{dE}{dx}\right)_{\rm rad} = \left(\frac{dE}{dx}\right)_{\rm coll} \qquad \text{for} \quad E = E_{\rm c}$$

Electrons/Positrons

$$\left(\frac{dE}{dx}\right)_{\rm tot} = \left(\frac{dE}{dx}\right)_{\rm rad} + \boxed{\left(\frac{dE}{dx}\right)_{\rm coll}}$$

Collision loss

You can follow the same computation of the Bethe Bloch, but:

- mass incident particle = mass target = m_e (i.e. incident electron will bounce around)
- incident particle and target are indistinguishable

The maximum energy transfer is now $W_{\rm max} = T_e/2$ where T_e is the kinetic energy of the incident electron

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{\tau^2(\tau+2)}{2(I/m_e c^2)^2} + F(\tau) - \delta - 2\frac{C}{Z} \right]$$

$\tau = T_e/m_e c^2$

$$F(\tau) = 1 - \beta^2 + \frac{\frac{\tau^2}{8} - (2r+1)\ln 2}{(\tau+1)^2} \quad \text{for } e^-$$

$$F(\tau) = 2\ln 2 - \frac{\beta^2}{12}\left(23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3}\right) \quad \text{for } e^+$$

Electrons/Positrons

$$\left(\frac{dE}{dx}\right)_{\text{tot}} = \boxed{\left(\frac{dE}{dx}\right)_{\text{rad}}} + \left(\frac{dE}{dx}\right)_{\text{coll}}$$

Radiation Loss : Bremsstrahlung $\qquad \sigma_{\text{brem}} \propto \left(\frac{e^2}{mc^2}\right)^2$

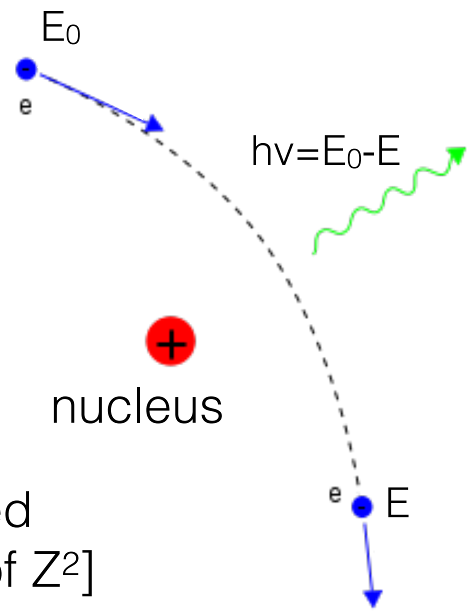
The only particle for which radiation loss is not negligible (up to several hundreds of GeV) is the electron. Already for muons the contribution is 40000 times lower (~100MeV/ 0.5MeV)^2

Bremsstrahlung depends on the electric field seen by the incident electron: the screening of the electrons around the nucleus play an important role.
Screening is described with

$$\xi = \frac{100\, m_e c^2 h\nu}{E_0 E Z^{1/3}}$$

$\xi \simeq 0$ $\quad$ E_0-E —>0 small screening

$\xi \gg 1$ $\quad$ E —>0 $\quad$ no screening



[the contribution of bremsstrahlung on electrons can be neglected but in the lightest nuclei. e-e bremsstrahlung goes as Z instead of Z^2]

Electrons/Positrons

The complete quantum mechanical description was first performed by Bethe and Heitler
Cross section for bremsstrahlung emission:

$$d\sigma = 4\,Z^2 r_e^2 \alpha \frac{d\nu}{\nu}\left\{(1+\varepsilon^2)\left[\frac{\phi_1(\xi)}{4}-\frac{1}{3}\ln Z - f(Z)\right] - \frac{2}{3}\varepsilon\left[\frac{\phi_2(\xi)}{4}-\frac{1}{3}\ln Z - f(Z)\right]\right\}$$

depends on 1/m² and the inverse of the photon energy

where:

- $r_e^2 = (e^2/mc^2)^2$
- $\varepsilon = E/E_0$
- $\phi_1(\xi)$, $\phi_2(\xi)$ = screening functions. Empirical expressions:

$$\phi_1(\xi) = 20.863 - 2\ln[1+(0.55846\,\xi)^2] - 4[1-0.6\exp(-0.9\,\xi) - 0.4\exp(-1.5\,\xi)]$$

$$\phi_2(\xi) = \phi_1(\xi) - \tfrac{2}{3}(1+6.5\,\xi+6\,\xi^2)^{-1}, \qquad (2.69)$$

where

$$\phi_1(0) = \phi_2(0) + \tfrac{2}{3} = 4\ln 183 \qquad \text{as } \xi \to 0$$

$$\phi_1(\infty) = \phi_2(\infty) \to 19.19 - 4\ln\xi \qquad \text{as } \xi \to \infty\,.$$

- f(Z) = correction factor to model the interaction of the emitting (a = Z/137)

$$f(Z) \simeq a^2[(1+a^2)^{-1} + 0.20206 - 0.0369\,a^2 + 0.0083\,a^4 - 0.002\,a^6]$$

Electrons/Positrons

The complete quantum mechanical description was first performed by Bethe and Heitler
Cross section for bremsstrahlung emission:

$$d\sigma = 4 Z^2 r_e^2 \alpha \frac{d\nu}{\nu} \left\{ (1+\varepsilon^2) \left[\frac{\phi_1(\xi)}{4} - \frac{1}{3} \ln Z - f(Z) \right] - \frac{2}{3} \varepsilon \left[\frac{\phi_2(\xi)}{4} - \frac{1}{3} \ln Z - f(Z) \right] \right\}$$

The energy loss by bremsstrahlung can be computed as:

$$-\left(\frac{dE}{dx}\right)_{\mathrm{rad}} = N \int_0^{\nu_0} h\nu \frac{d\sigma}{d\nu}(E_0, \nu)\, d\nu$$

where

N = total number of atoms/cm^3 $=\rho N_A/A$

$\nu_0 = E_0/h$

Note that $d\sigma \propto 1/\nu \Rightarrow$ the ν dependence in the integral vanishes

$$\boxed{-\left(\frac{dE}{dx}\right)_{\mathrm{rad}} = N E_0 \Phi_{\mathrm{rad}}}$$

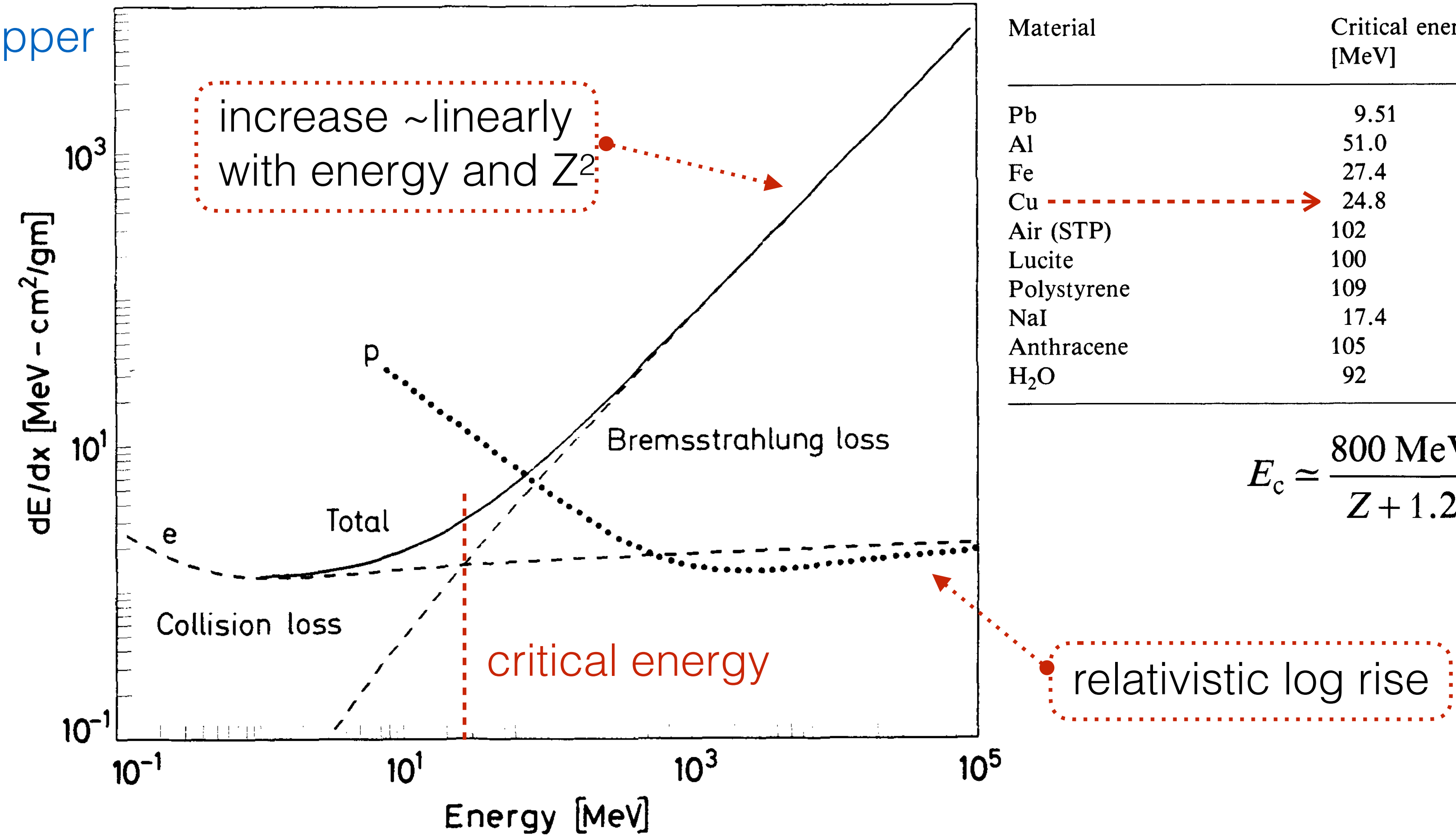
$$\Phi_{\mathrm{rad}} = \frac{1}{E_0} \int h\nu \frac{d\sigma}{d\nu}(E_0, \nu)\, d\nu = 4 Z^2 r_e^2 \alpha \left(\ln \frac{2E_0}{m_e c^2} - \frac{1}{3} - f(Z) \right)$$

[the contribution of bremsstrahlung on electrons can be neglected
but in the lightest nuclei. e-e bremsstrahlung goes as Z instead of Z^2]

Electrons/Positrons

Compare bremsstrahlung with collision loss:

electrons in copper

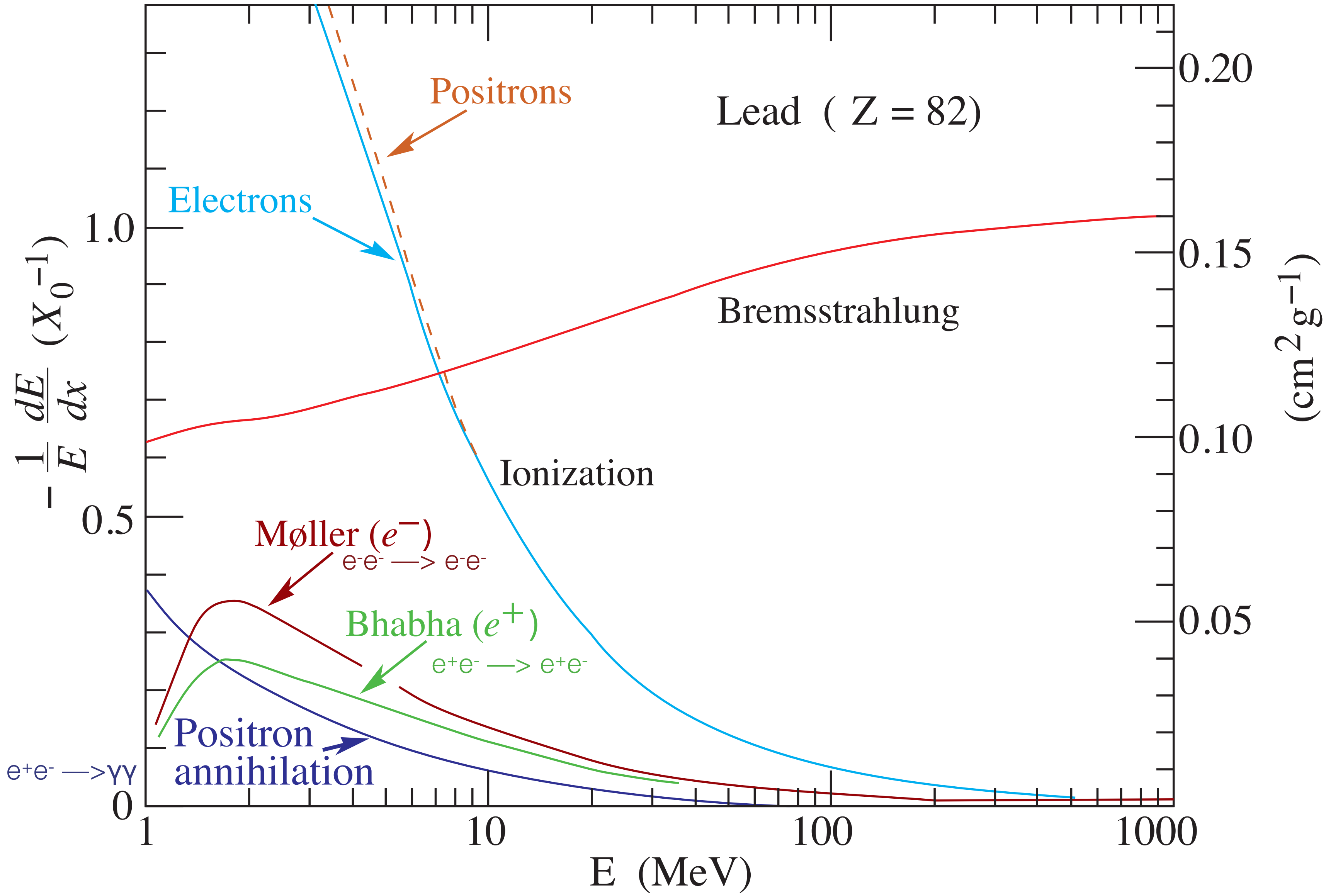


Material	Critical energy [MeV]
Pb	9.51
Al	51.0
Fe	27.4
Cu	24.8
Air (STP)	102
Lucite	100
Polystyrene	109
NaI	17.4
Anthracene	105
H_2O	92

$$E_c \simeq \frac{800\ \text{MeV}}{Z+1.2}$$

Energy loss by collision loss comes from the sum of a large number of scatterings.
Bremsstrahlung energy can be emitted all in one or few photons —> much larger fluctuations the energy in electron beams

Electrons/Positrons



Radiation length

Radiation length = distance over which the electron energy is reduced by 1/e due to radiation only

$$-\left(\frac{dE}{dx}\right)_{\mathrm{rad}} = N E_0 \Phi_{\mathrm{rad}} \quad \longrightarrow \quad E = E_0 \exp\left(\frac{-x}{X_0}\right) \qquad X_0 = 1/N\,\Phi_{\mathrm{rad}}$$

An approximated formula to compute it is useful to see what it depends on:

$$X_0 = \frac{716.4\ \mathrm{g/cm^2}\ A}{Z(Z+1)\ln(287/\sqrt{Z})}$$

The concept of radiation length is used to express material thickness in detectors, because is it roughly independent from the material type:

$$-\left(\frac{dE}{dx}\right)_{\mathrm{rad}} = N E_0 \Phi_{\mathrm{rad}} \quad \longrightarrow \quad -(dE/dt) \simeq E_0 \qquad \text{with } t = x / X_0$$

"fraction of a radiation length"

For mixtures/compounds we can use the same weighted average as for ionization:

$$\frac{1}{X_0} = w_1\left(\frac{1}{X_0}\right)_1 + w_2\left(\frac{1}{X_0}\right)_2 + \cdots$$

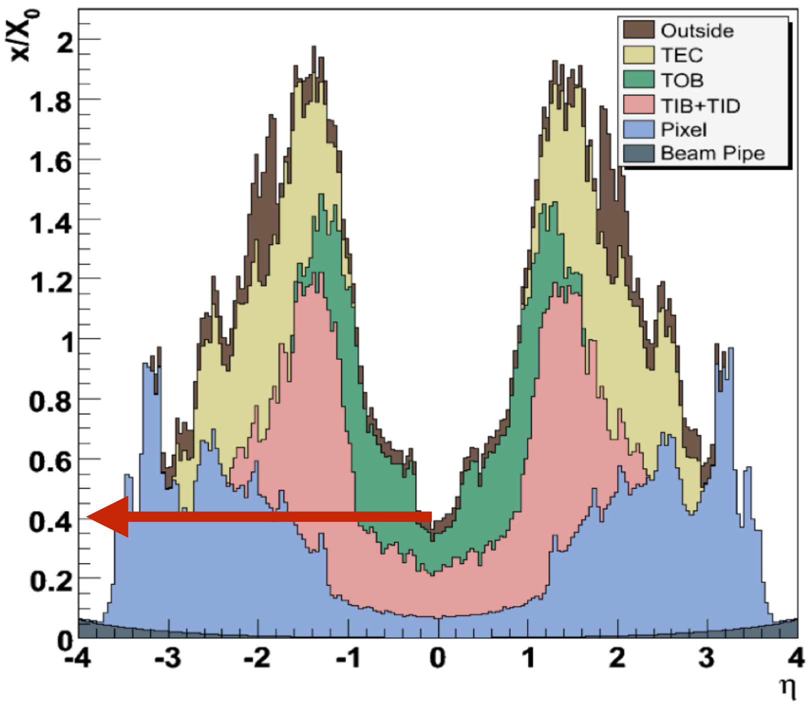
w_i = fraction by weight of the element in the mixture

Radiation length

The radiation length X_0 [g/cm2] is an intrinsic characteristic of the material, x/X_0 is a normalized unit of length used to measure the size of a piece of material.

Amount of material in the CMS tracker simulations expressed in "fraction of radiation length"

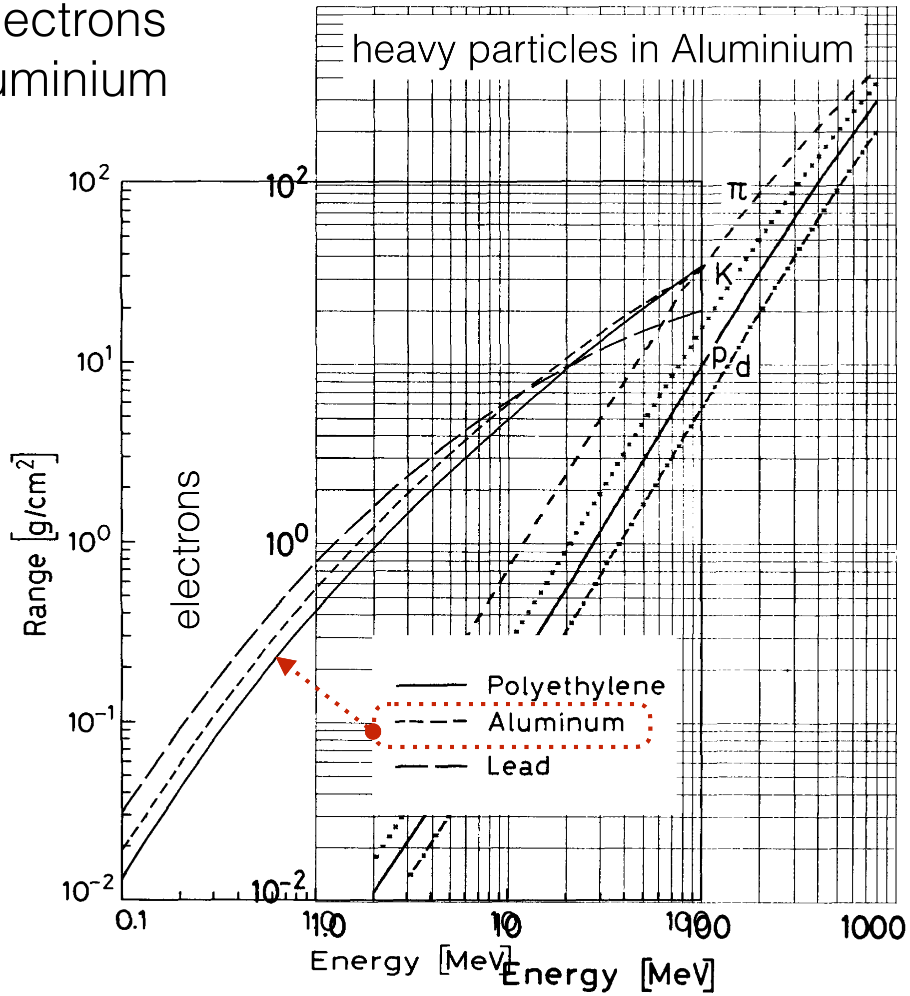
40% of a radiation length



We express the amount of material in a detector in fraction of a radiation length because it can be immediately translated to the effect it has on the traversing particle.
The actual physical size of the detector depends on the material used:
for the same radiation length you can have a short/thin (e.g. solid) or long/thick (e.g. gas) detector.

Range of electrons

Range comparison for electrons vs. heavy particles in Aluminium



Because of the much lighter mass electrons trajectories in matter are more affected by multiple scattering —> Path from the integral of dE/dx is not accurate and the energy fluctuations are large.

Multiple scattering

Elastic (Coulomb) scattering of the incident particle with nuclei (without spin effects) is simple Rutherford scattering.

$$\frac{d\sigma}{d\Omega} = z_2^2 z_1^2 r_e^2 \frac{(m_e c/\beta p)^2}{4 \boxed{\sin^4(\theta/2)}}$$

most of the collisions only make a small deflection

$m_{nucleus} >> m_{incident}$
energy transferred to the nucleus is negligible

The typical description of the scattering is divided in :

- Single scattering —> thin target/foil, mostly one scatter described by Rutherford
- Multiple scattering —> thick targets, many small-angle scatters: scattering distribution is Gaussian. Less frequent "hard" scatters produce non-Gaussian tails.
-

Multiple scattering

The typical description of multiple scattering is given the the gaussian approximation:

Defining $$\theta_0 = \theta^{\rm rms}_{\rm plane} = \frac{1}{\sqrt{2}}\theta^{\rm rms}_{\rm space}$$ $$\theta^2_{\rm space} \approx (\theta^2_{{\rm plane},x} + \theta^2_{{\rm plane},y})$$

we found that a good approximation for the 98% core of the gaussian is given by:

$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta c p}\, z\, \sqrt{x/X_0}\Big[1 + 0.038\ln(x/X_0)\Big]$$

incident particle

target radiation length

accurate to 11% or better for $10^{-3} < x/X_0 < 100$.

Assuming the deflections in the two planes are independent and identically distributed

$$\frac{1}{2\pi\,\theta_0^2}\exp\left(-\frac{\theta^2_{\rm space}}{2\theta_0^2}\right)d\Omega\ ,$$

$$\frac{1}{\sqrt{2\pi}\,\theta_0}\exp\left(-\frac{\theta^2_{\rm plane}}{2\theta_0^2}\right)d\theta_{\rm plane} \qquad d\Omega \approx d\theta_{{\rm plane},x}\, d\theta_{{\rm plane},y}$$

Energy loss distribution

So far we looked at the mean energy loss by an incident particle in a layer of material. Now we look at how the energy is actually statistically distributed.

The distribution is qualitatively different when thick or thin layers are considered:
- thick layers: distribution mostly gaussian created by several scatters
- think layers: gaussian core with large high energy tail created by single scatters

Defining $\kappa = \bar{\Delta}/W_{\max}$ as the ratio between the mean energy loss and the maximum allowed in a single collision, we talk about thin layer when κ< 10 it's considered.

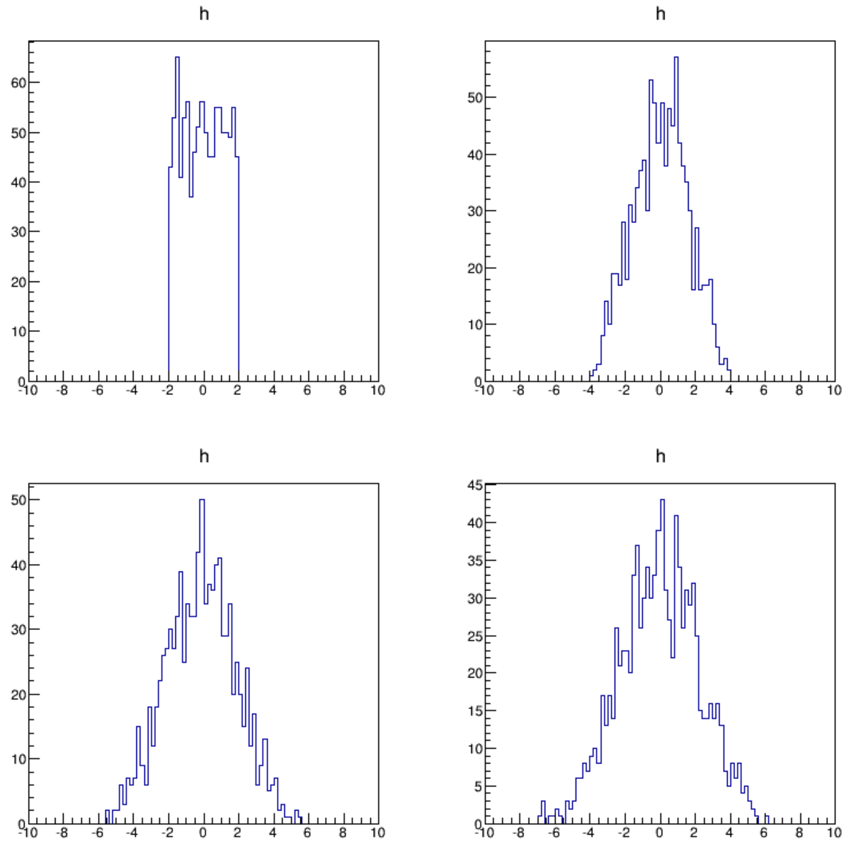
CLT: Central Limit Theorem
This is probably the most important theorem in statistics and it is the reason why the gaussian is so important

Take n independent variables x_i, distributed according to p.d.f.'s f_i having mean μ_i and variance σ_i^2, then the p.d.f. of the sum of the x_i, $S = \sum x_i$, has mean $\sum \mu_i$ and variance $\sum \sigma_i^2$ and it approaches the normal p.d.f. $N(S; \sum \mu_i, \sum \sigma_i^2)$ as $n \to \infty$.

CLT is valid when none of the variables dominate the sum - Lindberg criteria

Energy loss distribution

CLT: Central Limit Theorem



Sum of random variables from a uniform distribution in [−2,2] after the iterations 1 to 4.

Energy loss distribution

Thick layers: the total energy loss is the sum of a large number of collisions, none of which is dominating the sum,

In this conditions the energy is distributed as a gaussian:

$$f(\Delta) \propto \exp\left(\frac{-(\Delta-\bar{\Delta})^2}{2\sigma^2}\right)$$

where
Δ is the energy loss in the material
$\bar{\Delta}$ is the mean energy loss
σ is the standard deviation

The standard deviation can be computed (Bohr) for non relativistic incident particles as:

$$\sigma_0^2 = 4\pi N_a r_e^2 (m_e c^2)^2 \rho \frac{Z}{A} x = 0.1569 \rho \frac{Z}{A} x \text{ [MeV}^2\text{]}$$

N_a = Avogadro's number, r_e is the classical electron mass, m_e electron mass,
x,ρ,Z,A = thickness, density, Z, A of the target

and extended to relativistic particles as

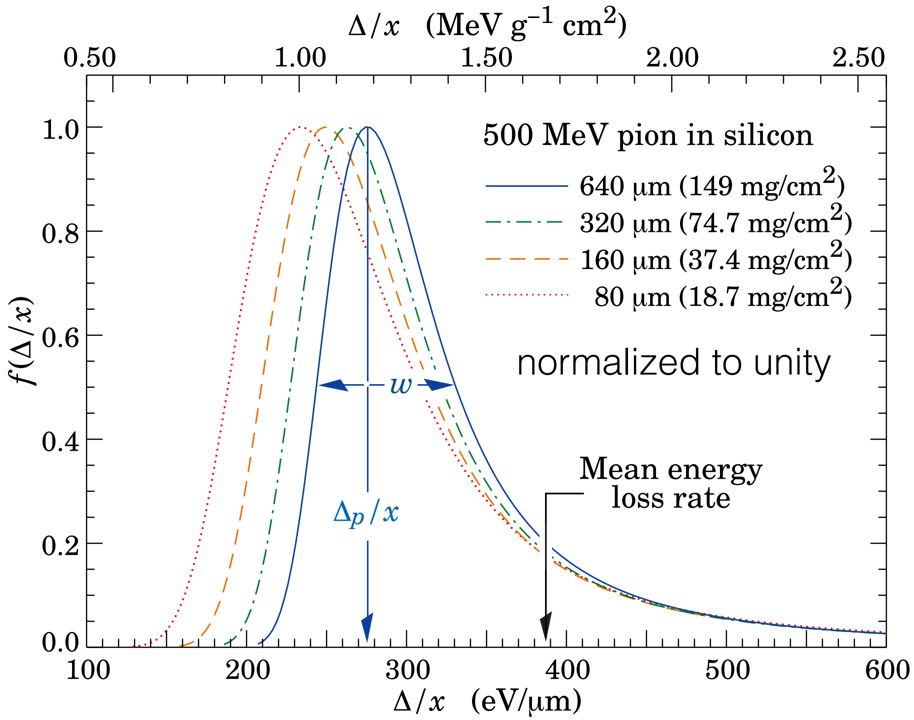
$$\sigma^2 = \frac{(1-\frac{1}{2}\beta^2)}{1-\beta^2}\sigma_0^2$$

Energy loss distribution

Thin layers: the number of collisions is small and CLT does not apply. There is the possibility that the energy deposited by a single scatter (with delta electrons emission) dominates the energy distribution.
In this case the distribution of the energy loss is given by the Landau-Vavilov theory:

$$p(x) = \frac{1}{\pi} \int_0^\infty e^{-t \log t - xt} \sin(\pi t)\, dt$$

NB: the mean and variance are not defined. For this reason in practical applications the distribution if often truncated at some high value of the energy.



Because of the large high energy tail, the distribution is asymmetric (skewed) and the mean energy loss is higher than the most probable (mode) energy loss

When quoting energy loss the most probable value should be used.

(more on this in later in the course)

Photons

Photon interactions

The photons we are interested in have energies > few keV (X-rays, gamma rays).

Photons are neutral, so there are no multiple small interactions with matter, when the photon interacts is gone. A practical consequence is that a photon beam cannot be degraded in energy, only in intensity:

$$I(x) = I_0 \exp(-\mu x)$$

where x is the thickness of the absorber and μ is the absorption coefficient which depends on the absorbing material and it's related to the total interaction cross section.

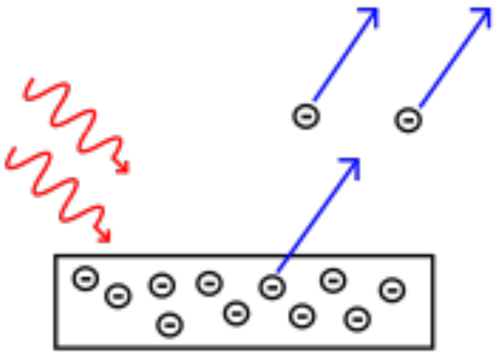
The main interactions above a few keV are:

- photoelectric effect
- Compton scatttering
- pair production (photon conversion)

Low energy interactions (Thomson - classical scattering with free electrons, Rayleigh - scattering with the whole atom or coherent scattering) are not relevant for the energy range we're interested in.

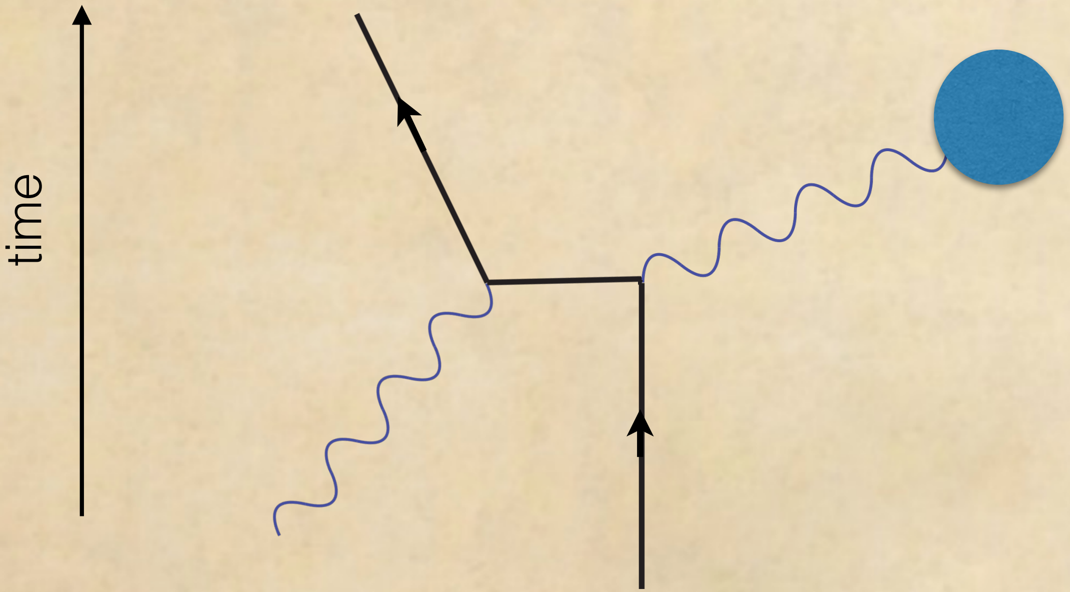
Photoelectric effect

Q: can you draw the Feynman diagram for the photoelectric effect?

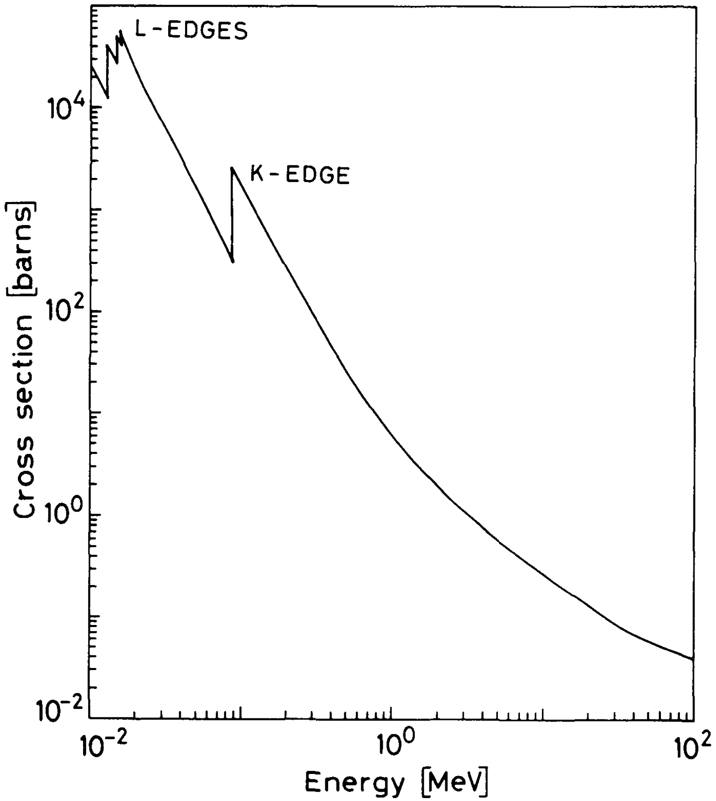


Photoelectric effect

Q: can you draw the Feynman diagram for the photoelectric effect?



Photoelectric effect



The cross section for the photoelectric effect shows the typical pattern of the binding energies of the various shells, K, L,M,…
For energies above the K-line the cross section goes as:

$$\Phi_{\text{photo}} = 4\,\alpha^4 \sqrt{2}\, Z^5\, \phi_0 (m_e c^2/h\nu)^{7/2} \text{ per atom },$$

$$\text{with } \phi_0 = 8\pi r_e^2/3 = 6.651 \times 10^{-25}\ \text{cm}^2;\ \alpha = 1/137\ .$$

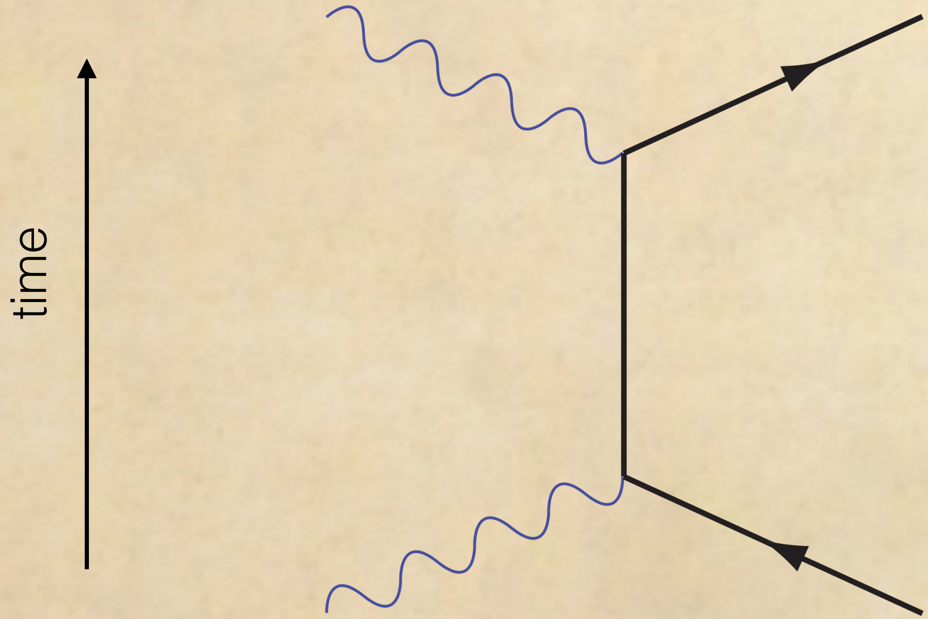
Note the dependence on the Z of the absorber: Z^5

Compton scattering

Q: can you draw the Feynman diagram for the Compton scattering ?

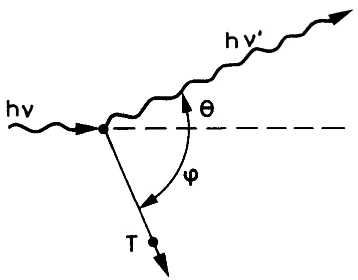
Compton scattering

Q: can you draw a Feynman diagram for the Compton scattering ?



Compton scattering

Applying momentum conservation, you get that the energy of the scattered photon and electrons are:



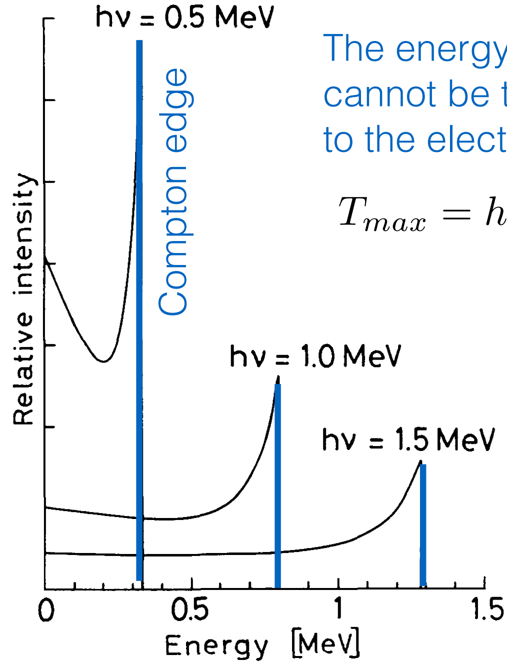
The electron is stopped by the material.

$$h\nu' = \frac{h\nu}{1+\gamma(1-\cos\theta)},$$

$$T = h\nu - h\nu' = h\nu\frac{\gamma(1-\cos\theta)}{1+\gamma(1-\cos\theta)},$$

$$\cos\theta = 1 - \frac{2}{(1+\gamma)^2\tan^2\varphi+1},$$

$$\cot\varphi = (1+\gamma)\tan\frac{\theta}{2},$$



The energy of the photon cannot be totally transferred to the electron

$$T_{max} = h\nu\frac{2\gamma}{1+2\gamma}$$

The cross section for this process can be computed in QED and it is given by the Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2}\frac{1}{[1+\gamma(1-\cos\theta)]^2}\left(1+\cos^2\theta+\frac{\gamma^2(1-\cos\theta)^2}{1+\gamma(1-\cos\theta)}\right)$$

or integrated over the full solid angle:

$$\sigma_c = 2\pi r_e^2\left\{\frac{1+\gamma}{\gamma^2}\left[\frac{2(1+\gamma)}{1+2\gamma}-\frac{1}{\gamma}\ln(1+2\gamma)\right]+\frac{1}{2\gamma}\ln(1+2\gamma)-\frac{1+3\gamma}{(1+2\gamma)^2}\right\}$$

Pair production

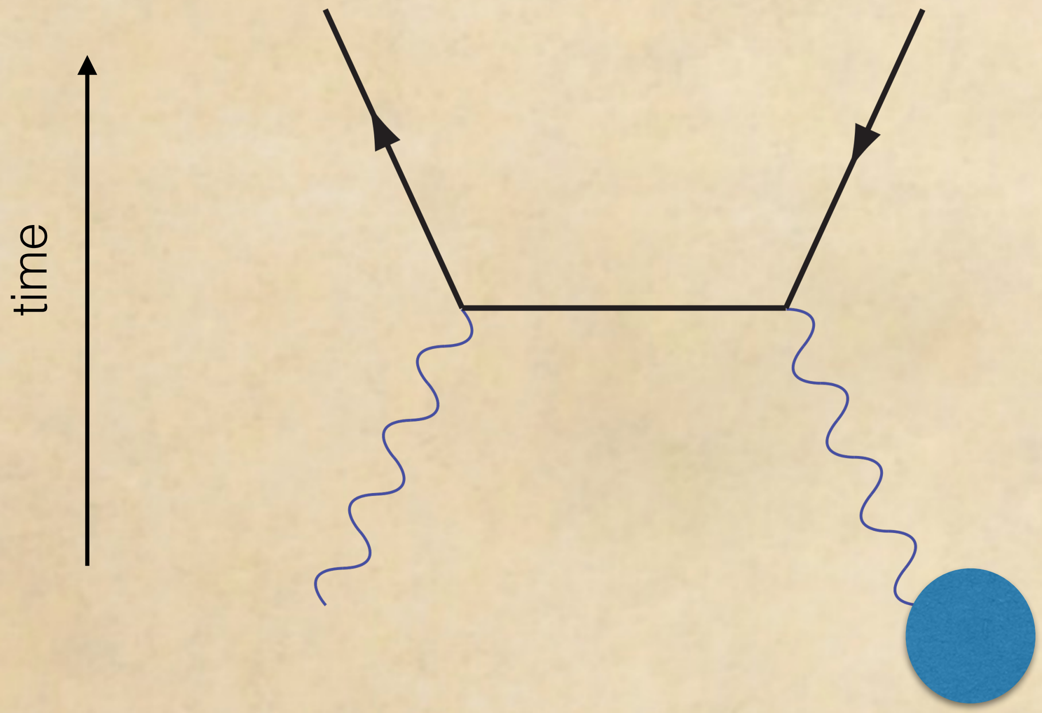
A photon producing an electron-positron pair in HEP is typically called a converted photon.

Q: can you draw the Feynman diagram for the pair production ?
what is the energy threshold for this process ?
Can it happen in vacuum ?

Pair production

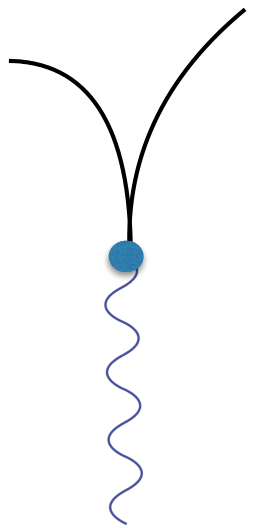
Q: can you draw the Feynman diagram for the pair production ?
what is the energy threshold for this process ?
At least twice the electron mass

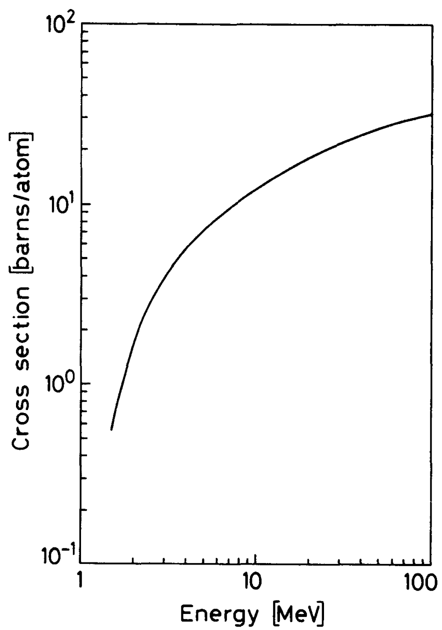
Can it happen in vacuum ?
No because of energy momentum conservation

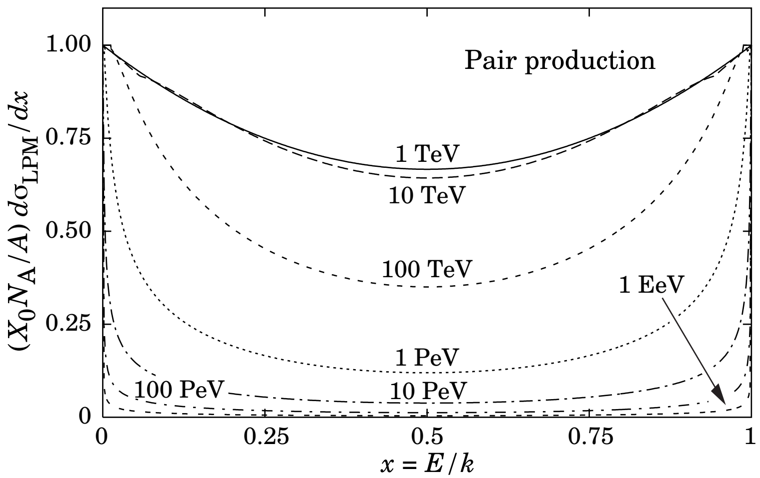


Pair production

A photon producing an electron-positron pair in HEP is typically called a converted photon.







The pair production raises steeply above twice the electron mass and it is practically flat above 1 GeV.

$$\frac{d\sigma}{dx} = \frac{A}{X_0 N_A}\left[1 - \tfrac{4}{3}x(1-x)\right] \qquad x = E/k$$

E =energy of the $e^-(e^+)$
k = photon energy

The probability for a photon "to convert" is:

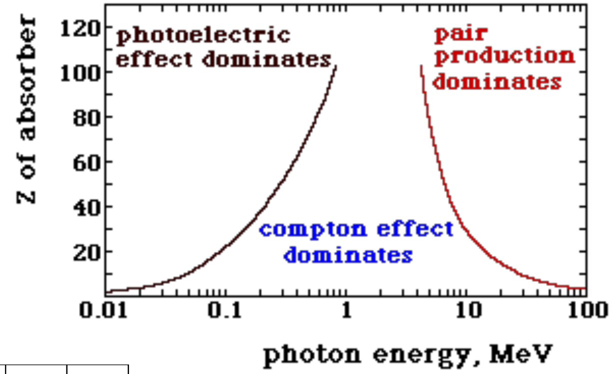
$$P = 1 - e^{-7/9\, x/X_0}$$

The conversion probability in the nuclear electric field is at least one order of magnitude larger than in the electron field (the ratio depends on the materials)

The probability distribution for the energy of the outgoing electron wrt the energy of the converting photon is almost flat at low energy while it peaks at 0/1 at very high energies

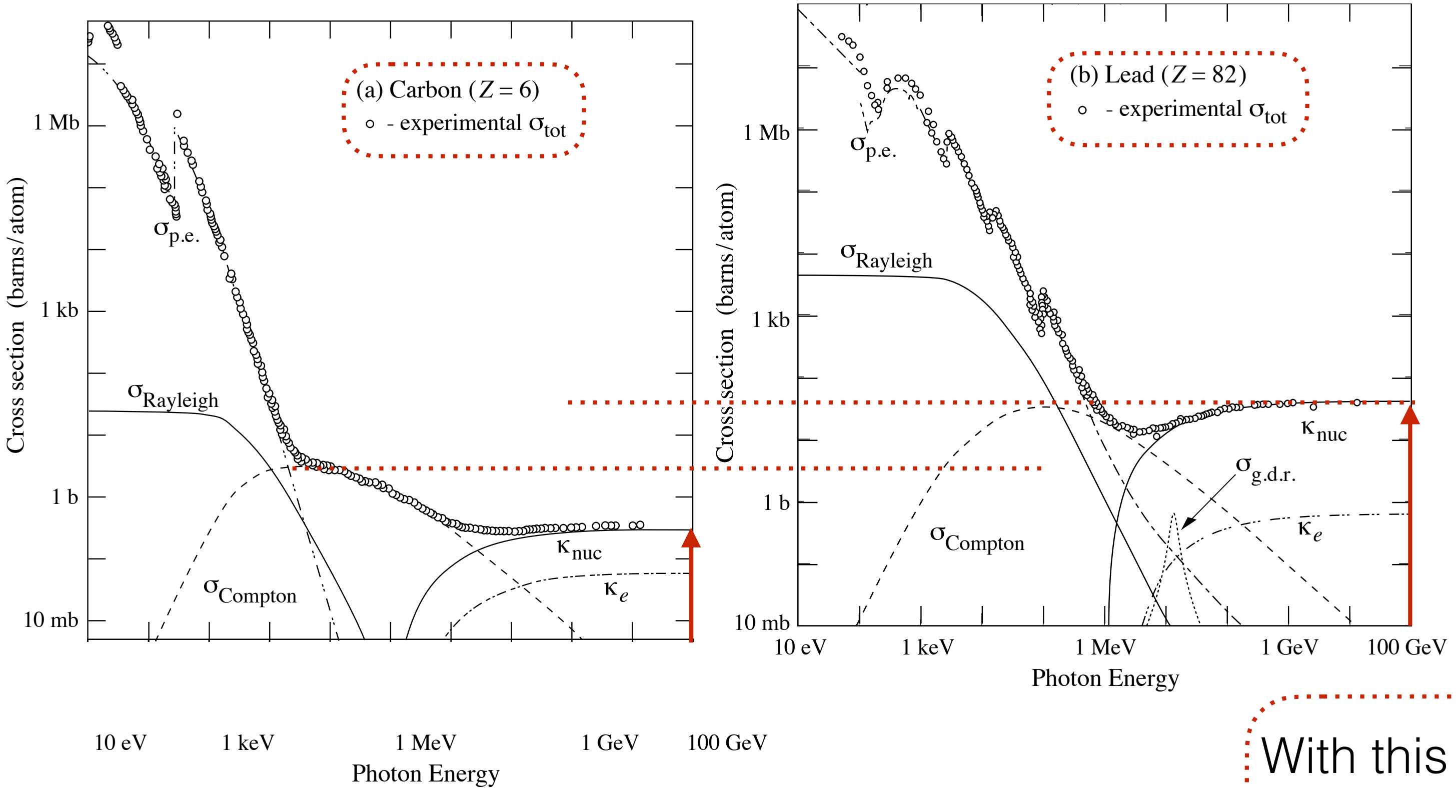
The e^+e^- tracks are parallel at the conversion vertex, because of the massless photon. For any decay of massive particles there is a finite angle between the decay products.

Photon interactions



Total absorption coefficient i.e. putting it all together

plots aligned on the vertical axis



$\sigma_{\text{p.e.}}$ = Atomic photoelectric effect (electron ejection, photon absorption)

σ_{Rayleigh} = Rayleigh (coherent) scattering–atom neither ionized nor excited

σ_{Compton} = Incoherent scattering (Compton scattering off an electron)

κ_{nuc} = Pair production, nuclear field

κ_e = Pair production, electron field

$\sigma_{\text{g.d.r.}}$ = Photonuclear interactions, most notably the Giant Dipole Resonance. In these interactions, the target nucleus is broken up.

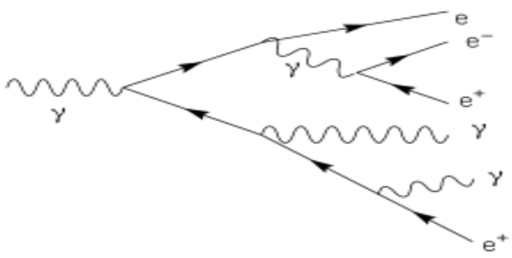
With this you then compute the attenuation as:

$$I/I_0 = \exp(-\mu x)$$
$$\mu = N\sigma = \sigma(N_a \rho / A)$$

Electromagnetic Showers

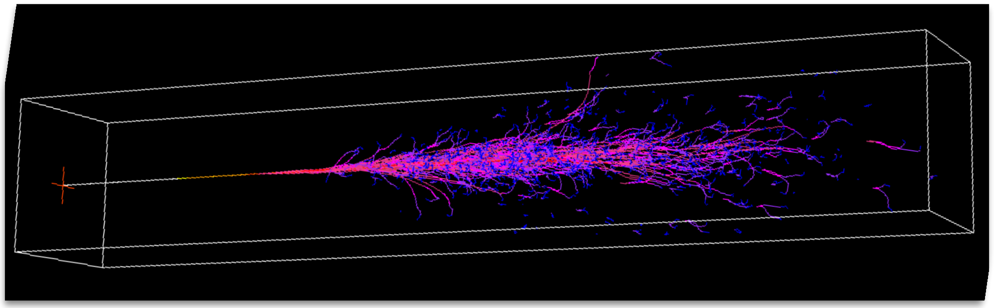
Electro-magnetic showers

The combined effect of photon pair production and electron bremsstrahlung creates the so called electromagnetic showers.



The shower can be initiated by either a photon or an electron.

The process continues until the energy of the electron-positron pairs drops below the critical energy. At that point the electrons will lose their energy mostly by collisions instead of bremsstrahlung and the shower dies out.



Q: how deep is a shower created by a 10 GeV photon wrt a 100GeV photons ?
Can you build a simple model for it ?

Simple shower model

Q: how deeper is a shower created by a 10 GeV photon wrt a 100GeV photons ?
Can you build a simple model for it ?

Begin with a photon of energy E_0
On average after 1 X_0 the photon will convert into an e+e- pair.
Total 2 particles each with energy $E_0/2$
after 2 X_0 the e+ and e- will both emit a photon with half the energy of the initial particle
Total 4 particles: 2 photons, e+ e- each with energy $E_0/4$
after 3 X_0 the photons will have converted into other e+e- pairs and the original pair emitted other 2 photons
Total 4 particles: 4 photons, 2e+ 2e- each with energy $E_0/8$
…
After n X_0 the number of particles will be $N = 2^n$ each with energy $E_0/2^n$

Assume the shower stops when $E_0/2^{n_{max}} = E_c$ then

$$n_{max} = \ln (E_0/E_c) / \ln (2)$$

and the maximum number of particle produced is $N_{max} \sim E_0/E_c$

The depth of a shower grows logarithmically with the energy of the incident particle.

Electro-magnetic showers

To have a better description of the electromagnetic shower we need to use Monte Carlo techniques.

To understand what is a Monte Carlo let's write a simple piece of pseudo-code for a photon initiated em-shower:

- set the energy of the incident photon
- slice the target in portions on depth dx: dx1…dxN (this is the granularity of the description)

for i =1..N:
  for each photon:
  - compute the probability P_c for a photon to convert in dx_i.
  - throw a random number x in [0,1]
    - if x > P_c:
      - add an electron positron pair in the list of particles
      - throw a random number in [0,1] and assign the electron energy as Egamma*x and for the positron Egamma(1-x)

  for each electron / positron:
  - compute the probability P_b to emit a bremsstrahlung of energy E_b
  - throw a random number x in [0,1]
  - if x > P_b:
    - add a photon to the list with energy E_b

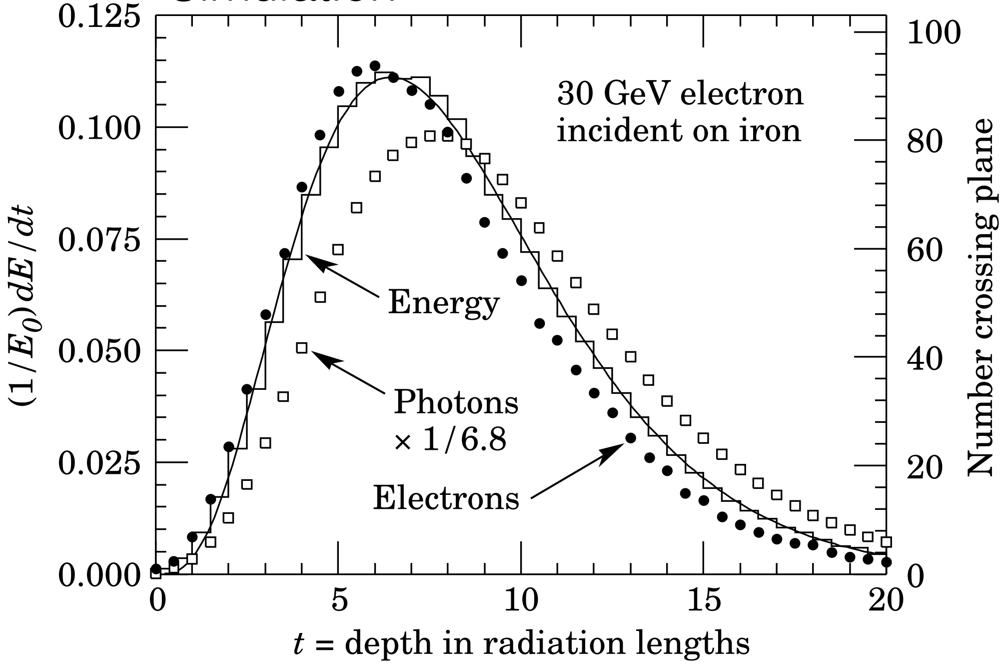
The use of random numbers allows to bring in the probabilistic component.

Electromagnetic showers

When describing the evolution of an electro-magnetic shower we usually use two scale variables: depth in radiation length and energy in terms of critical energy

$$t = x/X_0 \ , \qquad y = E/E_c \ ,$$

Simulation



The shower (after the first couple of radiation lengths) can be parametrised by:

$$\frac{dE}{dt} = E_0\, b\, \frac{(bt)^{a-1} e^{-bt}}{\Gamma(a)} \qquad (1)$$

The depth at which the maximum energy is deposited (shower max) is:

$$t_{\rm max} = (a-1)/b = 1.0 \times (\ln y + C_j) \ , \qquad j = e, \gamma \qquad (2)$$

where Cj = -0.5 (+0.5) for electron(photon) initiated showers

[to use (1), one finds (a – 1)/b from (2), then finds a either by assuming b ≈ 0.5 or by finding a more accurate value from tabulated data]

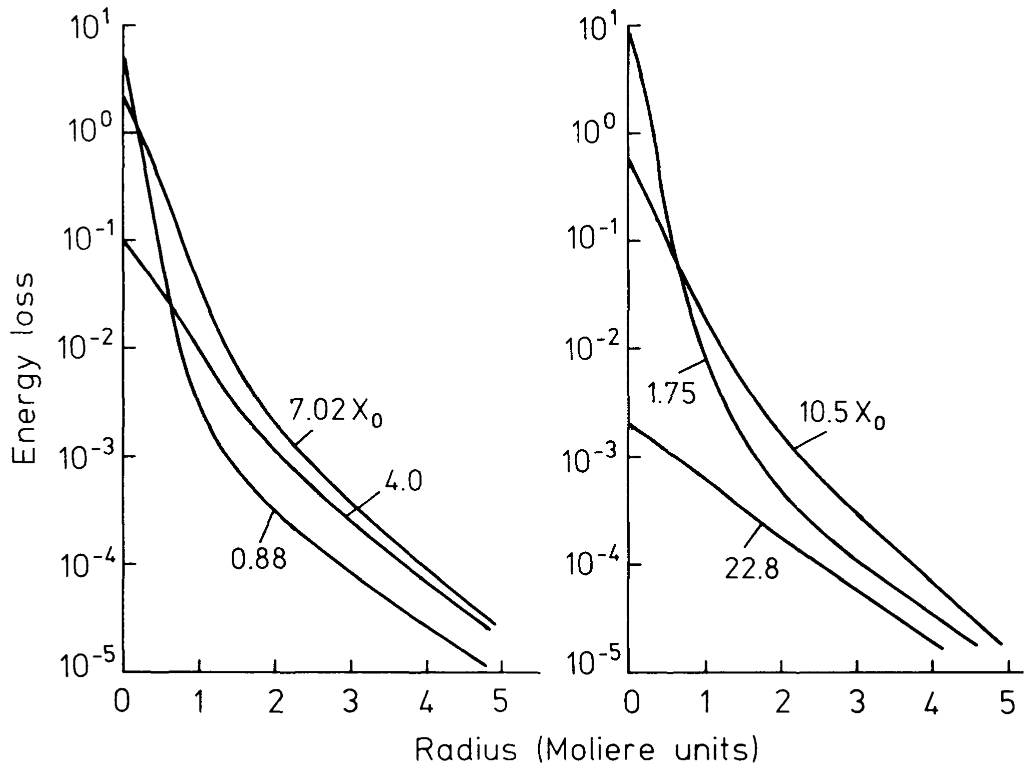
Electromagnetic showers

The transversal development of an em-shower can be described by the Moliere radius:

$$R_M = X_0\, E_s / E_c$$

where E_s = 21 MeV and E_c is the critical energy.
For mixtures and compound materials you can use again the Bragg additivity.

About 90% of the em-shower energy is contained within 1 R_M and 99% within 3.5 R_M.

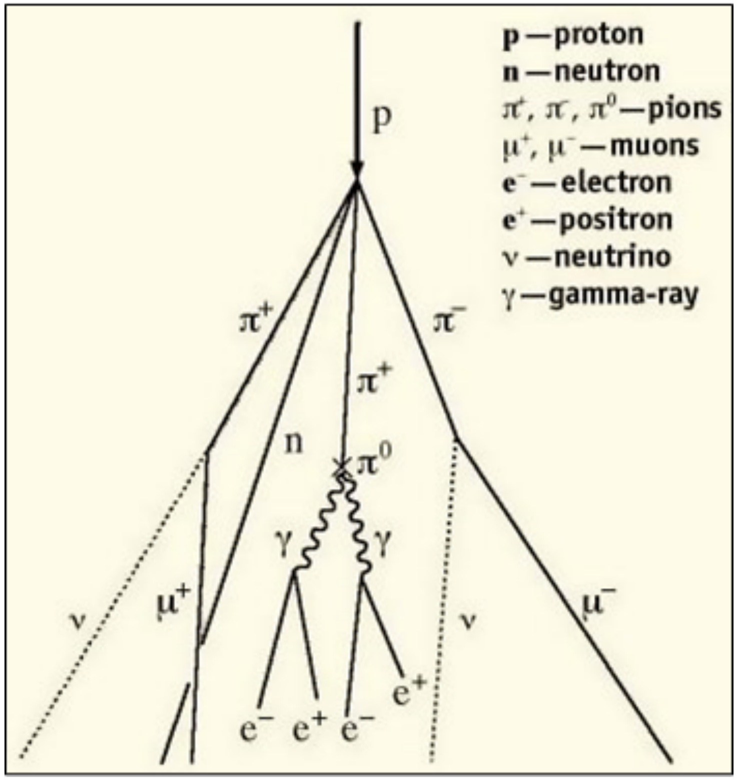


Transverse energy profiles for 1 GeV shower in lead, taken at different depths

Hadronic showers

Hadronic showers

Another cascade phenomenon is the production of hadronic showers.
The incident particle interacts through the strong force or electromagnetically with a nucleus creating hadrons and photons in the final state.



http://inspirehep.net/record/1416209/files/Hadron-Shower.png

The hadronic shower has a more complicated behaviour than an em shower. Typically we describe the total cross section as:

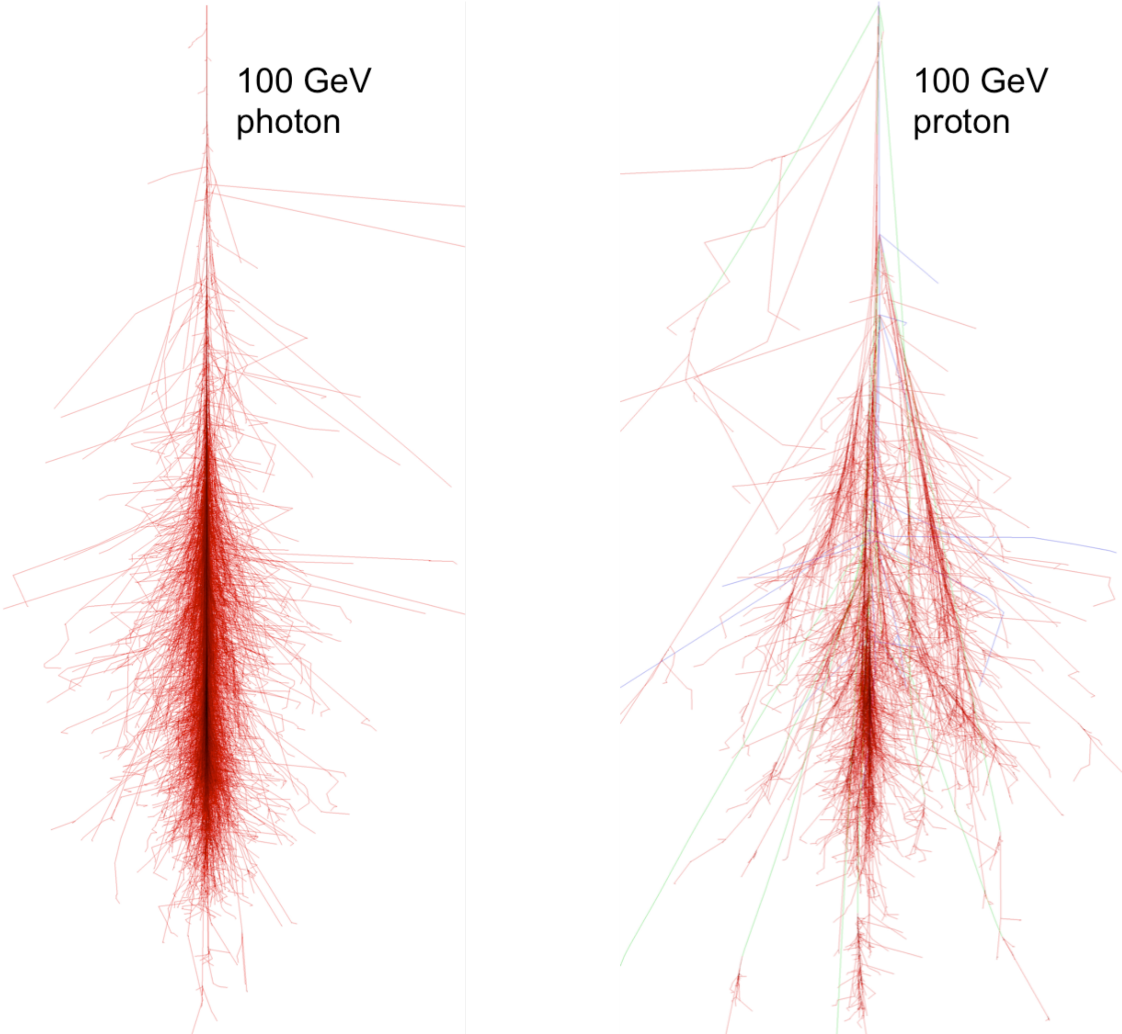
$$\sigma_{tot} = \sigma_{elastic} + \sigma_{inelastic}$$

The inelastic cross section is modelled by the nuclear interaction length: $\lambda_I = A/(N_A\, \rho\, \sigma_{inelastic})$ which play the same role as the electromagnetic interaction length

Hadronic showers tend to have a more complex shape than em showers

Because of the production of neutral pions (which decay to 2 photons) showers will always have an electromagnetic component.

Hadronic showers



https://inspirehep.net/record/1399130/plots

(more about hadronic showers later in the course)

Cherenkov radiation

Cherenkov radiation

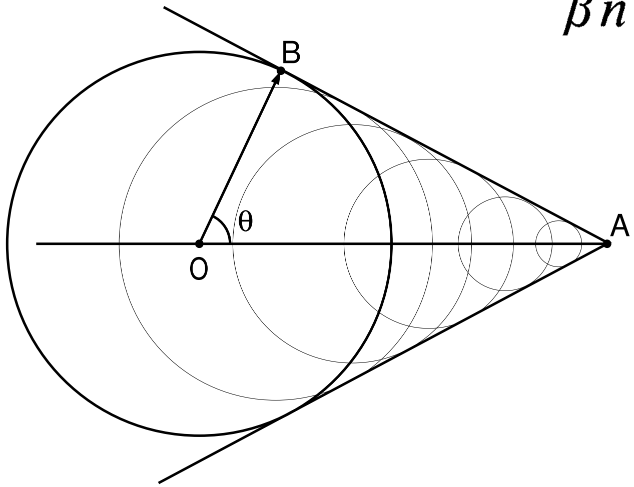
Cherenkov radiation is emitted by charged particles travelling faster than the speed of light in a material. If the refraction index of the material is n:

$$\beta c = v = c/n \qquad\qquad v_{\text{particle}} > c/n$$

i.e. there is a velocity threshold for the light to be emitted that depends on n.

Above that threshold the an electromagnetic shock wave (like the sonic boom) is created and a conical wavefront is emitted at a given angle

$$\cos\theta_C = \frac{1}{\beta n(\omega)}$$

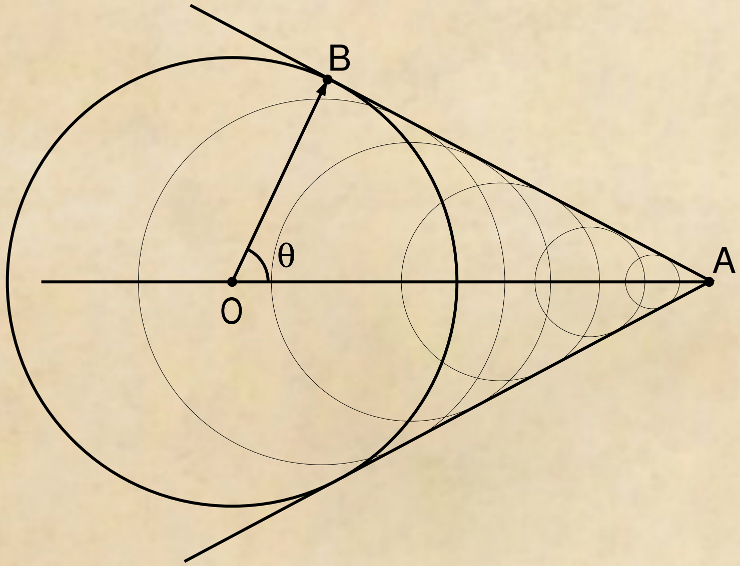


Q: where does the cosine comes from ?

Cherenkov cone

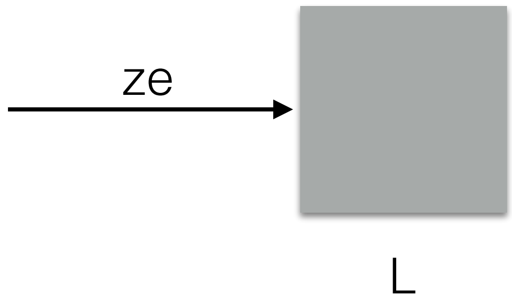
Q: where does the cosine comes from ?

a) Figure 2 shows a sketch of a typical Čerenkov wavefront. Suppose the particle travels from O to A in t seconds. The radiation sent out while it is at O froms a spherical suface with center at O and radius ct/n. The Čerenkov radiation wavefront which is tangent to all such spherical surfaces is a conic surface . In the triangle AOB, OB=ct/n, OA=$vt = \beta ct$, and so $\cos\theta = \mathrm{OB}/\mathrm{OA} = 1/(n\beta)$.



Cherenkov radiation

Consider a particle of charge ze, incident on a piece of material of length L
(simplified to a 2D problem)



$$\frac{d^2E}{d\omega\, d\Omega} = z^2 \frac{\alpha \hbar}{c} n \beta^2 \sin^2\theta \left| \frac{\omega L}{2\pi\beta c} \frac{\sin\xi(\theta)}{\xi(\theta)} \right|^2$$

where $\xi(\theta) = \frac{\omega L}{2\beta c}(1 - \beta n \cos\theta)$

[derivation e.g. Ch.14 Jackson]

As a function of the angle you see a diffraction-like pattern $\frac{\sin\xi(\theta)}{\xi(\theta)}$

with a maximum at the Cherenkov angle $\cos\theta_C = \frac{1}{\beta n(\omega)}$

Narrow peak if L >> λ —> it tends to a delta function

$\frac{\sin\xi(\theta)}{\xi(\theta)}$ —> δ(1-nβ cos ϑ)

For smaller L the peak broadens

Moreover n=n(ϑ), so each wavelength will have a different angle.

Cherenkov radiation

Integrating $\frac{d^2E}{d\omega\, d\Omega}$ over the solid angle and frequencies and dividing by L

we get to $\boxed{-dE/dx = z^2 \frac{\alpha \hbar}{c} \int \omega\, d\omega \left(1 - \frac{1}{\beta^2 n^2(\omega)}\right)}$

The energy loss increases with β —> so we can use it to measure the speed of particles. Knowing the momentum and the velocity we can get to the mass: PID (as Bethe Bloch)

The typical energy loss is of the order of 10^{-3} MeV cm^2/g, i.e. order of permill compared to ionization in a solid, but that still corresponds to a huge number of photons emitted

We can compute the number of photons emitted per unit length by integrating dE/dΩdω and dividing by L and ħω

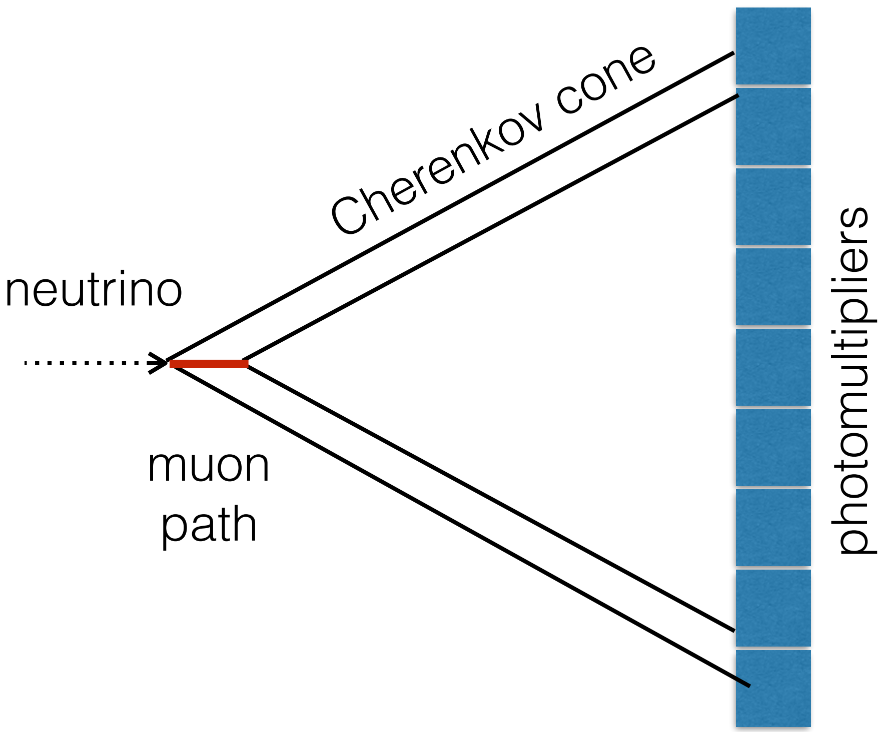
$$\frac{d^2N}{d\lambda\, dx} = \frac{2\pi z^2 \alpha}{\boxed{\lambda^2}} \sin^2\theta_C$$

Most of the energy is emitted in the UV !

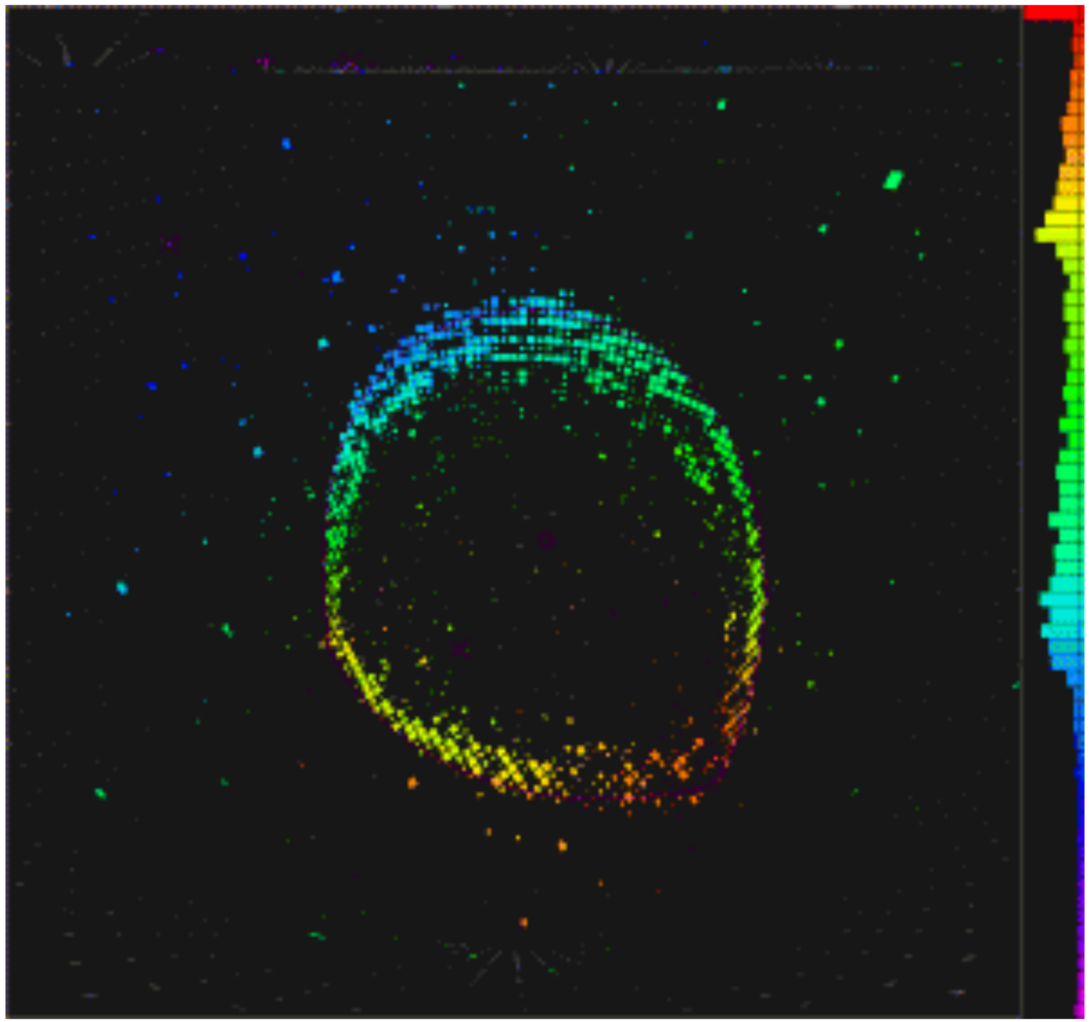
The typical devices used to detect Cherenkov radiations are photo-multipliers-tube (PMT) (—> see later in the course) which are sensitive to a range of wave length λ∈[350, 550]nm

$$\frac{dN}{dx} = 2\pi z^2 \alpha \sin^2\theta_C \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^2} = 475\, z^2 \sin^2\theta_C \text{ photons/cm}$$

Cherenkov ring in neutrino dets

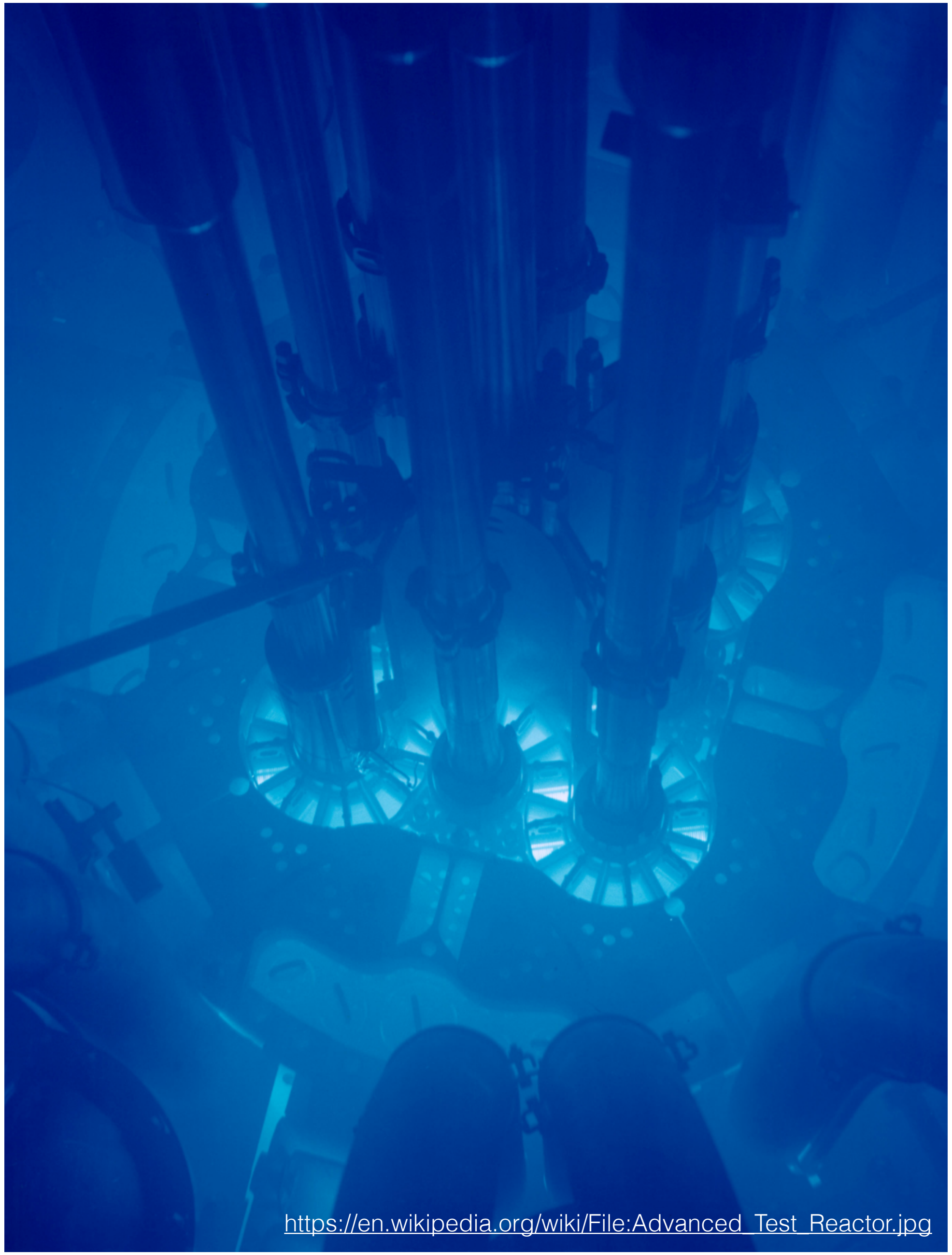


Muon event: Cherenkov light in SuperKamiokande



Cherenkov radiation

Cherenkov light is mostly emitted in the UV (promptly adsorbed by water).
What we see as blue is partly the "long" wavelength tail in the visible (UV—>blue)


https://en.wikipedia.org/wiki/File:Advanced_Test_Reactor.jpg

Transition radiation

Transition Radiation

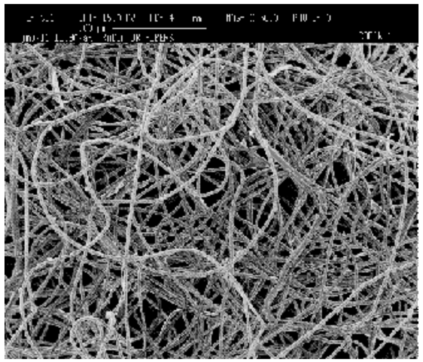
When an incident particle of charge ze crosses the boundary between two media with different refractive index it emits radiation. The energy of the photons emitted by a particle of charge ze crossing the boundary between vacuum and a medium with plasma frequency ω_p is:

$$I = \alpha z^2 \gamma \hbar\omega_p / 3$$

where $\hbar\omega_p = \sqrt{4\pi N_e r_e^3}\, m_e c^2/\alpha = \sqrt{\rho\ (\text{in g/cm}^3)\ \langle Z/A \rangle} \times 28.81\ \text{eV}$

Properties:

- energy of the photons is proportional to γ of the incident particle and typically in the X-ray range
- angle of emission is proportional to 1/γ
- number of radiated photons is αz^2. Because number of photons is so small (i.e. the probability to emit a TR photon is small), typically several interfaces are build using foils or fibres
- TR is extremely faint: effectively only relevant for γ >1000 particles



polyethylene fibres

From interactions to detectors

Now that you know how particles interact with matter you can understand how existing detectors work and invent your own.

The general goal of a detector is to record the passage of (energy deposited by) a particle by one or more of the previous mechanisms. The data will subsequently be analysed to gain information about the incident particle.

There is a huge number of different detectors. They can be roughly divided into:

- simple counters (no energy (just energy above a threshold), no position)
- position sensitive (position no energy)
  - in a magnetic field we can extract the momentum
- calorimeters (energy no position)
  - segmented calorimeter can arrive up to an “image” of the energy deposited

Combining momentum of a particle and its speed we can get to Particle identification

Detectors can be single units or composed by several sub-detectors. In latter case they are ordered:

- first tracking device which do not affect much the energy momentum of the particle
- then calorimeters which will stop the particle starting from the less penetrating
- then catch what penetrates the calorimeters

We will study various detector types when we will encounter them in the papers.

Detectors timeline

'11 cloud chambers Wilson
'28 Geiger tubes
'29 Coincidence Bothe
'34 PhotoMultipliers
'37 Nuclear emulsions
'52 Bubble chambers
'68 Multiwire Proportional chambers
'71 Drift Chambers
'74 Time Projection Chambers
'83 Silicon Strips / Pixels

Bibliography

Bethe-Bloch:
Leo "Techniques for Nuclear and Particle Physics Experiments" ch 2
PDG "Passage of particles through matter" ch 33