LEP Electroweak fits

Electroweak precision fit

Why high precision study of EWC (electroweak corrections) ?

- check the consistency of the gauge / Higgs sectors of the SM: not only a good model at low energies but that as a QFT describes experimental observations up to much higher scales.
- Infer the presence of new particles (fields) through quantum corrections (loops) on observables (top, Higgs, BSM)

Two ways to discover new physics: “direct” observation or observing deviations from theoretical predictions”

Observables	Loop corrections
masses coupling constants	at vertices
Branching ratios production cross sections	on propagators

Examples:

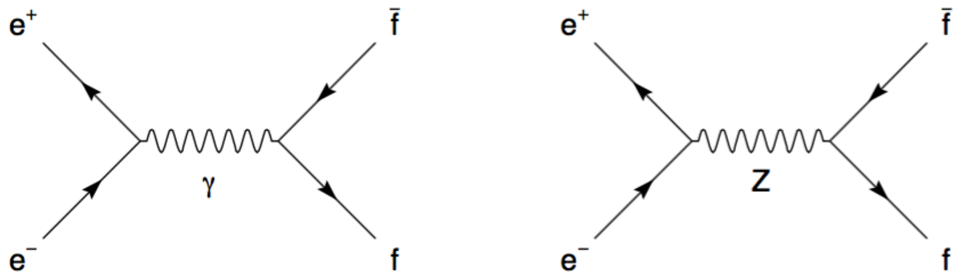
Take an observable whose high order corrections depend on m_{top}, m_{Higgs}

⇒ can infer those masses even before observing the particles !

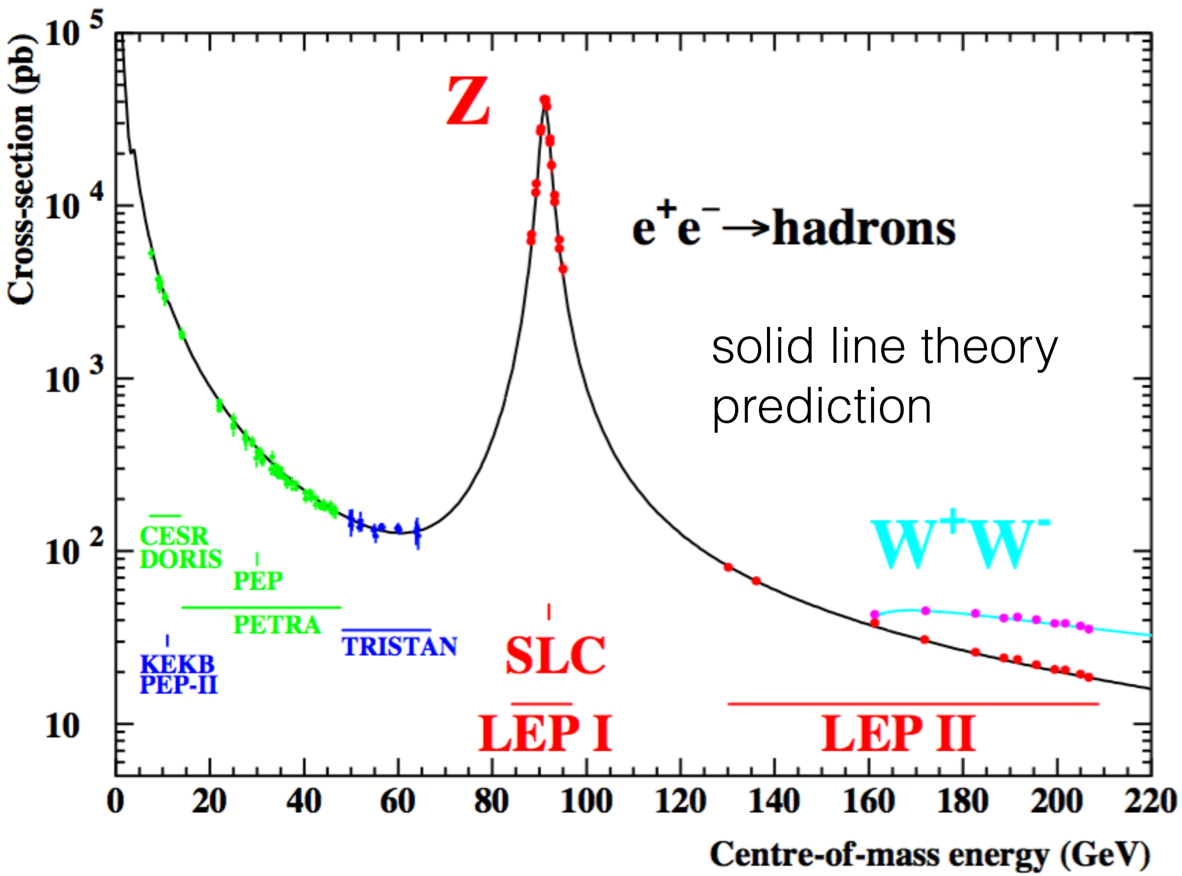
Flipping the argument, combined fits of of several observables are very stringent test of the theory/model producing the corrections

(we will go into more details about (pseudo-)observables when fitting the Higgs properties)

e^+e^- @ LEP/SLC



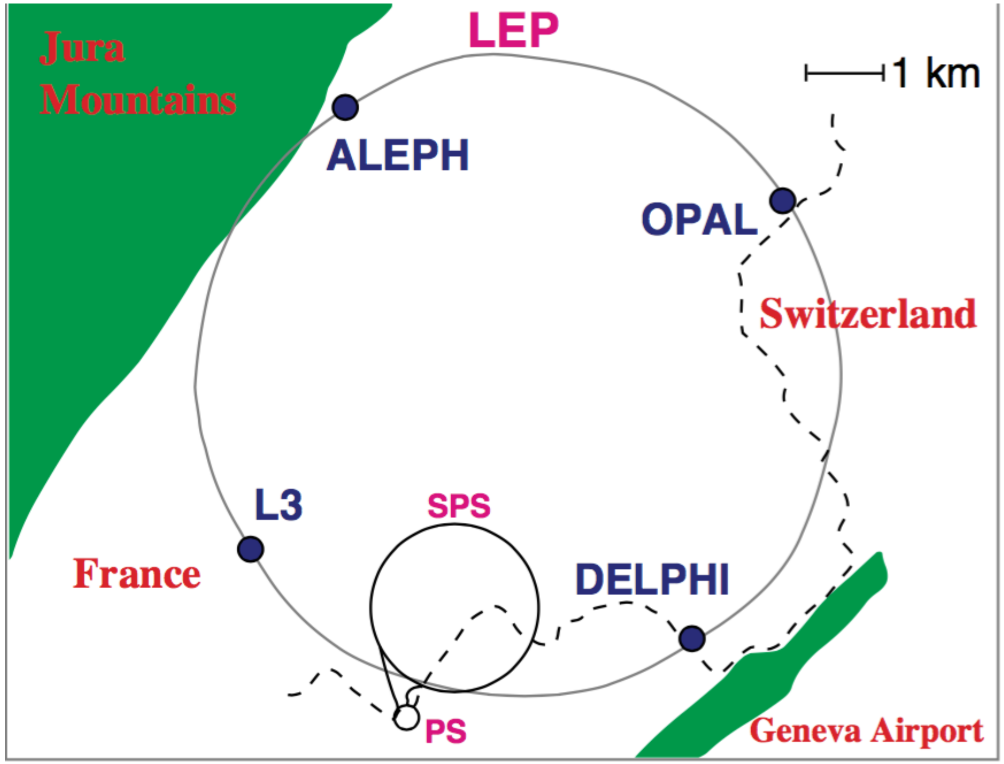
e+
f̄
e−
f
γ
e+
f̄
e−
f
Z



Cross-section (pb)
10 5
10 4
10 3
10 2
10
Z
e+e−→hadrons
solid line theory prediction
W+W-
CESR
DORIS
PEP
PETRA
TRISTAN
KEKB
PEP-II
SLC
LEP I
LEP II
0
20
40
60
80
100
120
140
160
180
200
220
Centre-of-mass energy (GeV)

e^+e^- @ LEP

beam energies at ~mZ/2   (LEP 1)
beam spot 150 um x 5 um
45kHz (4 bunches —> then 8)
125 MeV loss /turn because of
bremsstrahlung
Fantastic beam energy resolution:
2 MeV (~2 10^{-5} relative unc)

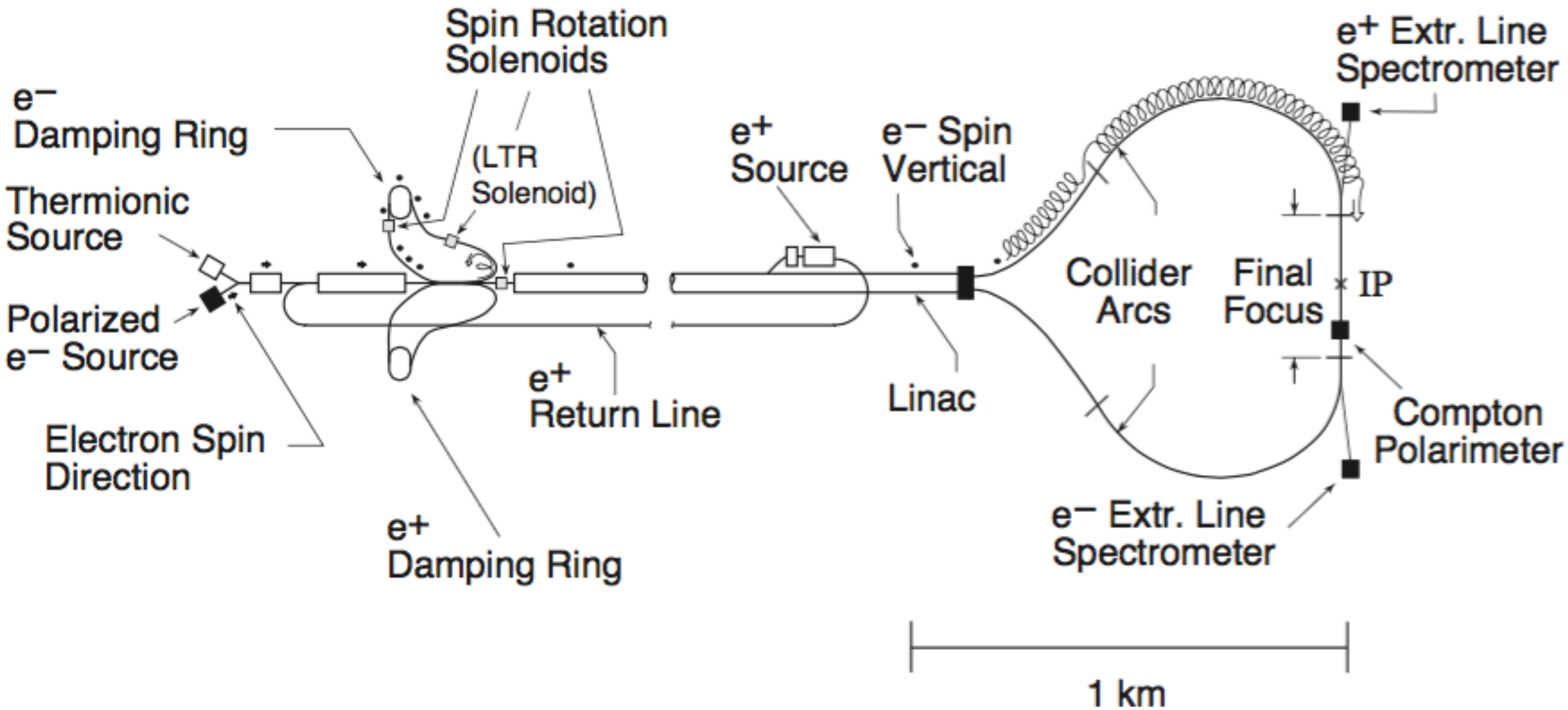


While the beam orbit length was constrained by the RF accelerating system, the focusing quadrupoles were fixed to the earth and moved with respect to the beam, changing the effective total bending magnetic field and the beam energy by 10 MeV over several hours. Sensitive to earth tides generated by the moon and sun, local geological deformations following heavy rainfall or changes in the level of Lake Geneva, electric trains

LEP: 7’000’000 Z   (1000 Z bosons/hour x 4 experiments when running at 2 10^{31} $cm^{-2}s^{-1}$)

e^+e^- @ SLC

SLC: 600'000 Z (longitudinal polarization)



Repetition rate **120 Hz**
Beam spot 1.5 um x 0.7 um (better selection of heavy quarks)
Polarized beams !

Basic measurements

Cross sections

$$\sigma = \frac{N_{\rm sel} - N_{\rm bg}}{\epsilon_{\rm sel}\mathcal{L}}$$

ε_{sel} = efficiency x acceptance

The Z couples with a mixture of vector and axial-vector couplings.

$$g_L^{\nu_e}\bar{\nu}_e\gamma_\mu(1-\gamma_5)\nu_e = g_L^{\nu_e}\bar{\nu}_{eL}\gamma_\mu\nu_{eL}$$

$$g_L^e\bar{e}\gamma_\mu(1-\gamma_5)e + g_R^e\bar{e}\gamma_\mu(1+\gamma_5)e = g_L^e\bar{e}_L\gamma_\mu e_L + g_R^e\bar{e}_R\gamma_\mu e_R$$

$$g_L^u\bar{u}\gamma_\mu(1-\gamma_5)u + g_R^u\bar{u}\gamma_\mu(1+\gamma_5)u = g_L^u\bar{u}_L\gamma_\mu u_L + g_R^u\bar{u}_R\gamma_\mu u_R$$

$$g_L^d\bar{d}\gamma_\mu(1-\gamma_5)d + g_R^u\bar{d}\gamma_\mu(1+\gamma_5)d = g_L^d\bar{d}_L\gamma_\mu d_L + g_R^u\bar{d}_R\gamma_\mu d_R.$$

This results in measurable asymmetries in the angular distributions of the final-state fermions, the dependence of Z production on the helicities of the colliding electrons and positrons, and the polarisation of the produced particles.

Basic measurements

Asymmetries

$$A_{\mathrm{FB}} = \frac{N_{\mathrm{F}} - N_{\mathrm{B}}}{N_{\mathrm{F}} + N_{\mathrm{B}}}$$

Forward / Backward

"forward" means that the produced fermion (as opposed to anti-fermion) is in the hemisphere defined by the direction of the electron beam (polar scattering angle $\theta < \pi/2$).

$$A_{\mathrm{LR}} = \frac{N_{\mathrm{L}} - N_{\mathrm{R}}}{N_{\mathrm{L}} + N_{\mathrm{R}}} \frac{1}{\langle \mathcal{P}_{\mathrm{e}} \rangle}$$

@ SLC

$N_L(N_R)$ is the number of Z bosons produced for left(right)-handed electron bunches, $\langle P_e \rangle$ is the magnitude of luminosity-weighted electron polarisation

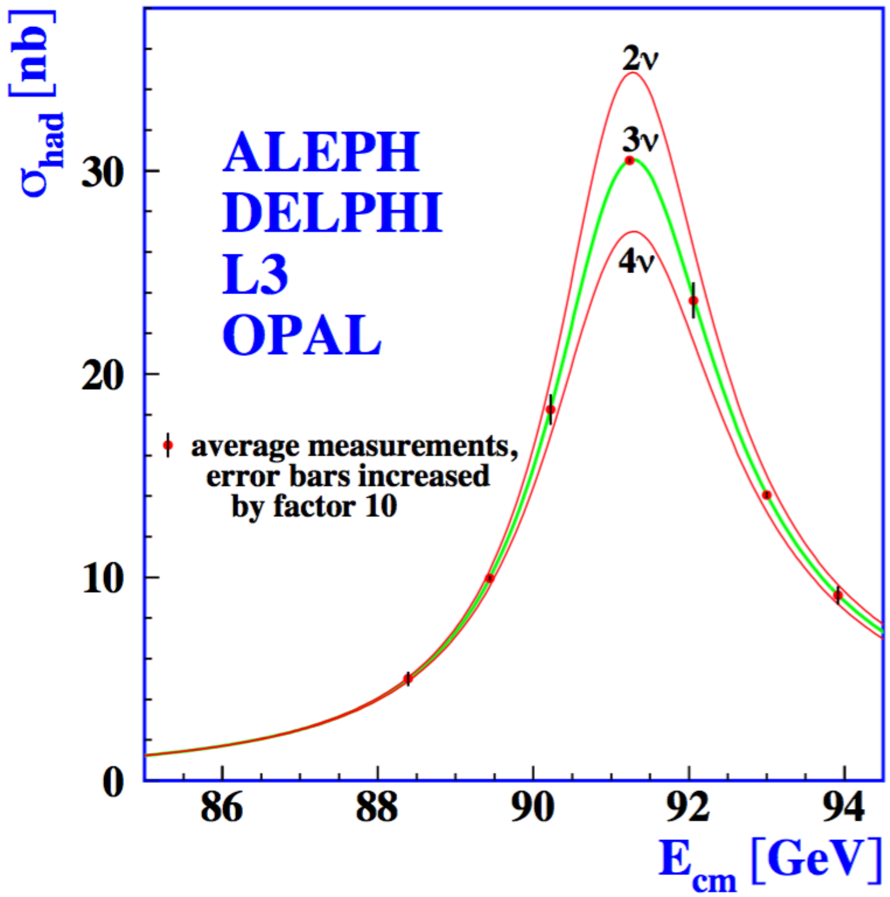
Example: number of (light) neutrino families

Determination of the number of light (i.e. kinematically accessible in Z decays) obtained by measuring the partial widths :

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{had} + \Gamma_{inv}$$

$$\Gamma_{had} = \sum_{q \neq t} \Gamma_{q\bar{q}}$$

$$\Gamma_{inv} = N_\nu \Gamma_{\nu\bar{\nu}}$$



SM tree level relations

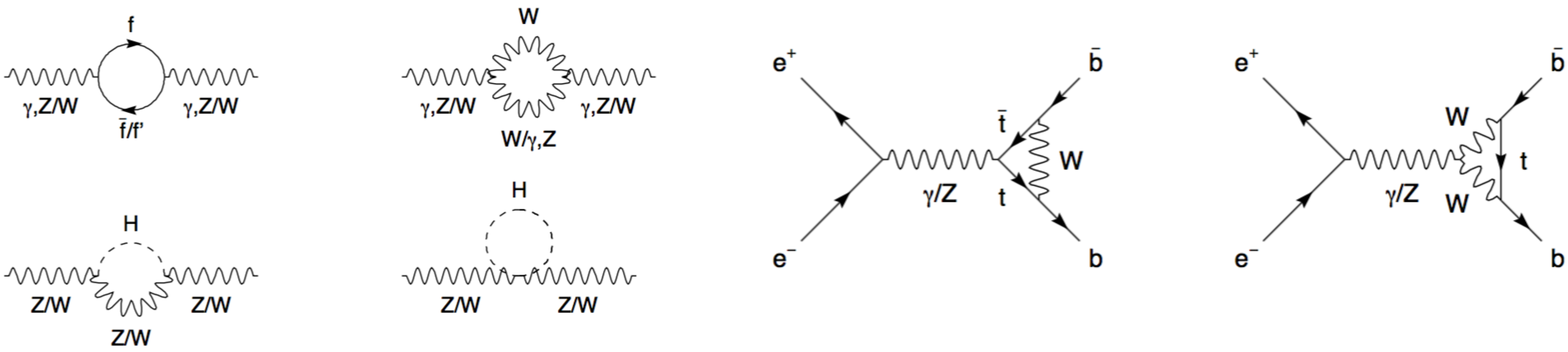
Relation between weak and e.m. couplings:

$$G_F = \frac{\pi\alpha}{\sqrt{2} m_W^2 \sin^2\theta_W^{tree}}$$

Relation between neutral and charged weak couplings:
(ρ is determined by the Higgs structure of the theory:
with only one Higgs doublet ρ = 1)

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2\theta_W^{tree}}.$$

Tree level relations are modified by radiative corrections to both the propagators and vertices



The effect of the corrections is O(%). If one can get both theoretical and experimental precisions to this level the effects of the loops can be tested.

Fit structure

5 input parameters from the Standard Model:

$$\alpha(m_Z) \quad \alpha_s(m_Z) \quad m_Z \quad m_{top} \quad m_H$$

In practice all the other parameters are either ~constant at the Z-pole or can be derived from these

Collect a (large) number of observables that depend on these inputs and fit them simultaneously to check if there is a (unique) set of values that can accommodate all measurements.

Build a X^2 fit from all the observables:

$$X^2 = \left(\frac{\text{observed - predicted}}{\text{uncertainty}}\right)^2$$

$O_1(\alpha, \alpha_s, m_Z, m_{top}, m_H \; ; \vec{x_1})$
$O_2(\alpha, \alpha_s, m_Z, m_{top}, m_H \; ; \vec{x_2})$
$O_3(\alpha, \alpha_s, m_Z, m_{top}, m_H \; ; \vec{x_3})$
…
$O_N(\alpha, \alpha_s, m_Z, m_{top}, m_H \; ; \vec{x_N})$

$$\frac{\partial X^2}{\partial \vec{p}} = 0$$

$$\vec{p} = (\alpha, \alpha_s, m_Z, m_{top}, m_H)$$

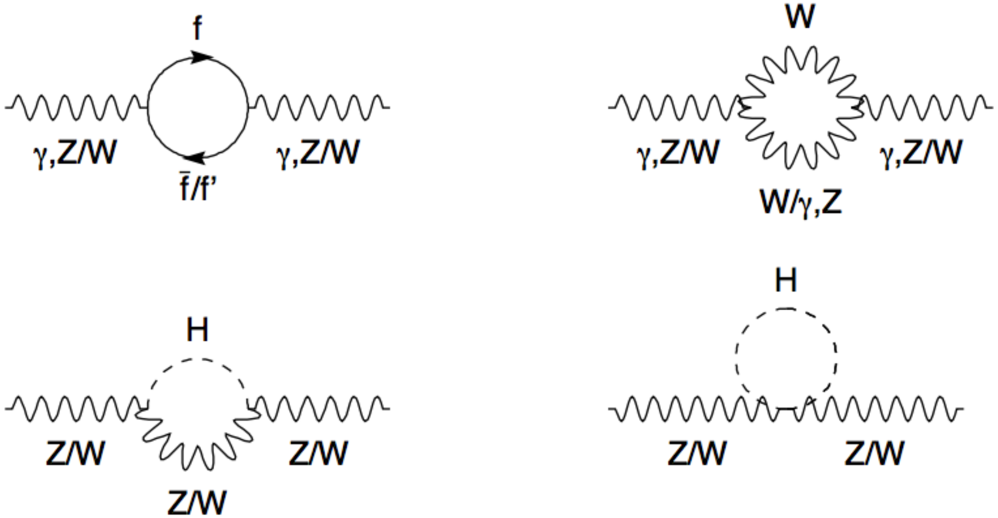
Based on the best fit values of the input parameters, predict the “SM expectation” for any observable and compare it with the measured values

m_{top} m_H

AT TREE LEVEL (Th. Lecture 1)

$$\rho := \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1$$

(consequence of using only 1 doublet in SM)



AT ONE LOOP

$$m_W^2 = \frac{m_Z^2}{2}\left(1 + \sqrt{1 - 4\,\frac{\pi\alpha}{\sqrt{2}\,G_F\,m_Z^2}\;\frac{1}{1-\Delta r}}\right)$$

WITH $\Delta r = \Delta\alpha + \Delta r_W$

m_{top} m_H

$$m_W^2 = \frac{m_Z^2}{2}\left(1 + \sqrt{1 - 4\frac{\pi\alpha}{\sqrt{2}G_F m_Z^2}\frac{1}{1-\Delta r}}\right)$$

WITH $\Delta r = \Delta\alpha + \Delta r_W$

$\Delta\alpha$ from RUNNING OF THE E.M. COUPLING

→ FERMIONS IN LOOPS 

$$\Delta\alpha(s) = \Delta\alpha_{lept} + \Delta\alpha_{top} + \Delta\alpha_{had}$$

from theory

from experiment
(low energy non-perturbative)

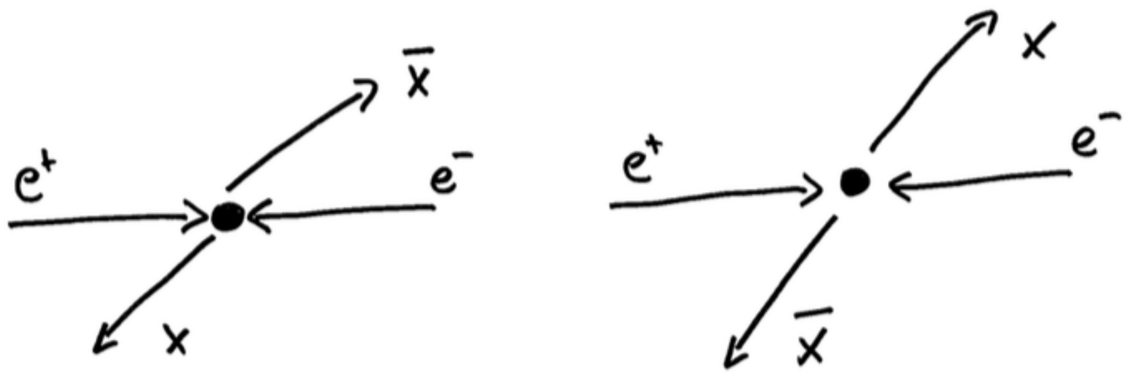
$$\Delta r_W(m_t, m_H) \simeq \frac{\alpha}{\pi \sin^2\theta_W}\left(-\frac{3}{16}\frac{\cos^2\theta_W}{\sin^2\theta_W}\cdot\frac{m_t^2}{m_W^2} + \frac{11}{24}\log\left(\frac{m_H}{m_Z}\right)\right)$$

Observables

LEP 1

- Z^0 BOSON $\quad m_Z \,, \Gamma_Z$
- $\sigma^0(e^+e^- \to q\bar{q})$
- RATIO OF FERMIONIC WIDTHS TO THE HADRONIC WIDTH $\quad R^0_f = \dfrac{\Gamma_f}{\Gamma_{q\bar{q}}} \quad f = b, c, \text{leptons}$
- FWD - BWD ASYMMETRY OF Z^0 DECAYS

$$A_{FB}^{0,x} = \frac{\sigma_F^x - \sigma_B^x}{\sigma_F^x + \sigma_B^x} \qquad x = b, c, \text{leptons}$$



FORWARD $\qquad\qquad$ BACKWARD

Observables

LEP 1

- POLARIZATIONS

$$A_x = \frac{g_{Lx}^2 - g_{Rx}^2}{g_{Lx}^2 + g_{Rx}^2} \qquad x = b, c, \text{leptons}$$

ASYMMETRY IN THE COUPLINGS TO LEFT-HANDED / RIGHT-HANDED FERMIONS

SLAC SLC/SLD POLARIZED BEAMS

LEP USE τ FOR LEPTONS

How do you measure the polarization of a tau ?

- EFFECTIVE EW-MIXING ANGLE

$$\sin^2\theta_{eff}^{lep}$$

LEP 2 / TeVatron

- W BOSON m_W, Γ_W

How do you measure the W mass ?

- top mass m_{top}

Measurements summary

	Measurement with Total Error	Systematic Error
$\Delta\alpha^{(5)}_{\mathrm{had}}(m^2_{\mathrm{Z}})$ [59]	0.02758 ± 0.00035	0.00034
m_{Z} [GeV]	91.1875 ± 0.0021	$^{(a)}0.0017$
Γ_{Z} [GeV]	2.4952 ± 0.0023	$^{(a)}0.0012$
σ^0_{had} [nb]	41.540 ± 0.037	$^{(a)}0.028$
R^0_ℓ	20.767 ± 0.025	$^{(a)}0.007$
$A^{0,\ell}_{\mathrm{FB}}$	0.0171 ± 0.0010	$^{(a)}0.0003$
+ correlation matrix Table 2.13		
$\mathcal{A}_\ell$ (P_τ)	0.1465 ± 0.0033	0.0015
$\mathcal{A}_\ell$ (SLD)	0.1513 ± 0.0021	0.0011
R^0_{b}	0.21629 ± 0.00066	0.00050
R^0_{c}	0.1721 ± 0.0030	0.0019
$A^{0,\mathrm{b}}_{\mathrm{FB}}$	0.0992 ± 0.0016	0.0007
$A^{0,\mathrm{c}}_{\mathrm{FB}}$	0.0707 ± 0.0035	0.0017
$\mathcal{A}_{\mathrm{b}}$	0.923 ± 0.020	0.013
$\mathcal{A}_{\mathrm{c}}$	0.670 ± 0.027	0.015
+ correlation matrix Table 5.11		
$\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$ ($Q^{\mathrm{had}}_{\mathrm{FB}}$)	0.2324 ± 0.0012	0.0010
m_{t} [GeV] (Run-I [212])	178.0 ± 4.3	3.3
m_{W} [GeV]	80.425 ± 0.034	
Γ_{W} [GeV]	2.133 ± 0.069	
+ correlation given in Section 8.3.2		

The most amazing results ever !

Take the high precision Z-pole measurements and fit simultaneously all 5 inputs:

X^2/ndof = 16/ 10 (probability 9.9%)

Parameter	Value	Correlations				
		$\Delta\alpha^{(5)}_{\text{had}}(m^2_Z)$	$\alpha_S(m^2_Z)$	m_Z	m_t	$\log_{10}(m_H/\text{GeV})$
$\Delta\alpha^{(5)}_{\text{had}}(m^2_Z)$	0.02759 ± 0.00035	1.00				
$\alpha_S(m^2_Z)$	0.1190 ± 0.0027	−0.04	1.00			
m_Z [GeV]	91.1874 ± 0.0021	−0.01	−0.03	1.00		
m_t [GeV]	173^{+13}_{-10}	−0.03	0.19	−0.07	1.00	
$\log_{10}(m_H/\text{GeV})$	$2.05^{+0.43}_{-0.34}$	−0.29	0.25	−0.02	0.89	1.00
m_H [GeV]	111^{+190}_{-60}	−0.29	0.25	−0.02	0.89	1.00

PDG $m = 173.21 \pm 0.51 \pm 0.71$ GeV

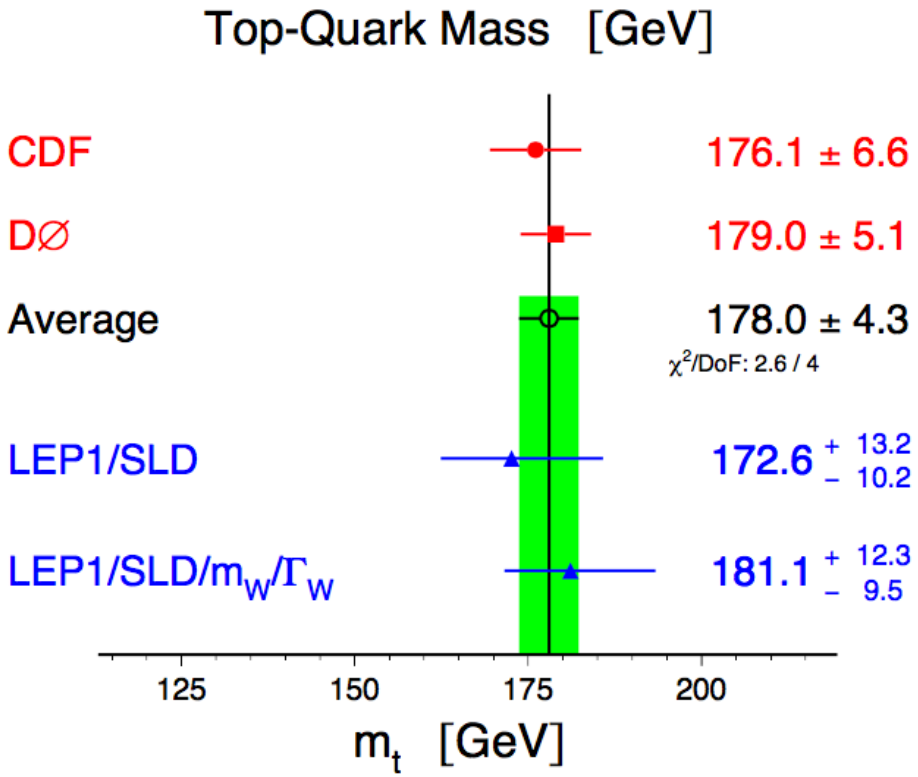
Today we discovered both:

$m_H = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.})$ GeV

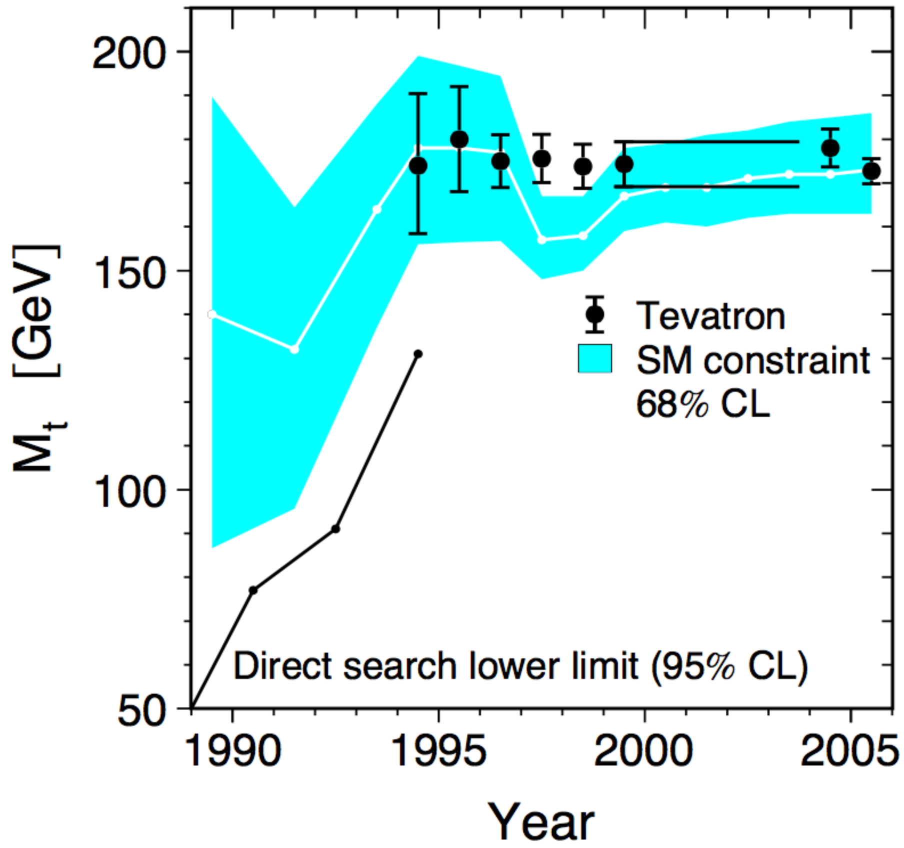
From these values we can extract all other SM parameters:

$m_W = 80.363 \pm 0.032$ GeV PDG Mass $m = 80.385 \pm 0.015$ GeV

m_{top}

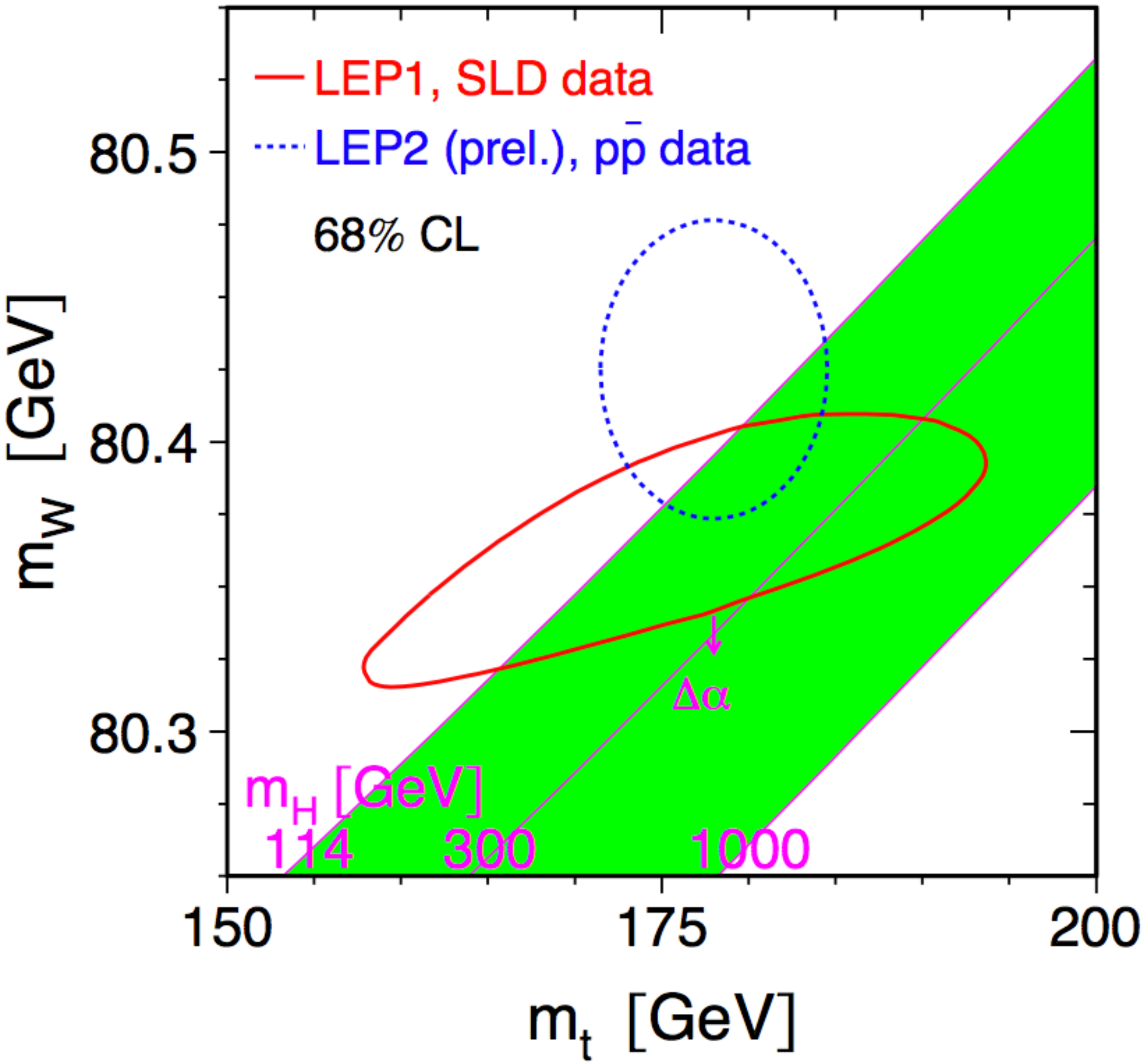


Top-Quark Mass [GeV]
CDF
176.1 ± 6.6
DØ
179.0 ± 5.1
Average
178.0 ± 4.3
χ²/DoF: 2.6 / 4
LEP1/SLD
172.6 + 13.2 − 10.2
LEP1/SLD/mW/ΓW
181.1 + 12.3 − 9.5
125
150
175
200
mt [GeV]



200
150
100
50
Mt [GeV]
Tevatron
SM constraint 68% CL
Direct search lower limit (95% CL)
1990
1995
2000
2005
Year

m_W vs. m_{top}


LEP1, SLD data
LEP2 (prel.), pp̄ data
68% CL
80.5
80.4
80.3
mW [GeV]
Δα
mH [GeV]
114
300
1000
150
175
200
mt [GeV]

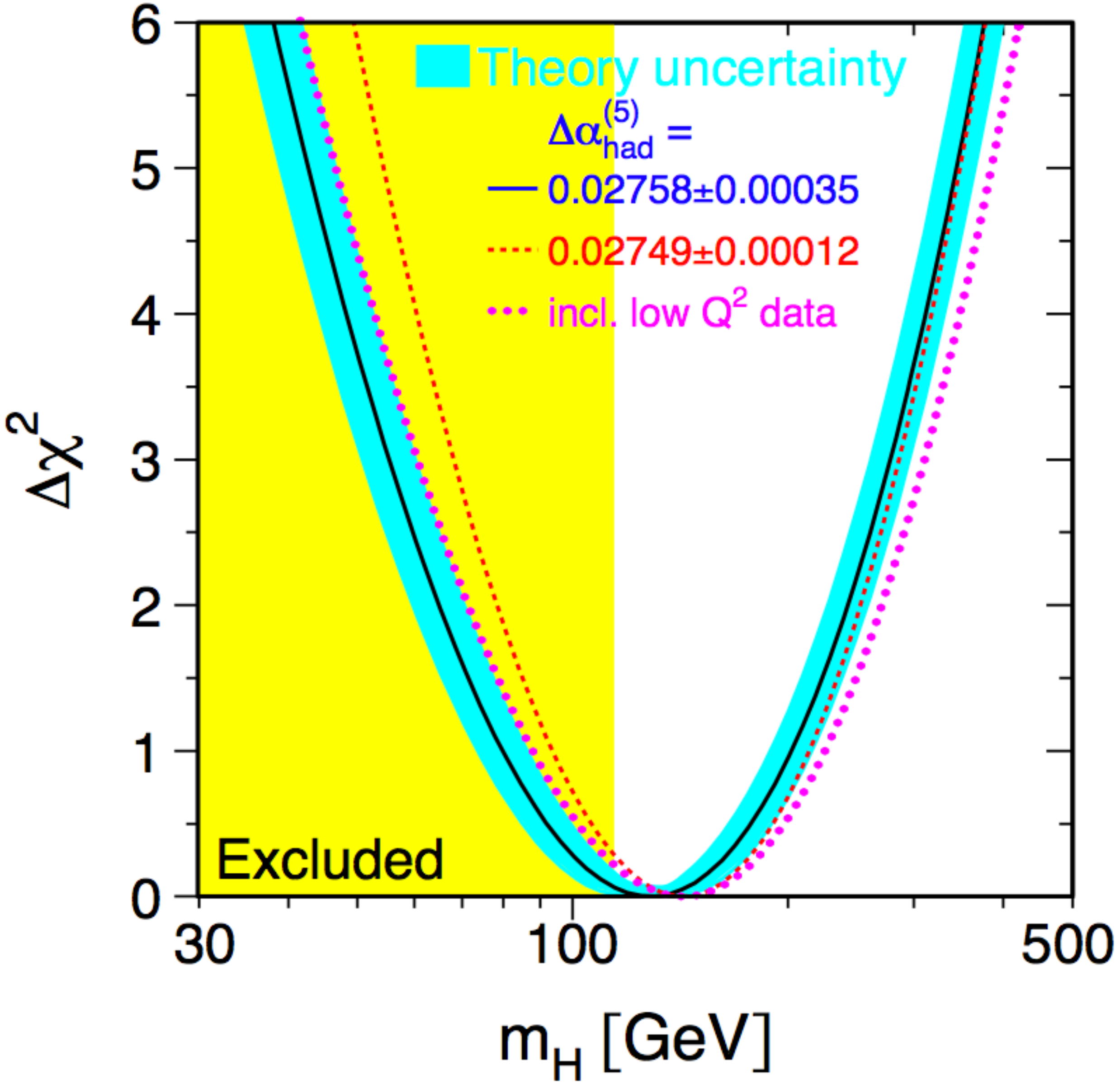
m_H @ LEP

Use the same inputs as before and add m_t, m_W, Γ_W from LEP2/TeVatron results

X^2/ndof = 18.3/ 13 (probability 15%)

Parameter	Value	Correlations				
		$\Delta\alpha^{(5)}_{\rm had}(m_Z^2)$	$\alpha_S(m_Z^2)$	m_Z	m_t	$\log_{10}(m_H/\mathrm{GeV})$
$\Delta\alpha^{(5)}_{\rm had}(m_Z^2)$	0.02767±0.00034	1.00				
$\alpha_S(m_Z^2)$	0.1188±0.0027	−0.02	1.00			
m_Z [GeV]	91.1874±0.0021	−0.01	−0.02	1.00		
m_t [GeV]	178.5±3.9	−0.05	0.11	−0.03	1.00	
$\log_{10}(m_H/\mathrm{GeV})$	2.11±0.20	−0.46	0.18	0.06	0.67	1.00
m_H [GeV]	129^{+74}_{-49}	−0.46	0.18	0.06	0.67	1.00

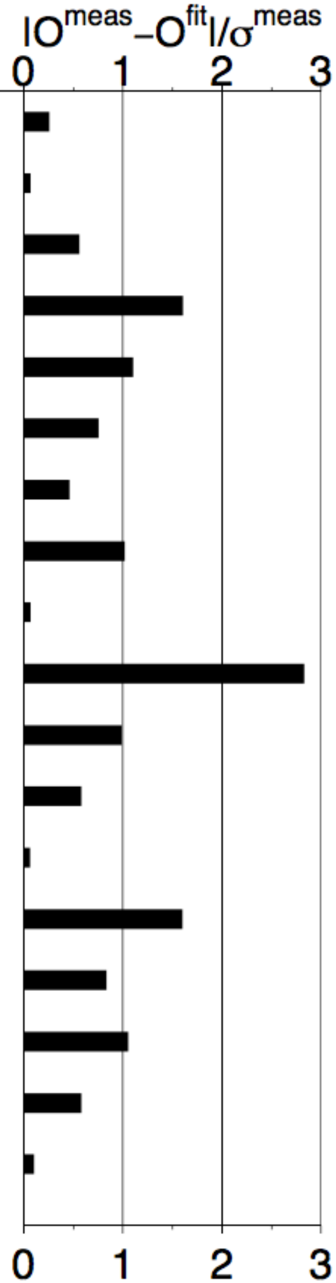
m_H @ LEP



Theory uncertainty
$\Delta\alpha_{had}^{(5)}$ =
0.02758±0.00035
0.02749±0.00012
incl. low Q² data
Excluded
$\Delta\chi^2$
6
5
4
3
2
1
0
30
100
500
m_H [GeV]

Pulls

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ_{had}^{0} [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5



Bibliography

Precision Electroweak Measurements on the Z Resonance
(our discussion is in chapter 1 and 8)

http://arxiv.org/abs/hep-ex/0509008

Fit procedure:

L. Lyons, D. Gibaut, and P. Clifford, Nucl. Instrum. Meth. A270 (1988) 110;
A. Valassi, Nucl. Instrum. Meth. A500 (2003) 391–405.