Elementary Particle and Nuclear Physics
Summary
Jacob Shapiro
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js@yashi.org

Abstract
A very rough translation of Prof. Klaus Stefan Kirch’s lecture notes from German to English in an attempt to prepare for his exam.

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</table>
1 Introduction

1.1 Motivation

I was forced to study this course as a prerequisite for my degree.

1.2 General

1.2.1 Units and Orders of Magnitudes

- $1eV = 1.602 \cdot 10^{-19} J$
- $1fm = 10^{-15} m$
- $1u = \frac{1}{12} M_{12C} \approx 1.66 \cdot 10^{-27} kg \approx 931.5 MeV/c^2$

1.2.1.1 Natural Units

- $h = 6.582 \cdot 10^{-16} eV \cdot s$, but if we set $\frac{1}{12} = 1 \iff 6.582 \cdot 10^{-16} eV \cdot \frac{1}{12} = 1$ from which we could get the relationship between $s$ and $eV$: $s = 1.519 \cdot 10^{15} (eV)^{-1}$.

- $c = 2.99 \cdot 10^{8} \frac{m}{s}$. Setting $\frac{1}{12} = 1$, we get a relationship between $m$ and $s$, which in turn gives us a relationship between $m$ and $eV$: $m = 5.08 \cdot 10^{9} (eV)^{-1}$
• $1eV = 1.6 \cdot 10^{-9} Joules = 1.6 \cdot 10^{-19} \frac{kg \cdot m^2}{s^2}$, from which we infer that $kg = 5.58 \cdot 10^{35} eV$.

• $1 \text{barn} = 10^{-24} cm^2 \implies (GeV)^{-2} = 0.389 \text{mb}$.

• $1 e(V^2) = 1.68 \cdot 10^{-2} Tesla$

### 1.2.2 Constants and Relations

• $\hbar = 6.582 \cdot 10^{-22} MeV s = 197 MeV fm c^{-1} \implies \hbar c \approx 200 MeV fm$

• $1 fm = 5.08 \cdot (GeV)^{-1}$.

• $\alpha = \frac{e^2}{4\pi\hbar c} \approx \frac{1}{137}$

• de Broglie wave length: $\lambda = \frac{h}{p} = \frac{2\pi\hbar}{pc}$

• Compton wave length: $\lambda = \frac{2\pi\hbar}{mc}$

### 1.2.3 Table of Nuclides

- $N =$ number of neutrons
- $Z =$ number of protons
- Black = stable nuclei
- Colored: unstable nuclei
– orange = $\beta^+$ decay
– blue = $\beta^-$ decay
– yellow = $\alpha$-decay

• above about $N = Z = 20$, more neutrons are necessary to balance the repulsive Coulomb force between the protons.

1.2.3.1 Isotopes of $Z$ protons The collection of all nuclei with $Z$ protons and different number of neutrons.

1.2.3.2 Isotones of $N$ neutrons The collection of all nuclei with $N$ neutrons and different number of protons.

1.2.3.3 Isobars of mass $A$ The collection of all nuclei with the same sum $A = N + Z$, but different numbers of protons and neutrons.

1.2.3.4 $\beta^-$-Decay
Let $A(Z, N)$ denote a nucleus of $Z$ protons and $N$ neutrons. The $\beta^-$-decay is the following process:

$$A(Z, N) \rightarrow A(Z + 1, N - 1) + e^- + \nu_e$$

1.2.3.4.1 Example–Neutron Decay $A(0, 1) \rightarrow A(1, 0) + e^- + \nu_e$ is the process of decay of one free neutron.

1.2.3.5 $\beta^+$-decay $A(Z, N) \rightarrow A(Z - 1, N + 1) + e^+ + \nu_e$

1.2.3.6 Double $\beta$-decay Using the semi-empirical mass formula we can employ the inequality $M(A, Z) \geq M(Z + 2, A) + 2m_e$ and thus get a relation on when a double beta decay is possible:

$$Z \leq \frac{\ln 2}{1 - 130.46A^{-2/3}}$$
1.2.3.7 α-decay \( A(Z, N) \rightarrow A(Z - 2, N - 2) + {}^4\text{He} \) (leptons are not mentioned in this reaction)

1.3 Special Relativity

- \( E_{\text{total}} = E_{\text{kinetic}} + m = \gamma m = \frac{m}{\sqrt{1 - \beta^2}} \)
- \( p = Ev = \gamma mv \)
- \( \beta = \sqrt{1 - \left( \frac{m}{E_{\text{total}}} \right)^2} \)
- \( x_{\mu} = \begin{bmatrix} t \\ x \end{bmatrix} \)
- \( p_{\mu} = \begin{bmatrix} E \\ p \end{bmatrix} \)
- \( a_\mu b^\mu = a_\mu b^\mu - a \cdot b \)
- \( p_{\mu}p^\mu = E^2 - p^2 = m^2 \) is the invariant mass. It stays the same regardless of reference frame.
- 4-momentum is conserved in reactions.
• $\Lambda_{x^k} := \begin{bmatrix} \gamma & -\gamma \beta & 0 & 0 \\ -\gamma \beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \implies x' = \Lambda x$

• For a charged particle in a magnetic field, $R = \frac{p}{qB} = \frac{m}{qB} \sqrt{\frac{T}{m} + \left(\frac{T}{m}\right)^2}$

### 1.3.0.8 Mandelstam Variables

They are used for scattering processes of two particles to two particles.

$s := (p_1 + p_2)^2 = (p_3 + p_4)^2$

$t := (p_1 - p_3)^2 = (p_2 - p_4)^2$

$u := (p_1 - p_4)^2 = (p_2 - p_3)^2$

• Easy to prove that $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$.

• For highly relativistic, $t \approx -4E_2E_4 \sin^2(\theta/2)$.

### 1.3.0.9 Motion of charged particle in Magnetic Field

$p = qBR \implies R = \frac{p}{qB} = \frac{\sqrt{(m+E_{kin})^2-m^2}}{qB}$

### 1.4 Statistics

Terms:
• FWHM: Width between two half maximum points.
• Expectation Value: \( E [X] := \int x f (x) \, dx \)
• Standard Deviation: \( STD := \sqrt{E \left( (X - E [X])^2 \right)} \)
• The error in \( n \) experiments, where each experiment has error \( \sigma \), is \( \sigma \sqrt{n} \).
• Covariance: \( Cov (X, Y) := E [(X - E [X]) (Y - E [Y])] \)

1.4.1 Gaussian Distribution
\[
f (x) := \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \frac{x^2}{\sigma^2}}
\]
Standard deviation = \( \sigma \), expectation value = 0, fwhm = \( 2\sqrt{2 \log(2)} \sigma \).

1.4.2 Binomial Distribution
\[
f (k; n, p) := \binom{n}{k} p^k (1 - p)^{n - k}
\]
Probability to get \( k \) “successes” in \( n \) trials, where \( p \) is the probability to get a single isolated success.

1.4.3 Poisson Distribution
Discrete distribution. If the (theoretical) average rate of events per time period \( T \) is given by \( \lambda \), the probability that \( k \) actual events take place (where \( k \in \mathbb{N} \cup \{0\} \)) during time period \( T \) (a random variable \( X \)) is given by \( P_{\text{Poisson}} [X = k] = e^{-\lambda} \frac{\lambda^k}{k!} \).
• \( E [X] = \lambda \)
• \( Var [X] = \lambda \)
• In the limit of \( \lambda \to \infty \), this distribution converges to the normal distribution.

1.4.4 Error Propagation
If \( u = f (x, y) \) is some variable that depends on two independent variables, then
\[
\sigma_u \approx \sqrt{\left( \partial_x f (\bar{x}, \bar{y}) \right)^2 \sigma_x^2 + \left( \partial_y f (\bar{x}, \bar{y}) \right)^2 \sigma_y^2}.
\]

1.4.5 Principle of Least Squares
Todo
1.4.6 \( \chi^2 \)-distribution and \( \chi^2 \)-test statistics

\[
\chi^2 = \sum_{i=1}^{N} \left( \frac{y_i - f(x; \alpha)}{\sigma_i} \right)^2
\]

Minimize \( \chi^2 \) by the parameters of \( f \) to get them.

Can also get the variance on these parameters as the covariance matrix.

\[
E(\chi^2_{\text{min}}) = N - P
\]

where \( N \) is the number of measurements and \( P \) is the number of parameters to be estimated. The probability that \( \chi^2 \) exceeds a certain value can be calculated.

1.4.7 Confidence Intervals

Confidence Intervals (CIs) are statistics tools used to express our estimate of a certain unknown parameter (which has some definite fixed value). The CI with confidence level \( CL \) for the estimate of the parameter \( w \) according to our observed outcome \( X \) can be defined as a random interval \([u(X), v(X)]\) for which the probability to include the real value \( w \) is equal \( CL \).

1.4.8 Method of Maximum Likelihood

In statistics, maximum-likelihood estimation (MLE) is a method of estimating the parameters of a statistical model. When applied to a data set and given a statistical model, maximum-likelihood estimation provides estimates for the model’s parameters.

We would like to estimate a parameter \( \alpha \) from \( n \) measurements. The probability distribution function is \( f(x; \alpha) \). Define \( L(\alpha) := \prod_{i=1}^{n} f(x_i; \alpha) \), differentiate \( \frac{\partial L}{\partial \alpha} = 0 \), find \( \alpha_{\text{max}} \).

1.4.9 Method of Least squares

\[
\chi^2 := \sum_{i=1}^{N} \left( \frac{y_i - f(x_i; \alpha)}{\sigma_i} \right)^2
\]

\[
\frac{\partial \chi^2}{\partial \alpha} = 0
\]

1.5 Clebsch-Gordan Coefficients

1.5.1 Theorem

Let \( j_1 > j_2 \). Then \( D_{j_1} \otimes D_{j_2} = D_{j_1+j_2} \oplus D_{j_1+j_2-1} \oplus \cdots \oplus D_{|j_1-j_2|} \).

Thus

\[
|j_1, j_2, j, m \rangle = \sum_{m_1, m_2, m_1+m_2=m} \langle j_1, j_2, m_1, m_2 | j_1, j_2, j, m \rangle |j_1, j_2, m_1, m_2 \rangle
\]

where \( \langle j_1, j_2, m_1, m_2 | j_1, j_2, j, m \rangle \) are the Clebsch-Gordan coefficients.

As an example, adding two spin-\( \frac{1}{2} \) angular momenta:
We can read off this table for example that $| j = 0, m = 0 \rangle = \frac{1}{\sqrt{2}} | m_1 = \frac{1}{2}, m_2 = -\frac{1}{2} \rangle - \frac{1}{\sqrt{2}} | m_1 = -\frac{1}{2}, m_2 = \frac{1}{2} \rangle$.

This table can also be constructed with the usage of the lower and raising operators.
1.6 Particles in the Standard Model

Lifetimes are mean lifetimes in seconds \( P(t) = e^{-t/\tau} \):

<table>
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<th>Leptons (spin 1/2)</th>
</tr>
</thead>
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<tr>
<td><strong>Generation</strong></td>
</tr>
<tr>
<td>first</td>
</tr>
<tr>
<td>second</td>
</tr>
<tr>
<td>third</td>
</tr>
</tbody>
</table>

*Neutrino masses are extremely small, and for most purposes can be taken to be zero; for details see Chapter.*
### Quarks (spin 1/2)

<table>
<thead>
<tr>
<th>Generation</th>
<th>Flavor</th>
<th>Charge</th>
<th>Mass</th>
<th>Mass*</th>
</tr>
</thead>
<tbody>
<tr>
<td>first</td>
<td>d (down)</td>
<td>-1/3</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>u (up)</td>
<td>2/3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>second</td>
<td>s (strange)</td>
<td>-1/3</td>
<td>120</td>
<td>1200</td>
</tr>
<tr>
<td></td>
<td>c (charm)</td>
<td>2/3</td>
<td>1200</td>
<td></td>
</tr>
<tr>
<td>third</td>
<td>b (bottom)</td>
<td>-1/3</td>
<td>4300</td>
<td>17400</td>
</tr>
<tr>
<td></td>
<td>t (top)</td>
<td>2/3</td>
<td>17400</td>
<td></td>
</tr>
</tbody>
</table>

*Light quark masses are imprecise and speculative; for effective masses in mesons and baryons, see Chapter 5.

### Mediators (spin 1)

<table>
<thead>
<tr>
<th>Force</th>
<th>Mediator</th>
<th>Charge</th>
<th>Mass*</th>
<th>Lifetime</th>
<th>Principal Decays</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>g (8 gluons)</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\gamma$ (photon)</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
<td></td>
</tr>
<tr>
<td>Weak</td>
<td>$W^\pm$ (charged)</td>
<td>$\pm 1$</td>
<td>80,420</td>
<td>$3.11 \times 10^{-25}$</td>
<td>$e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau, cX \rightarrow$ hadrons</td>
</tr>
<tr>
<td></td>
<td>$Z^0$ (neutral)</td>
<td>0</td>
<td>91,190</td>
<td>$2.64 \times 10^{-25}$</td>
<td>$e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q} \rightarrow$ hadrons</td>
</tr>
</tbody>
</table>

### Baryons (spin 1/2)

<table>
<thead>
<tr>
<th>Baryon</th>
<th>Quark Content</th>
<th>Charge</th>
<th>Mass</th>
<th>Lifetime</th>
<th>Principal Decays</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>$uud$</td>
<td>1</td>
<td>938.272</td>
<td>$\infty$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$udd$</td>
<td>0</td>
<td>939.565</td>
<td>885.7</td>
<td></td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>$uds$</td>
<td>0</td>
<td>1115.68</td>
<td>$2.63 \times 10^{-10}$</td>
<td>$\pi^-\pi^0$</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>$uus$</td>
<td>1</td>
<td>1189.37</td>
<td>$8.02 \times 10^{-11}$</td>
<td>$\pi^0, \pi^+$</td>
</tr>
<tr>
<td>$\Sigma^0$</td>
<td>$uds$</td>
<td>0</td>
<td>1192.64</td>
<td>$7.4 \times 10^{-20}$</td>
<td>$\Lambda \gamma$</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>$dds$</td>
<td>-1</td>
<td>1197.45</td>
<td>$1.48 \times 10^{-10}$</td>
<td>$\pi^-\pi^0$</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>$uss$</td>
<td>0</td>
<td>1314.8</td>
<td>$2.90 \times 10^{-10}$</td>
<td>$\Lambda^-\pi^+$</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>$dss$</td>
<td>-1</td>
<td>1321.3</td>
<td>$1.64 \times 10^{-10}$</td>
<td>$pK\pi, \Lambda\pi\pi, \Sigma\pi\pi$</td>
</tr>
<tr>
<td>$\Lambda^+$</td>
<td>$udc$</td>
<td>1</td>
<td>2286.5</td>
<td>$2.00 \times 10^{-13}$</td>
<td></td>
</tr>
</tbody>
</table>

### Baryons (spin 3/2)

<table>
<thead>
<tr>
<th>Baryon</th>
<th>Quark Content</th>
<th>Charge</th>
<th>Mass</th>
<th>Lifetime</th>
<th>Principal Decays</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta$</td>
<td>$uuu$, $uud$, $udd$, $ddd$</td>
<td>2,1,0,–1</td>
<td>1232</td>
<td>$5.6 \times 10^{-24}$</td>
<td>$N\pi$</td>
</tr>
<tr>
<td>$\Sigma^*$</td>
<td>$uus$, $uds$, $dds$</td>
<td>1,0,–1</td>
<td>1385</td>
<td>$1.8 \times 10^{-23}$</td>
<td>$\Lambda\pi, \Sigma\pi$</td>
</tr>
<tr>
<td>$\Xi^*$</td>
<td>$uss$, $dss$</td>
<td>0,–1</td>
<td>1533</td>
<td>$6.9 \times 10^{-23}$</td>
<td>$\Xi\pi$</td>
</tr>
<tr>
<td>$\Omega^-$</td>
<td>$sss$</td>
<td>−1</td>
<td>1672</td>
<td>$8.2 \times 10^{-11}$</td>
<td>$\Lambda K^-, \Xi\pi$</td>
</tr>
</tbody>
</table>
1.7 Elementary Particles

An elementary particle is such that cannot be decomposed into constituent particles.

1.7.1 Fermionic Elementary Particles

There are two types of particles: quarks and leptons. They are arranged into three families according to mass.

Muons have been discovered in cosmic rays.

Each of these particles has an anti-particle, marked with a bar above its symbol, or opposite electric charge.

---

### Pseudoscalar Mesons (spin 0)

<table>
<thead>
<tr>
<th>Meson</th>
<th>Quark Content</th>
<th>Charge</th>
<th>Mass</th>
<th>Lifetime</th>
<th>Principal Decays</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^\pm$</td>
<td>$ud, d\bar{u}$</td>
<td>1,-1</td>
<td>139.570</td>
<td>$2.60 \times 10^{-8}$</td>
<td>$\mu\nu_\mu$</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>$(u\bar{u} - d\bar{d})/\sqrt{2}$</td>
<td>0</td>
<td>134.977</td>
<td>$8.4 \times 10^{-17}$</td>
<td>$\gamma\gamma$</td>
</tr>
<tr>
<td>K$^\pm$</td>
<td>$u\bar{s}, s\bar{u}$</td>
<td>1,-1</td>
<td>493.68</td>
<td>$1.24 \times 10^{-8}$</td>
<td>$\mu\nu_\mu, \pi\pi, \pi\pi\pi$</td>
</tr>
<tr>
<td>K$^0, \bar{K}^0$</td>
<td>$d\bar{s}, s\bar{d}$</td>
<td>0</td>
<td>497.65</td>
<td>$\begin{cases} K^0_\ell : 8.95 \times 10^{-11} \ K^0_\mu : 5.11 \times 10^{-8} \end{cases}$</td>
<td>$\pi\pi, \pi\nu_\mu, \pi\nu_\mu, \pi\pi\pi$</td>
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<tr>
<td>$\eta$</td>
<td>$(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$</td>
<td>0</td>
<td>547.51</td>
<td>$5.1 \times 10^{-19}$</td>
<td>$\gamma\gamma, \pi\pi\pi$</td>
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<tr>
<td>$\eta'$</td>
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<td>957.78</td>
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<td>D$^{\pm}$</td>
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<td>$1.04 \times 10^{-12}$</td>
<td>$K\pi\pi, K\mu\nu_\mu, Ke\nu_e$</td>
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<td>D$^0, \bar{D}^0$</td>
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<td>D$^{*+}$</td>
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<td>1,-1</td>
<td>1968.2</td>
<td>$5.0 \times 10^{-13}$</td>
<td>$\eta_\pi, \phi\pi\pi, \phi\phi$</td>
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<tr>
<td>B$^{\pm}$</td>
<td>$\bar{t}b, b\bar{t}$</td>
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<td>$1.6 \times 10^{-12}$</td>
<td>$D^<em>\nu_e, D\nu_e, D^</em>\pi\pi\pi$</td>
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<td>B$^0, \bar{B}^0$</td>
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<td>$D^<em>\nu_e, D\nu_e, D^</em>\pi\pi\pi$</td>
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### Vector Mesons (spin 1)

<table>
<thead>
<tr>
<th>Meson</th>
<th>Quark Content</th>
<th>Charge</th>
<th>Mass</th>
<th>Lifetime</th>
<th>Principal Decays</th>
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<tbody>
<tr>
<td>$\rho$</td>
<td>$ud, (u\bar{u} - dd)/\sqrt{2}, d\bar{u}$</td>
<td>1,0,-1</td>
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<td>$K^*$</td>
<td>$u\bar{s}, d\bar{s}, s\bar{d}, s\bar{u}$</td>
<td>1,0,-1</td>
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<td>$\omega$</td>
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<td>D$^*$</td>
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<td>2008</td>
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<td>$D\pi, D\gamma$</td>
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<td>$Y$</td>
<td>$b\bar{b}$</td>
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<td>9460</td>
<td>$1 \times 10^{-20}$</td>
<td>$e^+e^-, \mu^+\mu^-, \tau^+\tau^-$</td>
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</tbody>
</table>
1.7.2 Bosonic Elementary Particles

The Gauge-Bosons are the mediators of interactions.

- The W-boson was first detected at CERN with the UA1 experiment by the decays $W^+\rightarrow e^+\nu_e$ and $W^\pm\rightarrow e^\pm\nu_e$. The mass of the W boson can be determined from the distribution of transverse momentum $p_t,e$ of the electron, or out of the $m_t$-distribution.

  - Jacobian Peak: A peak in a probability distribution which can be understood as due to the variation of a Jacobi determinant. For example, in the decay $W \rightarrow e^+\nu_e$: $\frac{d\sigma}{dp_t} = \frac{d\sigma}{d[\cos(\theta)]} = \frac{d\sigma}{d[\cos(\theta)]} \frac{2p_t}{M_W} \frac{1}{\sqrt{(M_W)^2 - p_t^2}}$

  This will give a peak at $p_t = M_W/2$.

- There are 8 gluons, because $SU(3)$ is represented by an adjoint representation of 8 dimensions.

1.7.3 Hadrons–Composite Particles

Hadrons are composed of several quarks (or anti-quarks).

- Hadrons, along with the valence quarks ($q_v$) that contribute to their quantum numbers, contain virtual quark-antiquark ($q\bar{q}$) pairs known as sea quarks ($q_s$). Sea quarks form when a gluon of the hadron’s color field splits; this process also works in reverse in that the annihilation of two sea quarks produces a gluon. The result is a constant flux of gluon splits and creations colloquially known as "the sea".[77] Sea quarks are much less stable than their valence counterparts, and they typically annihilate each other within the interior of the hadron. Despite this, sea quarks can hadronize into baryonic or mesonic particles under certain circumstances.

1.7.3.1 Baryons

Baryons are Fermionic Hadrons composed of 3 quarks. The nucleons, hyperons, etc are baryons.

- proton = uud
- neutron = udd

1.7.3.2 Mesons

Mesons are Bosonic Hadrons composed of one quark and one-antiquark. The pions, kaons, etc are mesons.

- The pions:
  - $\pi^+ = u\bar{d}$
  - $\pi^- = \bar{u}d$
  - $\pi^0 = \frac{1}{\sqrt{2}} (u\bar{u} - d\bar{d})$
• The kaons:
  
  \[ K^+ = u\bar{s} \]
  \[ K^- = \bar{u}s \]
  \[ K^0 = d\bar{s} \]

1.8 Radioactive Decay

\[ \frac{dN}{dt} = -\lambda N \]

1.9 Experiments

1.9.1 Annihilation

\[ e^- + e^+ \rightarrow \gamma + \gamma \]

22\text{Na} \rightarrow 22\text{Ne}^* + e^+ + \nu_e

22\text{Na} undergoes \beta^+ decay, which releases \( e^+ \). The material surround the source is abundant with \( e^- \) and so we get plenty of \( e^- - e^+ \) pairs which can annihilate. Two photomultipliers detect the incident photons and are connected to a coincidence box which only makes a sound when the two photomultipliers are triggers simultaneously (hence, stemming from the same event). To conserve momentum, the photons will come out back to back, because, approximately, the momentum before the annihilation is zero.

1.9.2 Cherenkov Counter

A device to detect particles.

1.9.2.1 Cherenkov Effect

When a charged particle passes through dielectric material at a speed greater than the phase velocity of light in that medium \( \frac{c}{n} \), electromagnetic radiation is emitted. The velocity of the particle is related
to the angle of the emitted EM radiation (between the path of the charged par-
ticle) by \( \beta = \frac{1}{n \cos (\theta)} \), where \( n \) is the refractive index of the incident material and \( \beta \) is the velocity (in units of \( c \)) of the particle. If the particle were moving slower than \( \frac{c}{n} \) then there’d be destructive interference and we’d observe no ra-
diation. However, when the speed of the particle is over \( \frac{c}{n} \) there is constructive interference.

So this can be exploited to detect certain particles, for example, which have the same energy, but different masses (and hence different \( \beta \)), because only if \( n\beta > 1 \) will we see Cherenkov radiation).

1.9.3 Electron Turbine

\[ r = \sqrt{\frac{2ml}{eB^2}} \]

1.9.4 Electron Diffraction

Bragg diffraction shows that electrons are also waves.

\[ n\lambda = 2d \sin (\theta) \]

1.9.5 Cloud Chamber / Wilson Chamber

On top of cold platter there’s alcohol steam and under the platter, there’s a wire grid hole, which “sucks up” the charges of the gas. Then accelerated particles can be sent through this apparatus, or, as shown in the lecture, inject radioactive material.

Particle detector for ionizing radiation: charged particles ionize gas atoms, which creates a mist. Apply magnetic field to detect charge sign.

“In its most basic form, a cloud chamber is a sealed environment containing a supersaturated vapor of water or alcohol. When a charged particle (for example, an alpha or beta particle) interacts with the mixture, it ionizes it. The resulting ions act as condensation nuclei, around which a mist will form (because the mixture is on the point of condensation). The high energies of alpha and beta particles mean that a trail is left, due to many ions being produced along the path of the charged particle. These tracks have distinctive shapes (for example, an alpha particle’s track is broad and shows more evidence of deflection by collis-
sions, while an electron’s is thinner and straight). When any uniform magnetic field is applied across the cloud chamber, positively and negatively charged particles will curve in opposite directions, according to the Lorentz force law with two particles of opposite charge.”

1.9.6 Gamma Spectroscopy

Observe the energy levels of something.
1.9.7 Radioactive Decay and Poisson Statistics

Part I
Particle Physics

2 Basics

2.1 Cross-Section and Luminocity

Read Griffiths chapter 6.

2.1.1 Cross-Section

We would like to describe a reaction of a beam of particles incident on some target.

- $n_a$ is the particle per unit volume density of the incident beam.
- $v_a$ is the velocity of each particle in the beam.
- $\phi_a := n_a v_a$ is thus the flux of incident particles.
- $n_t$ is the particle per unit volume density of the target.
- $A_t$ is the surface area of the target.
- $d$ is the thickness of the target.
- $N_t = n_t A_t d$ is the number of particles in the target.
- $\sigma$ is defined to be the projective surface of a single target particle. The total, or geometric cross-section.
- The number of reactions per unit time is thus $\dot{N} = \phi_a N_t \sigma$
- Thus $\sigma$ can also be thought of as $\frac{N}{\phi_a N_t}$, which is a measure of the probability of a reaction if an incident particle comes near a target particle.

2.1.2 Luminosity

Luminosity: the number of particles passing down the line per unit time per unit area.

- $\mathcal{L} := \phi_a N_t$
- $\dot{N} = \sigma \mathcal{L}$
2.1.3 Differential Cross-Section

- $\sigma := \int \frac{d\sigma}{d\Omega} d\Omega$

2.1.3.1 Doubled Differential Cross-Section

Also for energy values:

- $\frac{d^2\sigma(E, E', \theta)}{d\Omega dE'}$

2.2 Fermi’s Golden Rule

“A transition rate is given by the product of the phase space and the (absolute) square of the amplitude.”

The square of the transition matrix element is proportional to the probability that the particles of state $i$ goes over to the state $f$:

$M_{fi} := \langle \psi_f \mid H_{int} \mid \psi_i \rangle$

where $H_{int}$ is the Hamiltonian of the perturbation.

Thus the probability of transition between the states is given by:

$$\Gamma_{i\rightarrow f} = \frac{2\pi}{\hbar} |M_{fi}|^2 \rho(E_f)$$

where $\rho(E_f)$ is the energy state density.

The density of states is proportional to $p^2 \frac{dp}{dE}$ usually.

2.3 Feynman Diagrams

- Horizontal axis is time.
- Vertical axis is space.
- This sometimes gets reversed.
- Fermions are straight lines.
- Particles going against time are anti-particles.
- Photons are wavy-lines.
- Bosons of the Weak Interaction are dashed lines.
- Gluons are lines with loops.

1. Charge is conserved
2. Lepton number is conserved.
3. Baryon number is conserved.

- Particles that appear only inside the diagram are virtual particles.
• Charge is conserved along solid (Fermionic) lines.

• Virtual particles don’t obey the energy-momentum relation $E^2 = m^2 + p^2$

• Each vertex’ probability is proportional to a factor of $\sqrt{\alpha}$, where $\alpha$ is the coupling-constant of the interaction.

• In electron-positron annihilation, photons have a virtual mass.

2.4 Interactions

2.4.1 Model of Strong Interactions

The strong force between protons and neutrons, a residual force, is to the strong force, what the van der Waals force is to EM.

Nucleons exchange pions and thus form bound states.
Since the mass of the pion is \( \approx 140\text{MeV} \), we get a scale for the strength of the nuclear force.

One can fit a potential that obeys this scale, the Yukawa potential:

\[
U_Y (r) = \alpha_s \frac{\hbar c}{\lambda} e^{-\frac{r}{\lambda}}
\]

where \( \lambda = \frac{\hbar}{m_\pi c} \) is the Compton wave length of the exchanged particle and \( \alpha_s \approx 1/5 \) is the coupling constant of the strong interaction.

However, this is only an approximation: other mesons than pions can be
exchanged by the nuclear force, and the nuclear force is not a central force, there is also spin-spin and spin-orbit couplings.

The weak interaction is $10^{-12}$ orders of magnitude weaker than the strong interaction.

EM is $\frac{1}{137}$ weaker than the strong interaction
Gravitation is $10^{-39}$ orders of magnitude smaller than the strong interaction.

2.4.2 Weak Interactions

The weak interaction is responsible for decay and transition of particles. It has a very short range.

2.4.3 Gravitation

Gravitation has no effect on the phenomena we’re dealing with.

3 Scattering

Scattering is used to probe the inner structure of matter.

“The backscatter peak is caused by such primary gamma rays that have first interacted by Compton scattering in the surrounding material.” It is usually the case that most of the photons emerge with angle $\pi$ and energy $\frac{1}{2}m_e$.

- Compton scattering spectrum: $E' = \frac{E}{1 + \frac{E}{m_e}[1 - \cos(\theta)]}$ or $\lambda' - \lambda = \frac{1 - \cos(\theta)}{m_e}$

Incident electron is at rest.

3.0.3.1 Three Body Collision $A \rightarrow B + C + D$

- $E_B = \frac{m_A^2 + m_B^2 - (P_A - P_B)^2}{2m_A}$

- Maximal value for $E_B$ is $E_B = \frac{m_A^2 + m_B^2 - (m_C + m_D)^2}{2m_A}$.

- Minimal value for $E_B$ is $E_B = m_B$.

3.1 Bethe-Bloch Formula

“charged particles moving through matter interact with the electrons of atoms in the material. The interaction excites or ionizes the atoms. This leads to an energy loss of the traveling particle. The Bethe formula describes the energy loss per distance travelled of swift charged particles (protons, alpha particles, atomic ions, but not electrons—their mass is too small for this approximation) traversing matter (or alternatively the stopping power of the material). “

For a particle with speed $v$, charge $z$, and energy $E$, traveling a distance $x$ into a target of electron number density $n$ and mean excitation potential $I$, the relativistic version of the formula reads:
\[-\frac{dE}{dx} = \frac{4\pi}{m_e c^2} \cdot \frac{m_Z^2}{\beta^2} \cdot \left( \frac{e^2}{4\pi \varepsilon_0} \right)^2 \cdot \left[ \ln \left( \frac{2m_e c^2 \beta^2}{1 - \beta^2} \right) - \beta^2 \right]\]

where \( c \) is the speed of light and \( \varepsilon_0 \) the vacuum permittivity, \( = \frac{v}{c} \), \( e \) and \( m_e \) the electron charge and rest mass respectively.

- Observe that in the Bethe-Bloch formula the mass of the incident particle does not appear.
- Memorize that \(-\frac{dE}{dx} \propto \frac{Z^2}{\beta^2}\) or
  \[-\frac{dE}{dx} = C_1 \frac{Z_1^2}{\beta_1^2} \left[ \ln \left( \frac{C_2 \beta_1^2}{1 - \beta_1^2} \right) - \beta_1^2 \right]\]

- If two charged particles of mass \( m_1 \) and \( m_2 \) go and charge \( Z_1 e \) and \( Z_2 e \) go through the same material, their energy losses are related by the equation:
  \[ \frac{dE_2}{dx} = Z_2^2 \frac{dE_1}{dx} (m_1/m_2) \]
- For low energies, the range is proportional to \( \frac{E}{\pi^2} \).

### 3.1.1 Radiation Length and Mean Free Path

"In physics, the radiation length is a characteristic of a material, related to the energy loss of high energy, electromagnetic-interacting particles with it."

A property of the material that describes how much energy is lost for a high energy electron travelling in a medium:

\[-\left\langle \frac{dE}{dx} \right\rangle = \frac{E}{X_0}\]

Thus the energy of the particle is given by:

\[ E(x) = E_0 e^{-x/x_0} \]

Thus the mean free path is the average length an electron goes through in a medium until it collides ionizes an atom.

There’s also the pair-building mean free path, which is the length a photon goes before doing pair-production: \( X_p = \frac{2}{\beta} X_0 \).

Usually the mean free path is given together multiplied particle density already. Thus the actual mean free path, for lead for example, if it is given as 6.4g/cm² and the density of lead is 11.3g/cm³, is \( X_0 \approx 0.566 \text{cm} \).

There is a critical energy under which it no longer is possible to make pair-production or to do Bremsstrahlung and the energy of a particle is simply deposited as ionization. The number of particles at the end of such a cycle is roughly \( E_0 / E_C \).

In an EM calorimeter, \( \Delta E \approx 0.05\sqrt{E} \).
3.2 Elastic Scattering

3.2.1 Rutherford Scattering

In 1909 Rutherford sent \( \alpha \) particles at a thin gold foil.

\[
\frac{d\sigma}{d\Omega}_{\text{Rutherford}} = \left( \frac{Z_1 Z_2 \alpha \hbar c}{4E_{\text{kinetic}}} \right)^2 \sin^{-4}\left(\frac{\theta}{2}\right).
\]

Can be derived quantum mechanically from Fermi’s golden rule by assuming plane-waves and delta-function charge distribution.

3.2.2 Mott-Scattering (no recoil, with e-spin, no nucleon-spin)

“Mott scattering, also referred to as spin-coupling inelastic Coulomb scattering, is the separation of the two spin states of an electron beam by scattering the beam off the Coulomb field of heavy atoms. It is mostly used to measure the spin polarization of an electron beam.”

Mott scattering adds the effect of the spin of the electron to the Rutherford scattering:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Rutherford}} \left[ 1 - \beta^2 \sin^2\left(\frac{\theta}{2}\right) \right]
\]

The asteric means recoil of the target has not been taken into account.

3.2.2.1 Helicity

\[ h := \frac{s \cdot p}{|s||p|} = \pm 1 \] When \( h = 1 \) the particle is right handed. When \( h = -1 \) the particle is left handed. For highly relativistic particles, helicity is a conserved quantity.

3.2.3 Form Factor and Charge Distribution in the Nucleon

The form factor is the Fourier transform (into momentum-space) of the charge distribution of the target.

- \( \rho(r) = \rho_0 \exp\left(-\frac{r-a}{b}\right) \) where \( a = 1.07A^{1/3} \) and \( b \approx 0.54 \text{fm} \).
- Thus \( Ze = \int dV \rho \) and \( F(q^2) = \frac{1}{Ze} \int dV \rho(r) e^{-iqr} \)

It turns out, that in experiments, we don’t measure \( \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}}^* \), but something else. It is sensible to assume that what we measure is:

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{measured}} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}}^* |F(q)|^2
\]

we could assume fit the measurements for example to a charge distribution of the form:
\[ f(r) = \frac{f_0}{1 + e^{r/a}} \]

a kind of Fermi distribution.

From that we can deduce the relationship between the nucleus radius and the number of nucleons in it:

\[ R \approx 1.21 \text{fm} \cdot A^{\frac{1}{3}} \]

### 3.2.4 Mott Scattering (with recoil, with e-spin, no nucleon-spin)

If we take into account the recoil of the target:

\[
\left( \frac{d\sigma}{d\Omega} \right)_\text{Mott} = \left( \frac{d\sigma}{d\Omega} \right)_* \frac{E}{E'}
\]

where \( E \) is the initial energy of the incident particle and \( E' \) is the final energy of the incident particle.

### 3.2.5 Scattering of Point Spin-Particles (with Recoil, with both spins)

If we take the spin-spin interactions into account we get:

\[
\left( \frac{d\sigma}{d\Omega} \right)_\text{Point Spin-1/2} = \left( \frac{d\sigma}{d\Omega} \right)_\text{Mott} \left[ 1 + 2\tau \tan^2 \left( \frac{\theta}{2} \right) \right]
\]

where \( \tau := \frac{(p-p')^2 - (E-E')^2}{4M^2c^2} \) and \( M \) is the mass of the target. If we denote \( Q^2 \) as minus the transferred 4-momentum we get \( \tau = \frac{Q^2}{4M^2c^2} \).

#### 3.2.5.1 Nuclear Magneton

- \( \mu_N = \frac{e\hbar}{2M_p} \approx 3.1525 \cdot 10^{-14} \text{MeV}/T \)
- \( \mu_p = +2.79\mu_N \) for protons.
- \( \mu_n = -1.91\mu_N \) for neutrons.

### 3.2.6 Rosenbluth Formula (Griffiths Chapter 8.2)

But the nucleon is not a point particle, so we must correct for that with two form factors: one for the electric interaction and another for the magnetic interaction.

\[
\left( \frac{d\sigma}{d\Omega} \right)_\text{nucleon} = \left( \frac{d\sigma}{d\Omega} \right)_\text{Mott} \left[ A(Q^2) + B(Q^2) \frac{Q^2}{2M^2c^2} \tan^2 \left( \frac{\theta}{2} \right) \right]
\]
\[
\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \left( \frac{\theta}{2} \right) \right]
\]

- **Magnetischer Formfaktor:** \(G_M^2(Q^2) := B(Q^2)\),

- **Elektrischer Formfaktor:** \(G_E^2(Q^2) := A(Q^2)(1 - \tau) - \tau G_M^2(Q^2)\).

\[
\lim_{Q^2 \to 0} [G_E(Q^2)]^2 = \text{the electric charge.}
\]

- \(\lim_{Q^2 \to 0} [G_E(Q^2)]^2 = 1\) for the proton.
- \(\lim_{Q^2 \to 0} [G_E(Q^2)]^2 = 0\) for the neutron.

\[
\lim_{Q^2 \to 0} [G_M(Q^2)]^2 = \text{the magnetic moment.}
\]

- \(\lim_{Q^2 \to 0} [G_M(Q^2)]^2 = 2.79\) for the proton
- \(\lim_{Q^2 \to 0} [G_M(Q^2)]^2 = -1.91\) for the neutron.

\[
E' = \frac{E}{1 + (2E/Mc^2) \sin^2(\theta/2)}
\]

\[
\frac{d\sigma}{d\Omega} = \left( \frac{\alpha \hbar}{4\pi E \sin^2(\theta/2)} \right)^2 \frac{E'}{E} [2K_1 \sin^2(\theta/2) + K_2 \cos^2(\theta/2)]
\]

\[
K_1 = -q^2 G_M^2, \quad K_2 = \frac{(2Mc)^2 G_E^2 - [q^2/(2Mc)^2]G_M^2}{1 - [q^2/(2Mc)^2]}
\]

### 3.2.7 Experimental Determination of the Form Factors

The cross-section, for a fixed momentum transfer is measured, in various angles, and is fitted against the Rosenbluth-Formula: It is divided by the calculated Mott cross section and then plotted as a function \(\tan^2 \left( \frac{\theta}{2} \right)\): the slope and intersect can give the magnetic and electric form factors then:
By making the experiment with a fixed value of $Q^2$, one can try to fit a model of the ansatz for $G(Q^2)$, for instance, $G(Q^2) \propto \left(1 + \frac{Q^2}{a^2} \right)^{-2}$. Thus by making a Fourier transform we can get the charge distribution of a nucleon: $\rho(r) = \rho_0 e^{-ar}$ where $a \approx 4.3 \frac{1}{fm}$. 

\begin{center}
\includegraphics[width=0.5\textwidth]{image.png}
\end{center}
3.3 Inelastic Scattering

Griffiths: “Resonance - a special energy at which the particles involved 'like' to interact, forming a short-lived semibound state before breaking apart. Such ‘bumps’ in the graph of $\sigma$ vs. $E_{\text{kin}}$ are in fact the principal means by which short-lived particles are discovered.”

The name of a resonance denotes it invariant mass.

If $P$ is the 4-momentum of a target proton and $q$ is the 4-momentum of an exchanged particle, then the invariant mass, $W$, is defined as

$$W^2 := (P + q)^2$$

- the total mass available for the creation of new particles.

Define $\nu := \frac{Pq}{M}$ where $M$ is the mass of the proton. Then $\nu$ is Lorentz invariant, and (because $Q^2 := -q^2$):

$$W^2 = (P + q)^2 = P^2 + 2Pq + q^2 = M^2 + 2M\nu - Q^2$$

It is possible to show that $\nu$ is equal to the energy transfer from the electron to the proton, in the proton’s rest frame $\nu = E - E'$. 

---

Fig. 1.11 (a) In Rutherford scattering, the number of particles deflected through large angles indicates that the atom has internal structure (a nucleus). (b) In deep inelastic scattering, the number of particles deflected through large angles indicates that the proton has internal structure (quarks). The dashed lines show what you would expect if the positive charge were uniformly distributed over the volume of (a) the atom, (b) the proton. (Source: Halzen, F. and Martin, A. D. (1984) Quarks and Leptons, John Wiley & Sons, New York, p. 17. Copyright © John Wiley & Sons, Inc. Reprinted by permission.)
3.3.0.1 The Bjorken Scale Variable  The Bjorken Scale Variable $x$ is defined as $x := \frac{Q^2}{2M\nu}$ where $x \in [0, 1]$.

- For elastic scattering $W = M \implies Q^2 = 2M\nu \implies x = 1$
- For inelastic scattering $E' < E$.

The Bjorken scale variable is a measure of the elasticity of the scattering:

- For elastic scattering $x = 1$.
- For inelastic scattering $x < 1$.

$W^2 := (P')^2 = (P + q)^2 = P^2 + 2Pq + q^2 = M^2 + 2M\nu - Q^2 = M^2 + 2M\nu (1 - x)$

Thus if $x = 1 \implies W = M$, elastic.
If $x < 1 \implies W > M$, inelastic.

- The variable $x$ can be interpreted as the fraction of the proton momentum carried by the parton.

3.3.1 Width and Lifetime of Resonance

The cross section of a resonance with an energy $E_R$ can be approximated via the Breit-Wigner Distribution:

$$\frac{d\sigma}{d\Omega} \propto \frac{\Gamma^2}{(E - E_R)^2 + (\frac{\Gamma}{2})^2}$$

where $\Gamma$ is the width of the distribution.

From the Heisenberg uncertainty principle we can estimate $\Gamma \cdot \tau \approx \hbar$, where $\tau$ is the lifetime of the resonance.

- $\Gamma \propto \frac{1}{\tau} \propto m$, so the bigger the mass, the bigger the resonance width.
3.3.2 Modified Rosenbluth Formula and the Callan-Gross Relation

We would like to have a relation for the cross-section and form factors in non-elastic scattering.

We make the Ansatz:

\[
\left( \frac{d\sigma}{d\Omega} \right) = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ W_2 (Q^2, \nu) + 2W_1 (Q^2, \nu) \tan^2 \left( \frac{\theta}{2} \right) \right]
\]

where \(W_1\) and \(W_2\) are the structure functions as before. We define dimensionless functions:

\[
F_1 (x, Q^2) := MW_1 (Q^2, \nu)
\]

\[
F_2 (x, Q^2) := \nu W_2 (Q^2, \nu)
\]

where \(x := \frac{Q^2}{2M\nu}\) is the Bjorken scale variable.

If we put these definitions in the above ansatz we get the Modified Rosenbluth Formula:

\[
\left( \frac{d\sigma}{d\Omega} \right) = \left( \frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left[ \frac{1}{\nu} F_2 (x, Q^2) + \frac{2}{M} F_1 (x, Q^2) \tan^2 \left( \frac{\theta}{2} \right) \right]
\]

If we compare this formula to the scattering cross section of a spin-1/2 point particle we get:

\[
\frac{2\tau}{1} = 2\nu \frac{F_1}{M F_2}
\]

Which, if we define the \(\tau := \frac{Q^2}{4m^2}\) we get:

\[
F_2 (x) = 2xF_1 (x)
\]

Which is the Callan-Gross Relation.

If we were to experimentally find that this relation indeed holds, we could conclude that the constituents of the proton are point particles of spin-1/2.

3.3.3 Scale Refraction and Scale Invariance

? Not sure about this section...

When \(F_2\) does not depend on \(Q^2\) there is “scale breaking”.

The higher the \(Q^2\), the better the resolution and more likely it is to discern constituent particles of the proton.
3.4 Summary and Remarks

- Nucleons have point particle constituents. The inner structure of nucleons can be prescribed with the structure factors, which depend on the exchange of energy and momentum.

- If the constituents have spin-1/2 the Callan-Gross relation is observed.

- The relation between scattering neutrinos and electrons on neutrons is:

\[ F_{\nu,N}^2 = \frac{18}{5} F_{e,N}^2 \]

The factor \( \frac{18}{5} \) comes from the average of square of charge of proton vs. neutron, and then the average of that over proton and neutron. TODO

- From the fact that \( \int_0^1 F_{\nu,N}^2 (x) \, dx \approx 0.5 \), which is the part of the nucleon momentum which is carried by the quarks, we conclude that there are more constituents in the nucleon which don’t interact electromagnetically—the gluons. TODO

- There are 3 constituent quarks, each of mass approximately 300 MeV. This stands in contradiction of the pions which have two quarks and have a mass of less than 600 MeV, thus, there are also Valence Quarks which contribute to the observed properties such as charge and spin. TODO

- Scale Invariance / Scale violation: \( F_2 \) depends on \( Q^2 \! \)!

4 Collider Physics and Particle Detectors

4.1 Particle Accelerators

4.1.1 Van-De-Graff Accelerator

An electrostatic accelerator, can accelerate up to energies of 30 – 40 MeV.

A motor loads up charges (from earth) on a belt by spinning rubber near felt, this rubber belt then transmit the charges to a dome by the same mechanism.

4.1.1.1 Center of Mass Energy The center of mass energy is defined as the Lorentz-Invariant scalar product of the total four-momentum of the system:

\[ s := (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2E_1E_2 - 2p_1 \cdot p_2 \]

\[ \sqrt{s} := \sqrt{P_{\mu}P^{\mu}} \]

where \( P_\mu \) is the total 4-momentum of the system.
If the particles are in a frame of reference in which there’s no relative 3-momentum, then the momentum cannot reduce the total center of mass energy. Thus these type of experiments always provide larger center of mass energy.

4.1.1.2 Cyclotron “charged particles accelerate outwards from the center along a spiral path. The particles are held to a spiral trajectory by a static magnetic field and accelerated by a rapidly varying (radio frequency) electric field.”

The cyclotron frequency is: \( f = \frac{qB}{2\pi m} \). This frequency is independent of the radius of motion or the velocity. Thus in each revolution of the particle, the same frequency can be used to alternate the electric field so that the particle always gains kinetic energy and never loses it. The magnetic field makes the particle move in a spiral trajectory.

The upper limit on the energy of a cyclotron is \( 15\text{MeV} \) for protons and \( 10\text{MeV} \) for electrons.

4.1.1.2.1 Synchrocyclotron “A synchrocyclotron is a cyclotron in which the frequency of the driving RF electric field is varied to compensate for relativistic effects as the particles’ velocity begins to approach the speed of light.

\[
f = \frac{f_0}{\gamma} = f_0 \sqrt{1 - \beta^2}
\]

4.1.1.2.2 Isochronous Cyclotron “An alternative to the synchrocyclotron is the isochronous cyclotron, which has a magnetic field that increases with radius, rather than with time.”

4.1.2 Synchrotron

“the guiding magnetic field (bending the particles into a closed path) is time-dependent, being synchronized to a particle beam of increasing kinetic energy.”

4.1.2.1 Examples

1. LEP (Large Electron-Positron Collider) at CERN. Up to \( 206\text{GeV} \).
2. LHC (Large Hadron Collider) Proton-Proton collisions of up to \( 7\text{TeV} \). By 2014, the energies should reach \( 14\text{TeV} \).
4. DESY (Deutsches Elektronen-Syncrotron) in Hamburg. Also called HERA (Hadron-Elektron-Ring Anlage).
4.1.3 Usages outside of Particle Physics Experiments

- Synchrotron acceleration is a very good source of high energy UV and X-ray photons. These can be used to study atoms, molecules, and crystals.
- Age studies of $^{12}$C and $^{14}$C: Accelerator Mass Spectroscopy (also at the ETH).
- In medicine: radiation therapy.

4.2 Important Terms

- For a collider experiment, $\sqrt{s} = \sqrt{4E_{\text{beam}1}E_{\text{beam}2}}$
- For a fixed target experiment, $\sqrt{s} = \sqrt{2E_{\text{beam}}m_p}$

The specific luminosity $L$, is a measure of the frequency of a collision at the interaction point of an experiment. For a collider experiment, where two wave-packets with density $n_1$ and $n_2$ respectively, with frequency $f$ of collision, the specific luminosity is $L = f \frac{n_1 n_2}{4\pi \sigma_x \sigma_y}$, where $\sigma_x$ and $\sigma_y$ are the cross sections in the $x$ and $y$ directions, corresponding to the collision plane.

4.2.1 Example–Peak Luminosity LHC

In a real collider particles get lost and are inject, the luminosity varies with time. The highest reachable luminosity is called the “peak luminosity”. At the LHC it is around $2 \cdot 10^7 \frac{1}{\text{bs}}$. We can estimate the effective luminosity as $L_{\text{effective}} \approx \frac{1}{2} L_{\text{peak}}$. The number of events is given by $N = \sigma L$.

4.2.2 Example

The cross section for proton-proton collision at the LHC is around $100 m\text{b}$. The cross section for Higgs to four leptons is around $1 f\text{b}$. That would mean (with the above mentioned luminosity) around ten events per year.

4.3 Detectors

4.3.1 Structure

4.3.1.1 Tracking Chamber

Works like a cloud chamber. “We already know about the cloud chamber in the section about the experiments in the first chapter. The tracking chamber works exactly like that. Electrically charged particles ionize the gas in the chamber and thus bestowed upon ionization "clouds". Usually, however, semiconductor detectors are used today. Incident generate charged particles in a diode electron-hole pairs, which then generate the voltage which is applied a measurable current impulse. Additionally, a magnetic field is applied, so that the particles are deflected on circular paths.”
4.3.1.1 The Muon  For a muon with momentum 100GeV in a magnetic field of 1T we get a radius of 33m. The curvature is thus very weak. The sagitta of the muon is \( s = \frac{l^2}{8r} = 380\mu m \).

4.3.1.2 Electromagnetic Calorimeter–ECAL
- An example for the radiation length is \( X_0 \sim 1cm \).
- After 25\(X_0\), around 95% of the energy of the electrons and photons with an initial energy of 100GeV is absorbed.
- The number of produced particles per shower is about five to six times the radiation length.
- A shower has a transverse spread of only 3cm to 6cm.

4.3.1.3 Hadronic Calorimeter–HCAL  The Hadron Interaction Length, \( \Lambda \), is a used term instead of radiation length.
- The Hadron Interaction Length is in real experiments is around \( \Lambda = 10 - 20cm \)
- After 8\(\Lambda\) around 95% of the energy of Hadrons with initial energy of 100GeV is absorbed.
- The maximum number of produced particles per shower is around 2 – 3\(\Lambda\).
- A shower has a transverse spread of 30 – 60cm.

4.3.1.4 Muon Chamber  The muon chamber detects charged particles as well. Up to that point muons have not been absorbed. The Muon chamber allows to differentiate between muons and electrons and to determine the momentum of the muons.

4.3.2 Measurements

4.3.2.1 Neutrinos  Neutrinos cannot be detected directly so well, because they interact very seldom. But through the analysis of “missing energy” and “missing momentum” many parameters of the neutrinos can be determined. For that everything else about the reaction has to be measured, in order to invoke the conservation of energy and momentum. Other events can also account for the missing energy or momentum:
- Imperfect detector geometry means measurable particles don’t get measured.
- Failure of detector equipment.
- General measurement inaccuracies.
4.3.2.1.1 The Large Electron Collider at CERN  Electron-Positron Collisions. There were searches for $Z$-boson with $E_{CMS} = 100\text{GeV}$ and $W$-boson physics with $E_{CMS} = 206\text{GeV}$. Through these experiments the cross section and energy spectrum of $W$-$W$ bosons and certain decay branches were determined. In addition Supersymmetry was researched and the search for the Higgs Boson was conducted.

1988 Massen: $m_Z = 92.5 \pm 1.8 \text{ GeV}$, $m_W = 81.0 \pm 1.3 \text{ GeV}$
Abschätzung für die Anzahl verschiedener Neutrinos: $< 15 - 20.$
Weinbergwinkel (später): $\sin^2(\Theta_W) = 0.229 \pm 0.007.$

2000 Massen: $m_Z = 91.1876 \pm 0.0021 \text{ GeV}$, $m_W = 80.398 \pm 0.025 \text{ GeV}$
Abschätzung für die Anzahl verschiedener Neutrinos: $2.985 \pm 0.009.$
Weinbergwinkel (später): $\sin^2(\Theta_W) = 0.23119 \pm 0.00014.$

4.3.2.2 Measuring the Neutron Mass  The decay of tritium $3\text{H} \rightarrow 3\text{He} + e^- + \bar{\nu}_e$ is suitable to determine the neutrino mass.

\[
\sqrt{\frac{N(p_e)}{p^2}} (E_e) \propto \left[ 1 - \left( \frac{m_e}{E_0 - E_e} \right) \right]^{1/4} (E_0 - E_e)
\]
Experimental difficulties:

- The beta spectrum is steeply falling at the end point. As a consequence the counting rate is low, and the statistical error is big while the background becomes significant.

- Due to the finite detector resolution the Kurie plot is distorted (smeared) and has a high energetic tail, which extends beyond $E_0$.

- Electrons change their energy via internal scattering inside the source which increases the intrinsic spread of the measured energies of the electrons. This problem can be minimized using small sources but leads to a decreased counting rate.

- After the decay the daughter ions or molecules can be in an excited state with different binding energies. As a consequence we have a superposition of beta decay spectra.
• Sargent rule: in weak interactions the total transition rate is proportional to $Q^5$ where $Q$ is energy available in the reaction.

4.4 Up to Date Examples–LHC at CERN

4.4.1 The Higgs Boson

“The whole begins with the development of QED (quantum electrodynamics), which the electromagnetic interaction of massless particles (photons) completely describes. On the other hand you discover the short-range weak interaction, which is produced by massive spin-1 (vector) bosons (W and Z bosons). However, a consistent theoretical formulation of the weak interaction is possible only for massless interaction particles (Yang-Mills 1954). Peter Higgs solves this by introducing a symmetry breaking (Higgs mechanism) and a new spin-0 particle, the Higgs boson, predicted that gives the vector bosons of the weak interaction mass. Later succeeds then Glashow, Weinberg and Salam to unite the electromagnetic and weak interaction by mixtures of vector bosons and the photon construct (more on that later). These predictions are experimentally 1983/84 confirmed at CERN find this bosons. What is missing to complete the standard model is thus the Higgs boson. Some Possible actions in which a Higgs particle could arise, samples are given in Figure 4.6. Conversely, the theory predicts also decay modes and Relations, for a given energy / mass of the Higgs advance. The mass of the Higgs is unknown, so you have to search the whole energy spectrum. The theoretical branching ratios are shown in Figure 4.7. Two important observables in the Standard Model are the QED fine structure constant and the Weinberg angle (W), which is the mixing angle between photon and Z0 describes (we will introduce this detail later). According to the standard model, the experimentally measurable size $\sin^2(W)$ of the mass of the Higgs boson and the QED Feinstruktukkonsante (and the mass of the top quark) depends. We can thus testify about the Higgs boson make (because if the model should be correct).”

• Why is the decay channel $H \to ZZ$ often referred to as the “golden channel”?

1. Almost no background
2. No missing energy, the invariant mass is the mass of the Higgs Boson.
3. Easy to trigger.
4.4.2 Measurements at the LHC

- \( m_H = 114-140 \text{ GeV} \): Hauptzerfall: \( H \rightarrow \gamma \gamma \). Extrem schwierig und man benötigt mindestens 30 fb\(^{-1}\) für ein Signal.
- \( m_H = 135-155 \text{ GeV} \): Hauptzerfall: \( H \rightarrow ZZ^* \rightarrow 4 \) geladene Leptonen’ und \( H \rightarrow \ell \nu \ell \nu \). Signifikante Signale sind ab 10-20 fb\(^{-1}\) möglich.
- \( m_H = 155-180 \text{ GeV} \): Hauptzerfall: \( H \rightarrow WW \rightarrow \ell \nu \ell \nu \). Signifikante Signale sind ab 0.5-1 fb\(^{-1}\) möglich.
- \( m_H = 180-400 \text{ GeV} \): Hauptzerfall: \( H \rightarrow ZZ \rightarrow 4 \) geladene Leptonen’. Signifikante Signale sind ab 5-10 fb\(^{-1}\) möglich.
- \( m_H = 350-700 \text{ GeV} \): Hauptzerfall: \( qqH \rightarrow qqW \rightarrow q\bar{q}l\nu qg \) und \( H \rightarrow ZZ \rightarrow 4 \) geladene Leptonen’ und \( H \rightarrow ZZ \rightarrow \ell^+\ell^-\nu\nu \). Signifikante Signale sind ab 10 fb\(^{-1}\) möglich.

Im Juli 2012 wurde vom CERN bekannt gegeben, dass die beiden (unabhängigen) Experimente ATLAS und CMS ein neues Teilchen bei einer Masse von ungefähr 125-126 GeV entdeckt haben. Dieses Teilchen wird als möglicher Kandidat für das Higgs-Bozon gehandelt, es ist jedoch noch nicht mehr als die Masse dieses Teilchens bekannt und weitere Messungen sind nötig um die Eigenschaften dieses Teilchens zu bestimmen.

5 Baryons

Light baryons, made up of \( u \), \( d \) or \( s \) quarks.

5.1 Isospin

In the study of nucleons, it was found that protons and neutrons operate as identical particles under the strong interaction. In light of this, Heisenberg considered both particles as two states of the same particles, differing only by electric charge. The isospin is only defined for particles containing \( u \) and \( d \) quarks.

5.1.1 Definition of Isospin “\( z \)-component”

\[
I_z = I_3 = \frac{1}{2} \left[ (n_u - n\bar{u}) - (n_d - n\bar{d}) \right]
\]

where by \( n_i \) is the number of quarks of type \( i \).

5.1.2 Example

- Proton: \( I = \frac{1}{2}, I_z = \frac{1}{2} \).
- Neutron: \( I = \frac{1}{2}, I_z = -\frac{1}{2} \).

The \( z \)-component of the isospin is additive for a whole system: \( I_{z,\text{total}} = \sum_i I_{z,i} \).
5.1.3 Example

5.1.3.1 Mirror Nuclei

“Mirror nuclei are nuclei where the number of protons of element one (Z1) equals the number of neutrons of element two (N2), the number of protons of element two (Z2) equal the number of neutrons in element one (N1) and the mass number is the same.”

5.1.3.2 $^{14}_8\text{C}_6$ and $^{14}_6\text{O}_8$ are Mirror Nuclei

Mirror nuclei should have similar energy levels, because the strong interaction is invariant under isospin, and the only differences should stem from EM forces.

5.1.4 Remark

Experimentally it has been found that isospin is conserved by strong interactions. Thus it is a good “quantum number”.

5.2 Classification and Nomenclature

5.2.1 Strangeness

A strange quark has strangeness of $-1$. Thus the strangeness is the number of anti-quarks in the Baryon!

$$S := -(n_s - n\bar{s})$$

- It has been experimentally established that strangeness is a conserved quantum number in strong interactions, but not in weak interactions.
- Baryons with strangeness are called hyperons (but no charm or bottom).

5.2.2 Nucleons and Deltas

Nucleons are Baryons with isospin $I = \frac{1}{2}$ whereas $\Delta$-particles have isospin $I = \frac{3}{2}$.

5.2.3 Hyperon

<table>
<thead>
<tr>
<th>Name</th>
<th>$N$</th>
<th>$\Delta$</th>
<th>$\Lambda$</th>
<th>$\Sigma$</th>
<th>$\Xi$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isospin $I$</td>
<td>0/2</td>
<td>3/2</td>
<td>0</td>
<td>1</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>Strangeness $S$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
</tr>
</tbody>
</table>

5.2.4 The $\Delta$-particle

<table>
<thead>
<tr>
<th>$\Delta$</th>
<th>Isospin $I$</th>
<th>Ladung</th>
<th>Aufbau</th>
<th>$I_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta^{++}$</td>
<td>3/2</td>
<td>+2</td>
<td>$uuu$</td>
<td>+3/2</td>
</tr>
<tr>
<td>$\Delta^+$</td>
<td>3/2</td>
<td>+1</td>
<td>$uud$</td>
<td>+1/2</td>
</tr>
<tr>
<td>$\Delta^0$</td>
<td>3/2</td>
<td>0</td>
<td>$udd$</td>
<td>−1/2</td>
</tr>
<tr>
<td>$\Delta^-$</td>
<td>3/2</td>
<td>−1</td>
<td>$ddd$</td>
<td>−3/2</td>
</tr>
</tbody>
</table>
5.3 Conservation and Electric Charge

5.3.1 Baryon Number

The Baryon number is a quantum number, which, (at least as far as could be determined up until today), is always conserved. It is defined as

\[ B := \frac{n_q - n_{\bar{q}}}{3} \]

whereby \( n_q \) is total number of (any) quarks and \( n_{\bar{q}} \) is the total number of (any) anti-quarks.

- For example, \( pp \rightarrow \pi^+\pi^- \) is not allowed because baryon number is not conserved. But \( p\bar{p} \rightarrow \pi^+\pi^- \) is allowed, because baryon number before and after is zero.

5.3.2 Strangeness

The strageness is always conserved in the strong interaction. That means in transitions with exchange of gluons \( S \) is constant.

5.3.3 Electric Charge of the Baryons

The electric charge of the Baryons is given by:

\[ Q = I_3 + \frac{1}{2} (B + S) \]

5.4 Baryon Multiplets

Out of the \( sss \) delta particle, we see that there must another quantum number (due to Pauli’s exclusion principle): the color.

5.4.1 Baryon Decuplet–The Ten Fold Way

Baryons with total spin \( S = \frac{3}{2} \), orbital angular momentum of \( L = 0 \) and thus total angular momentum of \( J = \frac{3}{2} \) form a Baryon decuplet.

\( L = 0 \), so the space-wave-function is symmetric.

In order to reach \( S = \frac{3}{2} \), all spins have to be aligned at the same direction, and so the spin wave function is also symmetric.

If we have three quarks of the same flavor, then the color wave function must be anti-symmetric. This can be achieved in the following way:

\[ \phi \text{(color)} = \frac{1}{\sqrt{6}} \sum_{\alpha \in \{r, g, b\}} \sum_{\beta \in \{r, g, b\}} \sum_{\gamma \in \{r, g, b\}} \varepsilon_{\alpha\beta\gamma} |q_\alpha q_\beta q_\gamma \rangle \]

where \( \varepsilon_{\alpha\beta\gamma} \) is the anti-symmetric tensor (Levi-Civita tensor).
5.4.1.1 Example–Symmetrising a Wave Function  Symmetrizing the \( \Delta^+ = |uud\rangle \) wave function:

\[
\zeta (\text{flavor}) \chi (\text{spin}) = \frac{1}{\sqrt{3}} (|u_upd_upu\rangle + |u_upd_upu\rangle + |d_upu_upu\rangle)
\]

5.4.1.2 Remark  For three different quark flavors, there are 10 possible combinations:

5.4.2 Baryon Octet–The Eight Fold Way

Baryons with total spin \( S = \frac{1}{2} \), orbital angular momentum \( L = 0 \) and thus total angular momentum \( J = \frac{1}{2} \) form a Baryon octet.

\( L = 0 \), so the space-wave-function is symmetric.
In order for the total wave function to be anti-symmetric, we thus make, as before, the color wave function anti-symmetric and so the flavor-spin wave function should be symmetrized.

The spin is $S = \frac{1}{2}$, so the spin wave function is up,up,down, which is mixed, so the flavor wave function has to be mixed as well.

### 5.4.2.1 Example—Anti-symmetrizing the Proton

We consider the two spin-up $u$’s as one spin-1 system and the one spin-down $d$ as a spin-1/2 system (via the Clebsch-Gordan coefficients):

$$\chi_{\text{proton}} \left( J = \frac{1}{2}, m_J = \frac{1}{2} \right) = \sqrt{\frac{2}{3}} \chi_{uu} \left( 1, 1 \right) \chi_d \left( \frac{1}{2}, -\frac{1}{2} \right) - \sqrt{\frac{1}{3}} \chi_{uu} \left( 1, 0 \right) \chi_d \left( \frac{1}{2}, \frac{1}{2} \right)$$

where

$$\chi_{uu} \left( 1, 1 \right) = |u^\uparrow u^\uparrow\rangle$$

$$\chi_d \left( \frac{1}{2}, -\frac{1}{2} \right) = |d^\downarrow\rangle$$

$$\chi_{uu} \left( 1, 0 \right) = \frac{1}{\sqrt{2}} \left( |u^\uparrow u^\downarrow\rangle + |u^\downarrow u^\uparrow\rangle \right)$$

$$\chi_d \left( \frac{1}{2}, \frac{1}{2} \right) = |d^\uparrow\rangle$$

Thus

$$|p^\uparrow\rangle = \sqrt{\frac{2}{3}} |u^\uparrow u^\uparrow d^\downarrow\rangle - \sqrt{\frac{1}{6}} |u^\uparrow u^\downarrow d^\uparrow\rangle - \sqrt{\frac{1}{6}} |u^\downarrow u^\uparrow d^\uparrow\rangle$$

Now the state is symmetric for exchange of 1 and 2. To completely symmetrize it, we use the flavor symmetry:

$$|p^\uparrow\rangle = \frac{1}{\sqrt{18}} \left( 2 |u^\uparrow u^\uparrow d^\downarrow\rangle + 2 |u^\uparrow d^\uparrow u^\uparrow\rangle + 2 |d^\downarrow u^\uparrow u^\uparrow\rangle \right)$$

$$- |u^\uparrow u^\downarrow d^\uparrow\rangle - |u^\downarrow d^\downarrow u^\uparrow\rangle - |d^\uparrow u^\uparrow u^\uparrow\rangle$$

$$- |u^\downarrow u^\uparrow d^\uparrow\rangle - |u^\uparrow d^\downarrow u^\uparrow\rangle - |d^\uparrow u^\downarrow u^\uparrow\rangle$$.

Similarly the wave function for the neutron could be achieved.
5.4.2.2 The Octet

For a spin-1/2 point particle with $l = 0$ the magnetic moment is given by:

$$\mu = \frac{q\hbar}{2M}$$

Thus for quarks:

$$\mu_u = \frac{2}{3} \frac{e\hbar}{2m_u}$$
$$\mu_d = -\frac{1}{3} \frac{e\hbar}{2m_d}$$
$$\mu_s = -\frac{1}{3} \frac{e\hbar}{2m_s}.$$

Using the above wave function for the proton, we can calculate the expectation value for the magnetic moment:

5.5 Magnetic Moment

For a spin-1/2 point particle with $l = 0$ the magnetic moment is given by:

$$\mu = \frac{q\hbar}{2M}$$
\[ \langle p \mid \mu_z | p \rangle = \frac{1}{18} (4(\mu_u + \mu_u - \mu_d) + 4(\mu_u - \mu_d + \mu_u) + \cdots) \]
\[ = \frac{1}{18} (24\mu_u - 6\mu_d) \]

Thus for protons:
\[ \mu_p = \frac{1}{3} (4\mu_u - \mu_d) \]
and analogously for neutrons:
\[ \mu_n = \frac{1}{3} (4\mu_d - \mu_u) \]

Thus assuming that \( m_u \approx m_d \) we get:
\[ \frac{\mu_n}{\mu_d} = -\frac{2}{3} \]

The experimental result is:
\[ \frac{\mu_n}{\mu_p} = -0.685 \]

Thus the eight-fold way is a cool theory.

### 5.6 Mass of the Baryons

The mass of the Baryons is estimated partly by the mass of their constituents and partly by the spin-spin interactions of their constituents. Thus the mass of a Baryon is:

\[ M = m_1 + m_2 + m_2 + A \left( \frac{s_1 \cdot s_2}{m_1 m_2} + \frac{s_1 \cdot s_3}{m_1 m_3} + \frac{s_3 \cdot s_2}{m_3 m_2} \right) \]

whereby \( A \) is a constant and \( s_i \) is the spin of the \( i \)th quark.

Through experimentally measuring the mass of a Baryon we could “fit” the masses of the various quarks: \( m_u \approx m_d \approx 363 \text{ MeV}, m_s \approx 538 \text{ MeV} \).

(Derivation in POHV chapter 13.4 or 15.3)

### 5.7 π-Nucleon-Systems and Isospin Considerations

#### 5.7.1 Pions

The pions are mesons composed of:
\[ |\pi^+\rangle = |ud\rangle \]
\[ |\pi^-\rangle = |du\rangle \]
\[ |\pi^0\rangle = \frac{1}{\sqrt{2}} (|u\bar{u}\rangle - |d\bar{d}\rangle) \]
All pions have spin 0 and isospin 1. Thus the three pions form an isospin triplet:

\[ \pi^+ : I_3 = +1 \quad \pi^- : I_3 = -1 \quad \pi^0 : I_3 = 0. \]

The mass of the pions is approximately \(130 - 140\,\text{MeV}\).

### 5.7.2 \(\pi\)-N-Systems

The possible reactions for pion exchange between nucleons:

<table>
<thead>
<tr>
<th>Reaktion</th>
<th>(I)</th>
<th>(I_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastische Prozesse</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\pi^+ + p \rightarrow \pi^+ + p)</td>
<td>(\frac{3}{2})</td>
<td>(\frac{3}{2})</td>
</tr>
<tr>
<td>(\pi^- + n \rightarrow \pi^- + n)</td>
<td>(\frac{3}{2})</td>
<td>(-\frac{3}{2})</td>
</tr>
<tr>
<td>(\pi^- + p \rightarrow \pi^- + p)</td>
<td>(\frac{3}{2}, \frac{1}{2})</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>(\pi^+ + n \rightarrow \pi^+ + n)</td>
<td>(\frac{3}{2}, \frac{1}{2})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>Inelastische Prozesse</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\pi^- + p \rightarrow \pi^0 + n)</td>
<td>(\frac{3}{2}, \frac{1}{2})</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>(\pi^+ + n \rightarrow \pi^0 + p)</td>
<td>(\frac{3}{2}, \frac{1}{2})</td>
<td>(\frac{1}{2})</td>
</tr>
</tbody>
</table>

### 5.7.2.1 Isospin Mixing as Angular Momentum Addition

\[
|\pi^- p\rangle = \phi_{\pi^-}(1,-1) \otimes \phi_p \left( \frac{1}{2}, \frac{1}{2} \right) = \sqrt{\frac{1}{3}} \psi \left( \frac{3}{2}, \frac{1}{2} \right) - \sqrt{\frac{2}{3}} \psi \left( \frac{1}{2}, \frac{1}{2} \right)
\]

\[
|\pi^+ n\rangle = \phi_{\pi^+}(1,1) \otimes \phi_p \left( \frac{1}{2}, \frac{1}{2} \right) = \sqrt{\frac{1}{3}} \psi \left( \frac{3}{2}, \frac{1}{2} \right) + \sqrt{\frac{2}{3}} \psi \left( \frac{1}{2}, \frac{1}{2} \right)
\]

\[
|\pi^0 p\rangle = \phi_{\pi^0}(0,0) \otimes \phi_p \left( \frac{1}{2}, \frac{1}{2} \right) = \sqrt{\frac{2}{3}} \psi \left( \frac{3}{2}, \frac{1}{2} \right) - \sqrt{\frac{1}{3}} \psi \left( \frac{1}{2}, \frac{1}{2} \right)
\]

\[
|\pi^0 n\rangle = \phi_{\pi^0}(0,0) \otimes \phi_n \left( \frac{1}{2}, \frac{1}{2} \right) = \sqrt{\frac{2}{3}} \psi \left( \frac{3}{2}, -\frac{1}{2} \right) + \sqrt{\frac{1}{3}} \psi \left( \frac{1}{2}, -\frac{1}{2} \right).
\]

Thus the wave function of the proton and neutron are:

\[
|p\rangle = \phi_p \left( \frac{1}{2}, \frac{1}{2} \right) \equiv \psi \left( \frac{1}{2}, \frac{1}{2} \right) = -\sqrt{\frac{1}{3}} |\pi^0 p\rangle + \sqrt{\frac{2}{3}} |\pi^+ n\rangle
\]

\[
|n\rangle = \phi_n \left( \frac{1}{2}, \frac{1}{2} \right) \equiv \psi \left( \frac{1}{2}, -\frac{1}{2} \right) = \sqrt{\frac{1}{3}} |\pi^0 n\rangle - \sqrt{\frac{2}{3}} |\pi^- p\rangle.
\]
5.7.3 Pion Exchange

We can also calculate the same result as above in the following way:

The exchange of pions by two nucleons can be considered as a scattering process. We now proceed to estimate the transition matrix element of this scattering process.

\[
\begin{array}{ccc}
  p & a & p \\
  \\ & \pi^0 & \\
  \ & \ & \\
  p & a & p \\
  a) & & \\
  n & b & n \\
  \\ & \pi^0 & \\
  \ & \ & \\
  n & b & n \\
  b) & & \\
  n & c & p \\
  \\ & \pi^0 & \\
  \ & \ & \\
  n & c & p \\
  c) & & \\
  p & c & n \\
  \\ & \pi^0 & \\
  \ & \ & \\
  p & c & n \\
  d) & & \\
\end{array}
\]

Then requiring that \( a^2 = b^2 = ab + c^2 \) and also that \( c^2 \neq 0 \) (for charge conservation)

\[
|n\rangle = \phi \left( \frac{1}{2}, -\frac{1}{2} \right) = b|n\pi^0\rangle + c|p\pi^-\rangle
\]

So then for example, the neutron is

which gives us the same result as above.

6 Symmetries and Symmetry Breaking

6.1 Conserved Quantities

6.1.1 Noether’s Theorem

“any differentiable symmetry of the action of a physical system has a corresponding conservation law.”

6.1.2 Conserved Quantities in Quantum Mechanics

An observable \( \hat{O} \) is a conserved quantity if \( \frac{d}{dt} \langle \hat{O} \rangle = 0 \).
Schroedinger’s equation states that \(i\hbar \frac{d\psi}{dt} = \hat{H}\psi \implies -i\hbar \frac{d\psi^*}{dt} = (\hat{H}\psi)^* = \psi^* \hat{H}\).

Thus:
\[
\frac{d}{dt} \langle \hat{O} \rangle = \frac{d}{dt} \int \psi^* \hat{O} \psi d^3x = \int \left( \frac{d\psi^*}{dt} \hat{O} \psi + \psi^* \hat{O} \frac{d\psi}{dt} \right) d^3x = \int \frac{i}{\hbar} \psi^* (\hat{H}\hat{O} - \hat{O}\hat{H}) \psi d^3x = 0 \implies [\hat{H}, \hat{O}] = 0,
\]

### 6.1.3 Examples for Symmetries and their Corresponding Conserved Quantities

- Space translation symmetry: momentum conservation
- Rotation symmetry: angular momentum conservation
- Time symmetry: energy conservation
- Local gauge symmetry: electric charge conservation
- Inversion symmetry: Parity conservation

The last three symmetries are denoted by the operators \(T, C, P\):

- Time reversal: \(T : t \mapsto -t\)
- Charge conjugation: \(C : \text{particle} \mapsto \text{anti-particle}\)
- Parity: \(P : r \mapsto -r\)

- “The parity of an anti-fermion is opposite to the parity of a fermion, whereas the parity of an anti-boson is the same as the parity of a boson.”

<table>
<thead>
<tr>
<th>Erhaltungsgrössen</th>
<th>starke WW</th>
<th>schwache WW</th>
<th>e.m. WW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energie</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Spin</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Ladung</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Baryonzahl</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Leptonenzahl</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Strangeness</td>
<td>x</td>
<td>-</td>
<td>x</td>
</tr>
<tr>
<td>Isospin</td>
<td>x</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Parität</td>
<td>x</td>
<td>-</td>
<td>x</td>
</tr>
<tr>
<td>(CP)</td>
<td>x</td>
<td>-</td>
<td>x</td>
</tr>
</tbody>
</table>
6.2 Parity Violation

By mirroring, momentum changes sign, however, the spin does not (as it is a cross product) and so the helicity changes sign under parity mirroring.

- Maximal parity violation:
  - V-A: W couples only left handed Fermions and right handed anti-Fermions.
  - V+A: W couples only right handed Fermions and left handed anti-Fermions.

- No parity violation:
  - W couples to both left handed and right handed particles.

- From the SPS experiment of proton-anti-proton collisions, examining the angle distribution between the resultant positron and the anti-proton proves that the $W$ has spin 1 and it couples with $V \pm A$ that is, maximal parity violation.

6.2.1 The Wu-Experiment

![Diagram](image)

*Fig. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.*
The Coblat nuclei were cooled down, to insure that the spins are really aligned with the magnetic field (that is, without thermal distribution)—that is what energetically economical for them.

Thus:

Griffiths chapter 4.4: “Radioactive cobalt-60 nuclei were carefully aligned, so that their spins pointed in what was chosen to be the z-direction. Cobalt 60 undergoes beta decay: $^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- + \nu_e$. Wu recorded the direction of the emitted electrons. What she found was that most of them came out in the 'southerly' direction, opposite to nuclear spin.”

This proves that there is no parity symmetry in this process, for if there were, then there should be an approximately equal amount of electrons going off in the same direction as the nuclear spin.

Wikipedia: “It is now the number of in negative z-direction of emitted electrons is measured. One must distinguish here the following two orientations of the nuclei: Forward : The nuclear spins are aligned in the positive z-direction. In the negative z-direction detected electrons were emitted thus counter to the direction of spin (i.e. with negative helicity). This can be illustrated as follows (here the double arrow indicates a Spin-1/2 orientation, the single arrows for direction of motion):

Reverse : The nuclear spins have intrinsic angular momentum, under mirror symmetry the angular momentum vector, $p$, is still in the same direction relative to the direction of motion, $r$. To set up a "mirror" of the experimental set-up, it is therefore sufficient to rotate the nuclear spins with the magnetic field. There are then detected electrons, the spin vector in the same direction of the
momentum vector, with positive helicity. The nuclear spins are now aligned in the negative z-direction, with the electrons being emitted in the positive z-direction in the direction of the spin (i.e. with positive helicity) as follows:

As shown in the diagrams above, in both cases the spin and angular momentum vectors are conserved. If the parity operation was conserved, both scenarios above would be equally likely to happen: It would be the same number of electrons emitted in the direction of the nuclear spins in the opposite direction. Wu experimentally observed, however, that almost all the electrons are emitted against the direction of spin of the nuclei, which corresponds to a maximum parity violation."

• Difficulties:
  
  – Detector has to be placed near the Cobalt source since the $e^-$ have low energy, and thus a short range.
  – High polarization requires:
    * low temperatures
    * high magnetic field.

• The polarization of the Cobalt source was determined by the anisotropy of the gamma rays. A maximum polarization of 0.6 was achieved.

• If the weak interaction is parity invariant, the expected result is for there to be equal probabilities for an electron to be emitted in the same direction as the nuclear spin as to be emitted opposite the direction of the nuclear spin.

• Momentum gets flipped with parity transform, but angular momentum doesn’t, so if we make a parity transformation, the electrons should be emitted with the direction of the spin, but the spin should still point to the same direction, since its direction wouldn’t change under partly transformation.

6.2.2 The Parity of the Pion

The parity of the pion was measured in 1954 by stopping negative pions in a deuteron target, they form a pionic atom and the pion decays into the atomic S-state before the reaction $d^-$ takes place.

The overall symmetry factor for spin and angular momentum is $(-1)^{L+S+1}$. The parity of a two-particle state is $(-1)^L$ times the intrinsic parities.

6.2.3 Pion Decay (Griffiths page 132)

$\pi^\pm$ are the lightest hadrons, except for $\pi^0$, so they decay into leptons.
If the muon or electron had no mass, then this decay wouldn’t have been possible, because then it wouldn’t have been possible to switch to a frame of reference in which the particles are actually left handed. The portion of the muons that are left handed is \((1 - \beta)\).

### 6.2.3.1 Pion Production

In collision between protons and neutrons the following are possible:

\[
\begin{align*}
p + p & \rightarrow p + p + \pi^0 \\
p + p & \rightarrow p + \Delta^+ \rightarrow p + p + \pi^0 \\
p + p & \rightarrow n + \Delta^{++} \rightarrow n + p + \pi^+ \\
p + n & \rightarrow p + n + \pi^0 \\
p + n & \rightarrow n + n + \pi^+ \\
p + n & \rightarrow p + p + \pi^-
\end{align*}
\]

Observe: the formation of \(\pi^+\) and \(\pi^0\) is more probable than \(\pi^-\). In \(\pi^+\) and \(\pi^-\) the parity is also violated:

\[
\begin{align*}
\pi^+ & \rightarrow \mu^+ + \nu_\mu \\
\pi^+ & \rightarrow \mu^+ + \nu_\mu \\
\pi^- & \rightarrow \mu^- + \bar{\nu}_\mu \\
\pi^- & \rightarrow e^- + \bar{\nu}_e
\end{align*}
\]

Neutrinos are left handed and anti-neutrinos are right handed. Thus there
is only one type of reaction possible:

\[
\begin{array}{ccc}
\text{Muon} & \text{Pion} & \text{Neutrino} \\
\text{Helizität} & s_\mu & s_\pi = 0 \\
\bar{p}_\mu & \bar{p}_\pi & \bar{p}_\nu \\
\end{array}
\]

Measurements of the helicity of the muon allows us to determine the helicity of the anti-neutrino, by conservation laws.

### 6.3 Charge Conjugation

Only neutral particles can be eigenstates of the charge conjugation operator, for example, the neutral pion, or the photon.

\[
C|\pi^0\rangle = \eta_c|\pi^0\rangle \\
C|\pi^n\rangle = (-1)^n|\pi^n\rangle, \text{ for } n \text{ photons.} \\
C|\pi^0\rangle = C(|d\bar{d} + |u\bar{u}\rangle) = (|d\bar{d} + |\bar{u}u\rangle) = |\pi^0\rangle
\]

#### 6.3.1 Decay of the Pion

\[
\frac{R(\pi^0 \to \gamma + \gamma + \gamma)}{R(\pi^0 \to \gamma + \gamma)} \leq 4 \cdot 10^{-7}.
\]

This is because the initial state has \(C\) eigenvalue 1 whereas that of three photons has eigenvalue \(-1\), and the transition is \(C\)-invariant.

#### 6.3.2 Neutrino Symmetries

By combining \(C\) and \(P\) together, we get the \(CP\) operator:

\[
\begin{array}{c}
\bar{p} \rightarrow P \rightarrow \bar{p} \\
\end{array}
\]

as can be seen, the neutrino is \(CP\)-invariant.

### 6.4 Helicity of the Neutrino—the Goldhaber Experiment

The Goldhaber experiment allowed the determination of the helicity of the neutrino.
6.4.1 The Steps of the Experiment

6.4.1.1 Electron Capture Through a Weak Process

Then the excited Samarium emits a photon.

\[ ^{152}\text{Eu} + e^- \rightarrow ^{152}\text{Sm}^* + \nu_e \]

6.4.1.2 Angular Momentum

\[ J_z: 0 + 1/2 = 1 + (-1/2). \]

\[ \begin{array}{cccc|c|c|c}
^{152}\text{Eu} & + & e^- & \rightarrow & ^{152}\text{Sm}^* & + & \nu_0 & \rightarrow & ^{152}\text{Sm} & + & \nu_0 & + & \gamma \\
0 & + & +1/2 & \rightarrow & +1 & + & -\frac{1}{2} & \rightarrow & 0 & + & +1/2 & + & 1 & \Rightarrow H_\nu = -1 \\
0 & + & -\frac{1}{2} & \rightarrow & -1 & + & +\frac{1}{2} & \rightarrow & 0 & + & +\frac{1}{2} & + & -1 & \Rightarrow H_\nu = +1 \\
\end{array} \]

6.4.1.3 Spin Orientation

The Samarium and Neutrino could either both have helicity 1 or both have helicity \(-1\).

6.4.1.4 Photon Polarization

\[ ^{152}\text{Sm}^* \rightarrow ^{152}\text{Sm} + \gamma \]

We are interested in the case where the photon has the same momentum direction as the Samarium, in which case they have the same helicity.

6.4.1.5 Polarization Filter

Only photons with certain helicity pass through a magnetized iron.

6.4.1.6 Experimental Construction

6.4.1.7 Results

Only L.H. photons were measured. But the helicity of the photons is the same as the helicity of the neutrinos. Thus \( H_\nu = -1.00 \pm 0.3 \).
6.4.2 Measuring the Weak Interaction

\[ H = H_{EM} + H_{strong} + H_{weak} \]

The transition probability is proportional to \( H^2 \). Thus even though the weak int. is so weak, we can study it via the term \( H_{weak} H_{strong} \).

6.4.3 Eigen Parity

The eigen value of the parity operator for Bosons is the same as for anti-Bosons. For Fermions it is minus the anti-fermions.

6.5 The CPT Theorem

According to QFT \( \exists CPT \) symmetry in nature.

6.6 Kaons

\[ K^0 = d\bar{s} \]
\[ K^- = u\bar{s} \]
\[ K^+ = u\bar{u} \]

6.6.1 Production of Neutral Kaons

\[
\begin{align*}
S: & \quad p + n \longrightarrow p + \Lambda^0 + K^0 \\
& \quad E_{\text{min}} = 0.91 \text{ GeV}
\end{align*}
\]

\[
\begin{align*}
S: & \quad \pi^- + p \longrightarrow \Lambda^0 + K^0 \\
& \quad E_{\text{min}} = 0.91 \text{ GeV}
\end{align*}
\]

\[
\begin{align*}
S: & \quad \pi^+ + p \longrightarrow K^+ + K^0 + p \\
& \quad E_{\text{min}} = 1.5 \text{ GeV}
\end{align*}
\]

\[
\begin{align*}
S: & \quad \pi^- + p \longrightarrow \Lambda^0 + K^0 + n + n \\
& \quad E_{\text{min}} = 6.0 \text{ GeV}
\end{align*}
\]

6.6.2 Interaction with Material

Possible reactions with various energies, in which baryon number and strangeness must be conserved:
6.7 Parity Invariant 2-state system

6.7.1 Double Potential Well

Solution to the double potential well as a finite perturbation to the infinite two wells.

We get two states, symmetric and anti-symmetric, both of which are eigenstates of the parity operator.

6.7.2 $K^0 - \bar{K}^0$-System

(Griffiths 4.4.3.1)

We take the $K^0 - \bar{K}^0$ to be two states which are $CP$ eigenstates.

$CP|K^0\rangle = \eta|\bar{K}^0\rangle$

$CP|\bar{K}^0\rangle = \eta'|K^0\rangle$,

Because $(CP)^2 = 1$, we can choose $\eta = \eta' = -1$.

The Kaon can decay into two or three pions. Thus there was the idea that the kaons and anti-kaons can go back and forth from one another.

<table>
<thead>
<tr>
<th>$K^0$</th>
<th>Prozess</th>
<th>$\bar{K}^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0 + p \rightarrow K^0 + p$</td>
<td>Elastische Streuung</td>
<td>$\bar{K}^0 + p \rightarrow \bar{K}^0 + p$</td>
</tr>
<tr>
<td>$K^0 + n \rightarrow K^0 + n$</td>
<td>Ladungsaustausch</td>
<td>$\bar{K}^0 + n \rightarrow \bar{K}^0 + n$</td>
</tr>
<tr>
<td>$K^0 + p \rightarrow K^+ + n$</td>
<td>Hyperonenerzeugung</td>
<td>$\bar{K}^0 + p \rightarrow \Lambda^0 + \pi^+$</td>
</tr>
</tbody>
</table>

| $K^0 + p \rightarrow \bar{\Lambda}^0 + 2p + \pi^-$ |

$\Lambda^0 + \pi^+$
Two pions have parity 1 whereas three pions have parity $-1$. Both have $C = 1$. Thus $K_1$ decays into two pions whereas $K_2$ decays into three pions.

These two eigenstates have different lifetimes, because the $2\pi$ is much faster, because the energy released is greater. So if we start with a beam of $K^0 = \frac{1}{\sqrt{2}} (K_1 + K_2)$ the $K_1$ component will decay away quickly and down the line there will be a beam of pure $K_2$.

Observe: $K_1$ and $K_2$ are not anti-particles of one another, rather, each is its own anti-particle.

Kaons are produced in the strong interaction, as eigenstates of strangeness, but they decay in the weak interaction, as eigenstates of $CP$.

- $CP(\pi^0) = -1$, $CP(\pi^0\pi^0) = 1$
- $CP(\pi^\pm\pi^\pm) = 1$
- $CP(\pi^0\pi^0\pi^0) = -1$
- $CP(\pi^\pm\pi^\pm\pi^0) = -1$
• Thus $K_1$ decays weakly into two pions and $K_2$ decays weakly into three pions, the phase space available for two pions is larger, so the decay of the $K_1$ is faster, hence it has a shorter lifetime.

• $\tau_{K_1} \approx 8.96 \cdot 10^{-11} s$

• $\tau_{K_2} \approx 5.18 \cdot 10^{-8} s$

6.7.3 Regeneration of Kaons

If you wait long enough, you get only $K_2$ in your beam. But you could regenerate $K_1$. If you shoot the $K_2$ beam at a material that has different cross section between $K_0$ and $\bar{K}_0$.

6.8 Indirect CP-Breaking with Neutral Kaons

Experimental data shows that for a pure $K_2$ there are still some $2\pi$ decays: the $K_2$ is not a pure eigenstate of $CP$.

$$K_L \rightarrow \pi + \pi + \pi \quad (32\%)$$

$$K_L \rightarrow \pi + e + \nu \quad (41\%)$$

$$K_L \rightarrow \pi + \mu + \nu_\mu \quad (27\%)$$

With B-mesons direct $CP$ violation was observed.

7 Particle Physics at PSI

7.1 Proton Accelerator

The proton accelerator at the PSI can accelerate particles to energy of up to $590 MeV$. The current of the resulting particle-current is $I_p = 2.2 mA$. The resulting power is $P = 1.3 MW$. Protons are shot at a target, so that pions are produced, which can then used for further experiments or the following reaction can be researched:

$$p + N \rightarrow x + y + \pi$$

Provided the produced particles are charged, they can be deflected by a magnetic field. At the PSI the following questions are examined:

• What makes leptons so fundamental? Something about being point particles: a muon cannot be distinguished from an electron, other than being heavier.

• The process $\mu \rightarrow e + \gamma$ is not allowed in the standard model. But the process $\mu \rightarrow e + \nu + \bar{\nu}$ exists. Thus there must be a conserved quantity which is forbidden in the first process and allowed in the second.
• Up to now the following relation for both processes has been observed: \( \frac{R(\mu \rightarrow e + \gamma)}{R(\mu \rightarrow e + \nu + \bar{\nu})} < 10^{-11} \).

• Investigation of the muon decay allows us to determine the lifetime of the muon:

\[
\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} F(p) \left( 1 + \frac{3}{5} \frac{m_\mu^2}{M_W^2} \right)
\]

whereby \( G_F \) is the Fermi constant and \( M_W \) is the effective coupling constant for the weak interaction.

• With these known constants the lifetime of the muon was found to be \( \tau_\mu = 2.2\mu s \).

### 7.2 Muonic-Hydrogen Lambshift

In 2010 the radius of the proton was newly determined at the PSI. The result deviates strongly from the value measured up to that point.

#### 7.2.1 Lamb Shift

Vacuum fluctuations lift the degeneracy of the levels at the \( n, l \) energy level in the hydrogen atom. The shift is proportional to the mass of the electron. Thus replacing the electron in the atom with the muon, we get a much larger shift.

#### 7.2.2 The Proton Radius

Because the proton is not a point particle, the Coulomb potential \( \frac{1}{r} \) does not describe it well for \( r \rightarrow 0 \). So we use another potential. If we include the hyperfine structure, we can shoot a laser to raise the muonic hydrogen from \( 2S \) to \( 2P \) with the right energy and through the Lamb shift ascertain the proton radius.

### 7.3 CP-Violation of Ultra Cold Neutrons

Based on the inspection of the electric dipole moment of neutrons it is possible to verify the \( CP \)-violation. Despite the fact that the neutron is electrically neutral, it can still have an electric dipole moment, independently of the distribution of charge. If the neutron has an electric dipole moment, then the parity and time-reversal are violated.

#### 7.3.1 Matter-Anti-Matter Assymetry

Sacharow postulated 3 criteria to explain the assymetry between matter and anti-matter:

• Thermodynamic out-of-equilibrium during the big bang
• Violation of the baryon number conservation
• Violation of the $C$ and $CP$ symmetry.

7.3.2 Ultra Cold Neutrons (UCN)

“To demonstrate the CP violation, so the electrical dipole moment has to be
determined. These neutrons are slowed down to a few meters per second. This
is done at the moment at PSI (see slides). Wherein the dipole moment is
then measured by placing the UCN in a uniform electric and magnetic field.
The magnetic field which oscillates time-reversal is generated. By reversing the
polarity of the electric field, a space inversion is created. By measuring the
so-called Larmor frequency, the dipole moment can be determined. So far, the
dipole moment of about $2.9 \cdot 10^{-26} e \text{ cm}$ could be determined accurately. At PSI,
the UCN to help this upper bound further and correct site down.”

8 Properties of the Weak Interaction

(chapter 9 in Griffiths)

Because the cross section for scattering experiments of the weak interaction is
very small, the properties of the weak interaction are measured through decays.

8.1 Universality of the Charged Weak Interaction

8.1.1 $\beta^+$ decay

$\langle du \rangle u \rightarrow \langle du \rangle d + W^+ \rightarrow \langle du \rangle d + e^+ + \nu_e$ with lifetime of around 103s.
8.1.2 $\mu^+$-decay

$\mu^+ \rightarrow +\bar{\nu}_{\mu} + W^+ \rightarrow \nu_{\mu} + e^+ + \nu_e$ with lifetime of around 2.2\(\mu s\).
8.1.3 $\tau^+$-decay

$\tau^+ \rightarrow +\bar{\nu}_\tau + W^+ \rightarrow \bar{\nu}_\tau + e^+ + \nu_e$ or $\tau^+ \rightarrow +\bar{\nu}_\tau + W^+ \rightarrow \bar{\nu}_\tau + \mu^+ + \nu_\mu$ with lifetime of around $3 \cdot 10^{-13}$ s. The branching ratios for both is around 0.18.

8.1.4 The Universality

The matrix element of the weak interaction is all the same for the above interactions. The only difference is in the phase space volume.

The relation between lifetime and transition probability is $\tau \propto \frac{1}{\Gamma_{i \rightarrow f}}$, thus, $\tau \propto \frac{1}{p_{\text{max}}}$ where $p_{\text{max}}$ is the maximal momentum of the positron.

$$\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} |M_{fi}|^2 \rho(E_f)$$

$$\rho(E_f) = \frac{d\sigma}{dE_f} = \int d^3 p_e d^3 p_\nu \frac{p_\nu^{\delta}}{p_{\text{max}}} \frac{p_e^{\delta}}{p_{\text{max}}} \frac{d^3 (p_e/p_{\text{max}})}{d^3 (E_e/E_{\text{max}})} = p_{\text{max}} \cdot I,$$

the matrix element is the same for all three decays. Thus the only difference is in the phase space room, which is proportional to $p_{\text{max}}$. If we plot $\log(\tau) \propto p_{\text{max}}$, we will see the slope is indeed $-5$ which confirms that fact.

$$M_{fi} \propto g \cdot \frac{1}{Q^2 + M_W^2} \cdot g, \text{ where } g \text{ is the weak charge. From that we find that } g \sim e.$$  

8.1.4.1 The Fermi Constant

The Fermi constant $G_F$ is defined as $G_F := \sqrt{\frac{2}{\pi}} \left( \frac{\alpha g^2}{e^2 M_W c^2} \right)$ It is the effective coupling constant if the decay was a 4-particle decay (without the W) (Griffiths 9.2).
8.1.5 Summary

The universality of the weak charge is the same for quarks and leptons (verified experimentally)!

8.1.6 Example–The Decay of the Tau

The decay of the tau is about 3 times more likely into a pion and something than it is for a lepton (either electron or muon), which can be explained by the fact that quarks come in three colors so they are three times as likely.

8.2 Quark Mixing

(Griffiths 9.5)

There are six types of quarks, by rising mass:

\[
\begin{pmatrix}
(u) \\
(d) \\
(c) \\
(s) \\
(t) \\
b
\end{pmatrix}.
\]

They are eigenstates of the strong force. In the strong force only transitions within a family are allowed. But the weak force does not respect that.

8.2.1 Quark Mixing Matrix

8.2.2 Cabbibo-Kobayashi-Masukawa Matrix Theory and Experiment

We can estimate the ratios between different entries in the CKM matrix by considering the branching ratios of different reaction:

\[
\left| \frac{V_{us}}{V_{cb}} \right| \approx \sqrt{\frac{\Gamma(\text{reaction involving } b \to u)}{\Gamma(\text{reaction involving } b \to c)}}
\]

This is because at each vertex, the coupling constant will be \( g_W \cdot |V_{bc}| \), for an exchange between \( b \to c \) for example.

The CKM matrix is mostly diagonal, but not quite.

8.2.3 GIM Mechanism

In quantum field theory, the GIM mechanism (or Glashow–Iliopoulos–Maiani mechanism) is the mechanism by which flavour-changing neutral currents (FC-NCs) are suppressed. It also explains why weak interactions that change strangeness by 2 (\( S = 2 \) transitions) are suppressed while those that change strangeness by 1 (\( S = 1 \) transitions) are allowed. The mechanism was put forth by Sheldon Lee Glashow, John Iliopoulos and Luciano Maiani in their famous paper "Weak Interactions with Lepton–Hadron Symmetry" published in Physical Review D in 1970.[1] At the time the GIM mechanism was proposed, only three quarks (up, down, and strange) were thought to exist. Glashow and James Bjorken predicted a fourth quark in 1964,[2] but there was little evidence for its existence.
The GIM mechanism however, required the existence of a fourth quark, and the prediction of the charm quark is usually credited to Glashow, Iliopoulos, and Maiani.

8.2.4 CP-Violation Revisited

8.3 ElectroWeak Unification

8.3.1 Nueteral Weak Interaction: The $Z^0$ Boson

Detected in the decay of $Z^0 \to e^+ + e^-$.

8.3.2 Weak Iso-spin and the Weinberg Angle

Weak iso-spin is zero for right-handed fermions. This involves weak-quark eigenstates (rotated by the CKM matrix). It’s $+\frac{1}{2}$ for the “not-negatively” charged fermions and $-\frac{1}{2}$ for the negatively charged fermions. It’s $-1$ for $W^-$ and $+1$ for $W^+$. As a triplet, we expect there to be also a $W^0$ with weak iso-spin $|1, 0 >$ and $B^0$ with weak iso-spin $|0, 0>$.  

\[ |\gamma > = \cos (\theta_W) |B^0 > + \sin (\theta_W) |W^0 > \]

\[ |Z^0 > = - \sin (\theta_W) |B^0 > + \cos (\theta_W) |W^0 > \]

where $\theta_W \approx 30^\circ$.

Then $e = g \cdot \sin (\theta_W)$.

8.4 Neutrino Oscillations

Today we know of three different species of neutrinos, depending on whether they are produced in association with an electron ($e$), a muon ($\mu$) or a tau ($\tau$) lepton. For simplicity, let us consider only the first two species. The $|e\rangle$ and the $|\mu\rangle$ states are created by weak interaction (weak eigenstates). But it is now known that free neutrinos do not stay in the state in which they were originally created but they oscillate between the different flavours. The weak eigenstates ($|e\rangle$ and $|\mu\rangle$) are thus not identical with the eigenstates of the free neutrino hamiltonian (energy eigenstates). The energy eigenstates are denoted $|1\rangle$ and $|2\rangle$, with the corresponding eigenvalues $E_1$ and $E_2$. The relation between the weak eigenstates ($|e\rangle$ and $|\mu\rangle$) states and the energy eigenstates ($|1\rangle$ and $|2\rangle$) is given by:

\[ |\nu_e > = \cos (\theta) |\nu_1 > - \sin (\theta) |\nu_2 > \]

\[ |\nu_\mu > = \sin (\theta) |\nu_1 > + \cos (\theta) |\nu_2 > \]

If, for example, at time 0 there was an electron neutrino, the probability to have a muon neutrino vs. time is:

\[ |\langle \nu_\mu | \psi (t) \rangle |^2 = [\sin (2\theta)]^2 \left[ \sin \left( \frac{E_1 - E_2}{\hbar} t \right) \right]^2 \]

The length at which the probability to find a muon neutrino is thus:

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\[ L_{\text{max}} = \frac{\hbar \pi 2 \rho_0 c}{(m_1^2 - m_2^2)} \]

Part II
Nuclear Physics

9 Nucleon Model

9.1 Introduction

9.1.1 Charge Density Distribution in Nucleon

The charge distribution of a nuclide looks like this:
\[ \rho (r) = \frac{\rho_0}{1 + e^{(r - c)/a}} \]
whereby \( \rho_0, a \) and \( c \) are experimental parameters.

For big nuclides \( c = 1.07 \, fm \cdot A^{1/3} \) and \( a = 0.54 \, fm \).
Because the nucleus is basically incompressible, it can be deduced that the strong interaction is repulsive at very small distances.

9.1.2 Binding Energy in Nucleon

Binding energy per nucleon in the nucleus:
The lightest stable nucleus is the deuteron (neutron and a proton). $^4\text{He}$ has a very high binding energy.

## 9.2 Semi-Empirical Mass Formula (Liquid Drop Model)

$$M(A, Z) = NM_n + Z(M_p + m_e) - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N - Z)^2}{4A} + \delta \frac{\Delta}{A^{1/2}}$$

with $N = A - Z$. Assume the following values for the parameters:
- volume term $a_v = 15.67 \text{ MeV}$
- surface term $a_s = 17.23 \text{ MeV}$
- Coulomb term $a_c = 0.714 \text{ MeV}$
- asymmetry term $a_a = 93.15 \text{ MeV}$
- pairing term $\delta = -11.2, 0, 11.2 \text{ MeV}$ for (Z,N) even-even, odd-even, odd-odd nuclei.

$M(A, Z) := (A - Z) m_n + Z m_p - \Delta M$

$\Delta M$ is called the mass defect, which is also the definition of the binding energy.

The mass of the nucleus is lower than the mass of its constituents, implying that there should a positive binding energy.

The semi empirical mass formula has several terms:

1. The Volume Term
The nucleus is considered to be a liquid-drop, and so, the binding energy should be proportional to the volume of the drop. As the density is more or less homogenous, the volume is proportional to $A$.

$$a_V \cdot A$$

where $a_v \approx 15.7 MeV$

2. The Surface Term
   
   TODO

3. The Coulomb Term
   
   TODO

4. The Assymetry Term
   
   TODO

5. The Paring Term
   
   TODO

• For example, using the formula, we can calculate that the reaction $n_{thermal} + ^{235}_{92}U \rightarrow ^{131}_{50}Sn + ^{102}_{42}Mo + 3n$ releases 176.59 MeV. The kinetic energy of thermal neutrons is $\sim 0.025 eV$.

• We could also compute the minimal ratio $\frac{Z^2}{A}$ for spontaneous fission: $\frac{Z^2}{A} > 48$. This is assuming a deformation in the nucleus of the form:

  - Ellipsoid with $a := R(1 + \varepsilon)$, $b := R\left(1 - \frac{1}{2}\varepsilon\right)$, where $\varepsilon$ is the deformation parameter. Thus the volume and surface term change:

    $$a_c A^{2/3} \rightarrow a_c A^{2/3} \left(1 + \frac{2}{5}\varepsilon^2 + \ldots \right)$$

    $$a_c \frac{Z^2}{A^{1/3}} \rightarrow a_c \frac{Z^2}{A^{1/3}} \left(1 - \frac{1}{5}\varepsilon^2 + \ldots \right)$$

  - Then we require that the binding energy of the deformed state is higher than the binding energy with zero deformation ($\varepsilon = 0$).

9.2.1 Binding energy

9.2.2 Bethe-Weizsaecker-Formula

9.3 Fermi Gas Model

Assumptions of the model:
• The nucleons move freely in a potential box with no collisions. The potential on any one nucleon is caused by all other nucleons.

• Nucleons are spin-1/2 particles, and so, they must obey the Fermi-Dirac statistics, and correspondingly the Pauli exclusion principle.

• The potential is more or less constant in the inner of the nucleus and at the edge rapidly falling, so it is approximately by a box potential.

\[ \frac{dN}{dE} = \frac{a^3 m^{3/2} \sqrt{E}}{\sqrt{2 \pi^3 h^3}} \]

\[ \mathcal{E}_F = \frac{\hbar^2}{2m} \left( \frac{3}{2} \pi^2 n \right)^{3/2} \]

The factor of 1/2 is because we put both proton and neutrons into the same state.

9.3.0.1 Explain the Asymmetry term  TODO (not a very illuminating explanation...)

9.3.1 Average Energy of a Particle in a Fermi Gas

\[ \langle E \rangle \approx (20 \text{MeV}) \times \left[ A + \frac{5}{9} \left( \frac{(N-Z)^2}{A} \right) \right] \]
9.4 Nuclear Shell Model

There are magic numbers in which nucleons are more stable (for either the number of protons or neutrons or both) 2, 8, 20, 28, 50, 82, 126. There are also half magic numbers, 40 and 64.

Evidence for nuclear shell model:

1. Large number of nuclides with proton or neutron magic numbers.
2. Small cross section for neutron absorption.
3. First excited level is high.
4. Separation energy should be higher.

9.4.1 Separation Energy

The separation energy is the energy required to remove one proton or neutron.

The picture is explained due to the fact that protons “happily” interact with neutrons, but not with themselves.

There is a gap in the separation energy around the magic numbers.

9.4.2 Hartree-Fock Method

A many-particle perturbative method. Not sure how much is expected to know of this—it is highly technical.

9.4.3 Schroedinger Equation

It is assumed about the potential that:

1. It is radially symmetric: $V(r) = V(r)$
2. The nucleons are considered to be independent of each other. Each one is affected by its own potential.
Due to 1., there could be no quadrupole moment: \([H, \mathbf{L}] = [H, \mathbf{L}^2] = 0\)

\[
H\psi = \left[ \frac{\hbar^2}{2m} \frac{\partial^2}{\partial r^2} + \frac{\mathbf{L}^2}{2mr^2} + V(r) \right] \psi = E\psi
\]

Thus:

\[
\psi(r) = R_{nl} (r) Y_l^m (\theta, \varphi)
\]

and so:

\[
\left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial r^2} + \frac{\hbar^2}{2m} \frac{l(l+1)}{r} + V(r) \right] \cdot r \cdot R_{nl} (r) = E_{nl} \cdot r \cdot R_{nl} (r)
\]

9.4.4 The Potential: 3D Harmonic Oscillator

\[
V(r) = \frac{1}{2} m \omega^2 r^2
\]

\[V(r) = \begin{cases} 
-V_0 \left( 1 - \frac{r^2}{R^2} \right) & r < R \\
0 & r > R
\end{cases}\]

\[E = \hbar \omega \left( N + \frac{3}{2} \right), \quad N \in \mathbb{N}\]

\[N = 2(n-1) + l, \quad n \in \mathbb{N}\]

For every level \(l\) the occupation possible is \(2 \sum_{l=1}^{n} (2l + 1)\).

<table>
<thead>
<tr>
<th>(N)</th>
<th>Kernkonfiguration (Orbitale)</th>
<th>Parität</th>
<th>Anzahl degenerierter Nukleonen</th>
<th>Gesamt</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(1s)</td>
<td>+</td>
<td>(2)</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>(1p)</td>
<td>-</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>(2s, 1d)</td>
<td>+</td>
<td>(2+10=12)</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>(2p, 1f)</td>
<td>-</td>
<td>(6+14=20)</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>(3s, 2d, 1g)</td>
<td>+</td>
<td>(2+10+18=30)</td>
<td>70</td>
</tr>
<tr>
<td>5</td>
<td>(3p, 2f, 1h)</td>
<td>-</td>
<td>(6+14+22=42)</td>
<td>112</td>
</tr>
<tr>
<td>6</td>
<td>(4s, 3d, 2d, 1i)</td>
<td>+</td>
<td>(2+10+18+26=56)</td>
<td>168</td>
</tr>
</tbody>
</table>
This only fits for the first three magic numbers.
One possibility is to modify the potential, to the Wood-Saxon Potential:

\[ V(r) := -V_0 \frac{1}{1 + e^{(r-R)/a}} \]

But even this potential doesn’t produce the magic numbers.

### 9.4.5 Spin-Orbit Coupling

Consider the interaction of the spin-orbit of a nucleon:

\[ V(r) = V_{\text{center}} + V_{ls}(r) \frac{(l \cdot s)}{\hbar^2} \]

\[
\begin{pmatrix}
\frac{(l \cdot s)}{\hbar^2} \\
V_{ls}(r)
\end{pmatrix} = \begin{cases} 
1/2 & j = l + 1/2 \\
-(l + 1)/2 & j = l - 1/2 
\end{cases}
\]

Thus

\[ \Delta E_{ls} = \frac{2l+1}{2} \langle V_{ls}(r) \rangle \]
9.4.5.1 Intruder In the 50 shell, the $1g_{9/2}^+$ level has positive parity:

States with different parity cannot mix.
9.4.6 Applications of the Shell Model
9.4.7 Valence Nucleons

10 Deformed Nuclei and Collective Nuclear Excitations

Electrical quadrupole moment means that the charge distribution is not spherically symmetric:

The odd moments (dipole and octupole) vanish due to parity conservation, and so the quadrupole is the measure of non-sphericity:

$$Q_{ij} = \int \rho \left( 3r_i r_j - r^2 \delta_{ij} \right) d^3r$$

10.1 Quadrupole Moments

10.2 Rotations Model
10.2.1 Rotations Model
10.2.2 Classical Rotor
10.2.3 No Excitations–Symmetric Case
10.2.4 Excitations–Asymmetric Case

10.3 Multipol Transitions
10.3.1 Characteristic Selection Rules
10.3.2 Transition Probability

10.4 Giant Resonance

Giant Dipole Resonance
The primordial synthesis of light elements in the early Universe (D, 3He, 4He and 7Li) is a process which is sensitive to many aspects of the physics model that describes it, such as the number of neutrino species in the theory. The yield of the most stable element 4He is determined by a competition between the expansion rate of the Universe, the rates of the weak interactions that interconvert neutrons and protons, and the rates of the nuclear reactions that build up the 4He nuclei.

1. a) At high temperatures \( t < 1 \text{s}, kT > 1 \text{MeV} \), neutrons and protons can interconvert via weak interactions: \( n + e^+ + p + e, n + e + p + e, \text{and } np + e + e \). As long as the interconversion rate of neutrons and protons is faster than the expansion rate of the Universe, the neutron-to-proton ratio tracks its equilibrium value, exponentially decreasing with temperature (Boltzmann factor). Calculate the \( n/n_p \) ratio at the end of this period (use \( kT = 720 \text{ keV} \)).

\[
\frac{n}{n_p} = e^{-(m_n - m_p)/k_BT} \implies \frac{n}{n_p} \simeq \frac{1}{6}.
\]

2. b) Once the interconversion rate becomes less than the expansion rate, the \( n/n_p \) ratio effectively freezes-out \( t \sim 1 \text{s}, kT = 720 \text{ keV} \), thereafter decreasing slowly due to free neutron decay \( n + p + e + e \). Calculate the \( n/n_p \) ratio after \( t \sim 100 \text{ seconds} \), the time after which nucleosynthesis can start. (neutron lifetime \( n = 887 \text{s} \)).

\[
N = N_0 e^{-t/\tau} \implies \frac{n}{n_p}(t) = \frac{\left(\frac{n}{n_p} \right)e^{-t/\tau}}{n_p + \left(\frac{n}{n_p} \right)(1 - e^{-t/\tau})} = 0.146 \simeq \frac{1}{7}.
\]
3. c) The nucleosynthetic chain, whereby protons and neutrons fuse to D (np), and further to 4He (npp), starts as soon as deuterium becomes stable against photo-dissociation (t ~ 3 min, kT ~ 100 keV). Since there are less neutrons than protons, there will be n/2 4He nuclei formed. The mass fraction of helium in the Universe (neglecting the other light elements) can be written as $Y_P = M_{He}$, where $M_X = n Xm_X$ is the total mass of X in the $M_{He} + M_{H}$ universe. Calculate this fraction $Y_P$ at the beginning of nucleosynthesis. Hint: calculate $n_{He}$ using the previously found n/np ratio. $n_{H}$

There are two neutrons necessary for each helium, and for each $n_n$, there exists a proton, necessary for helium. Thus, $n_{He}/n_{H} = \frac{1}{2}n_{n}/n_{n} - n_{n} \approx \frac{1}{2}$.

$Y_{P} = \frac{n_{He}m_{He}n_{He}}{n_{H}m_{H} + n_{H}m_{H}} \approx 0.25$

4. d) A full calculation of the primordial helium abundance in the Standard Model yields $Y_P = 0.246 + 0.013(N^{3}) + 0.18 n^{887} + 0.01 \ln(\phi^{5})$ where $N$ is the effective number of light neutrino families, and is the baryon-to-photon ratio in units of 1010. Given that the observational upper bound on $Y_P$ is 0.248, calculate the upper bound on $N$ for $= 3$. Compare this result to the LEP result from the Z width measurement.

$\Rightarrow N_{v} < 3.54$

The combined LEP result for $N$ is 2.984 ± 0.008.

11.3.1 Production of Alcohol and other Light Elements
11.3.2 Generation of Heavy Ions and SYNTHESEPFADE??
11.3.3 Primordial Element Synthesis

12 Star Evolution, Cosmology and Dark Matter
12.1 Star Evolution
12.1.1 The Sun

In heavy stars the dominant mechanism is the CNO (Carbon-Nitrogen-Oxygen) cycle, in which the fusion process is 'catalyzed' by small amounts of those three elements.

However, in the sun (and other relatively light stars), the dominant route is the pp-chain.

- The sun shines with about $4 \cdot 10^{26} W$.
- The energy released per $^4 He$ nucleus is $\sim 24.75 MeV$.
- The mass of the sun is $\sim 2 \cdot 10^{30} kg$, of which about half are protons.
- Thus the sun could shine for $\sim 4 \cdot 10^{10}$ more years.
12.1.1.1 The pp-chain

1. Two protons make a deuteron:
   (a) \( \beta^+ \) decay: \( p + p \rightarrow d + e^+ + \nu_e \) (weak int.)
   (b) \( e^- \) capture: \( p + p + e^- \rightarrow d + \nu_e \) (weak int.)

2. Deuteron plus proton makes \( ^3He \):
   \( d + p \rightarrow ^3He + \gamma \) (strong int.)

3. \( ^3He \) makes \( \alpha \)-particle or \( ^7Be \) (Beryllium):
   (a) \( \beta^+ \) decay: \( ^3He + p \rightarrow \alpha + e^+ + \nu_e \) (weak int.)
   (b) \( ^3He + ^3He \rightarrow \alpha + p + p \) (strong int. and EM)
   (c) \( ^3He + \alpha \rightarrow ^7Be + \gamma \) (strong int. and EM)

4. \( ^7Be \) makes \( \alpha \)-particles:
   (a) \( e^- \) capture: \( ^7Be + e^- \rightarrow ^7Li + \nu_e \) (weak int.)
   (b) \( ^7Li + p \rightarrow \alpha + \alpha \) (strong or EM)
   (c) \( ^7Be + p \rightarrow ^8B + \gamma \)
   (d) \( ^8B \rightarrow ^8Be^* + e^+ + \nu_e \)
   (e) \( ^8Be^* \rightarrow \alpha + \alpha \)

There are 5 possible reactions that result in a neutrino, and each of these neutrinos has a characteristic spectrum:
12.1.1.2 The Davis Experiment Chlorine plus a neutrino turns into argon, so if you count the number of argon atoms you get the number of neutrinos. It turns out (the Davis Homestake experiment) that only a third of the theoretically expected neutrinos arrived at the detector. This was explained by neutrino oscillations (and the fact that the experiments are sensitive only to one type of neutrinos—νe).

12.1.1.3 The Super-Kamiokande Experiment Uses water. This experiment is sensitive to all three neutrino types.

Via elastic neutrino-electron scattering (νe + e → νe + e) the emitted electron is detected by the Cherenkov radiation it emits in water.

12.1.1.4 The Sudbury Neutrino Observatory (SNO) Uses heavy water (D2O)
Via elastic neutrino-electron scattering (this is sensitive to all types of neutrinos) and two other reactions which are sensitive only to electron neutrinos: $\nu_e + d \rightarrow p + p + e$ and $\nu + d \rightarrow n + p + \nu$

12.1.2 Neutron Stars

12.2 Basics of Cosmology
12.2.1 Birth of the Universe
12.2.2 Friedmann Equation
12.2.3 Flow Equation
12.2.4 The Hubble Law

12.3 Time Evolution of Density in the Universe
12.3.1 Material
12.3.2 Radiation
12.3.3 Composition

12.4 Dark Matter
12.4.1 The Problem of the Cosmological Constant
12.4.2 Contributions to the Density of the Universe
12.4.3 Evidence for Dark Matter

Opening Monologue for the Test–Neutral Kaon Decay

1. According to the standard model, there are four types of interactions: electromagnetic, gravity, weak and strong. Each type of interaction has a force carrier—the gauge Bosons—and Fermions, which are charged for these interactions: quarks and leptons. The most famous lepton is the electron whereas the quarks do not seem to exist by themselves: they usually appear in “packages” of quark anti-quark (called mesons, which are Bosons) or three quarks (called baryons, which are Fermions). The nucleon (proton or neutron) is a baryon. Together the mesons and baryons are called hadrons.

2. One example of mesons are the neutral kaons: $|K^0 > = |d\bar{s} >$ and $|\bar{K}^0 > = |\bar{d}s >$, which have a mass of roughly $500 MeV$. They were first discovered in cosmic rays but can also be produced by strong decays such as colliding two protons or a proton and a pion.
3. Charge conjugation and parity are two interesting quantum mechanical operators. The former swaps particles for anti-particles whereas the latter changes the position of a particle from $r$ to $-r$. It was initially believed that physics was invariant under the operation of either one of these operators. Experiments, like Wu’s in the 1950s, showed us that this is not the case for the parity operator. Then it was believed that physics should be invariant under the composed operator $C \circ P$, is is mostly the case but still not precisely. Our only hope to recover symmetry is the CPT theorem (which includes the time reversal operator) of quantum field theory, which is believed to be an exact law of nature.

4. In quantum field theory, the intrinsic parity of a Fermion must be opposite to the intrinsic parity of an anti-Fermion. Otherwise, it is by convention that quarks are defined to have positive parity. Additionally parity is a multiplicative quantity of a composite system, and the parity of the spherical harmonics is $(-1)^l$ (where $l$ is the orbital angular momentum of the system)–so for a composite system we must multiply by this factor. In conclusion, for baryons the parity is positive whereas for mesons it is negative, in the ground state ($l = 0$).

5. Not every particle will be an eigenstate of the charge conjugation operator. Only those particles which are their own anti-particles can be that, such as the photon or some mesons, such as the $\pi^0$. For a meson, charge conjugation eigenvalues will be $(-1)^{l+s}$. In addition charge conjugation is also a multiplicative number.

   - The charge conjugation eigenvalue of the neutrino gives a left-handed anti-neutrino, which does not occur, so the weak interaction is not charge conjugation invariant.

6. Let’s compute the composed $C \circ P$ operator eigenvalues for the $K^0$ and $\bar{K}^0$:

   - $P(K^0) = -K^0$ whereas $P(\bar{K}^0) = +\bar{K}^0$
   - $C(K^0) = \bar{K}^0$ whereas $C(\bar{K}^0) = K^0$
   - So $CP(K^0) = -\bar{K}^0$ and $CP(\bar{K}^0) = -K^0$.

7. So we see $K^0$ and $\bar{K}^0$ are not eigenstates of the $CP$ operator, but we can construct linear combinations of them which will be eigenstates:

   $K_1 := \frac{1}{\sqrt{2}} (K^0 - \bar{K}^0)$ and $K_2 := \frac{1}{\sqrt{2}} (K^0 + \bar{K}^0)$

Then $CP(K_1) = K_1$ and $CP(K_2) = -K_2$ by direct computation.

8. Experiments have showed us that in general the neutral kaons decay into either two or three pions.
9. What are the \( CP \) values of the two and of the three pion systems?

\[
P(\pi) = -1 \text{ and } C(\pi^0) = (-1)^{l+s} = 1
\]

Thus \( CP(\pi\pi) = C(P(\pi\pi)) = C((-1)(-1)\pi\pi) = C(\pi\pi) = (+1)(+1) = 1 \)

Similarly, \( CP(\pi\pi\pi) = -1 \).

10. If \( CP \) is to be a conserved quantity in the weak interaction, \( K_1 \) may only decay to \( CP = 1 \) states (two pions) whereas \( K_2 \) could only decay to \( CP = -1 \) states (three pions).

11. Experiments show that is generally the case (albeit not precisely). Observe that the decay of \( K_1 \) to two pions is much faster, because the energy released is greater (mass of one pion is \( \approx 130 \text{MeV} \)) and so there is more phase space available. In comparison, the mass of three pions is larger and so the decay rate of \( K_2 \) will be slower. If we start with a beam of \( K_0 \) particles (produced in the strong interaction, which preserves strangeness, so the products will be strangeness eigenstates) we will see after a while only \( K_2 \) particles left in the beam. The lifetimes are given by \( \tau_{K_1} \approx 10^{-10}s \) and \( \tau_{K_2} \approx 10^{-8}s \).

12. Cronin and Fitch showed in 1964 that after a long while \( K_2 \) can decay into two pions as well. This is a very small effect, but clearly shows that \( CP \) is not conserved in the weak interaction. Thus it would appear that the long-lived kaon state is actually a mixture of \( K_1 \) and \( K_2 \) which can be written as:

\[
|K_L> = \frac{1}{\sqrt{1+|\epsilon|^2}} (|K_2> + \epsilon |K_2>) \quad \text{where } \epsilon \text{ is a small parameter which determines the mixing.}
\]

This can be accommodated for by adding a small phase factor to the Cabibbo-Kobayashi-Maskawa matrix.

13. While \( |K_L> \) prefers to decay to three, rather than two, pions, it can also decay into \( \pi^+e^-+\nu_\epsilon \) or into \( \pi^-e^++\nu_\epsilon \). Note that these two possibilities are \( CP \) conjugates of each other. It turns out that the branching ratios for these two possibilities are not the same, hence, another evidence of \( CP \) violation by the weak interaction.

14. If we let the \( K_2 \) beam interact with matter we’ll regenerate some \( K_1 \) again. Note that \( \sigma (\bar{K_0}) < \sigma (K^0) \).

15. By observing strangeness oscillations we can estimate the mass difference between the \( K_1 \) and the \( K_2 \) states. It turns out that \( \Delta m \approx 3 \cdot 10^{-6}\text{eV} \).

The probability to find \( \bar{K_0} \) if we started with \( K_0 \) goes like \( P(\bar{K_0}) \propto \exp\left(-\frac{1}{2} \left(\Gamma_1 + \Gamma_2\right) t \right) \cos (\Delta m t) \).
Part III

From Test Protocols

13 Things to Memorize

13.1 Memorize Leisurely

1. Alpha decay, like other cluster decays, is fundamentally a quantum tunneling process. Unlike beta decay, alpha decay is governed by the interplay between the nuclear force and the electromagnetic force.

2. Age of the solar system \( \sim 10^9 \) years.

Fig. 4.12 Feynman diagrams contributing to \( K^0 = \bar{K}^0 \). (There are others, including those with one or both \( u \) quarks replaced by \( c \) or \( t \).)
14 Things To Do and Open Questions

1. How do \( \nu \) come to be? Beta decay?

2. Which \( \nu \) type is most prevalent?
3. Are $\mu$ stable? No.

4. Ratio of $\nu$ types in the atmosphere.

5. Where do $\nu$ come from? Uranium fission? How does the reaction sequence happen (first Uranium is excited, then ...)

6. See of quarks..?

7. Feynman diagram of $e^- - e^+$

8. Inverse $\beta$ decay Feynman diagram

9. What spin does the $W$ have? (It’s a Boson) How to measure it? Homework exercise.


12. Electron capture?


14. There are situations (A=76) where only double beta decay is helloed.

15. How to measure the mass of neutrinos? (HWS10E04) Why use Triton?? What’s the current limit on the mass of the neutrinos?

16. How to measure neutrino oscillations? (KAMLAND)

17. What is the typical error of a particle counter?

18. How to produce pions?? cosmic rays or proton collisions.

19. How to accelerate protons? How does a linear accelerator works?

20. How does a synchrotron werk?

21. How to excite protons? (deep inelastic scattering)

22. Which delta resonances exist? 1232 (they are an isospin quadruplet)

23. How do the pions ($\pi^\pm$, $\pi^0$) decay? and with which interaction?

24. What’s the angular momentum of $^3He$, $^3H$, $^4He$?
25. Be able to say what’s the total angular momentum of a particular nuclide ($^{113}$Nb for instance)

26. How does a given nuclide decay?

27. How to detect neutrinos? (neutrinos interact with deutrium)

28. Which conservations laws *always* hold? (take table from somewhere)

29. Theoretical and experimental proof of the TCP theorem.

30. CP violation with the Kaon decay.

31. Spontaneous decay: $\alpha$, $\beta$, $\gamma$ decays. (does it necessitate deformation?)

32. Time scales for each interaction.

33. Fermi / Gamow-Teller decays (by spin of decay)

34. Prerequisites for each scattering (Rutherford, Mott, etc..)

35. How do different form factors look like for various charge distributions..

36. Why is the form factor independent of $Q^2$? (point-like charge distribution!)

37. Dependence of $Q^2$ on $x$.

38. In beta decay the coupling constant on the vertices are *not* the same: for the quarks it’s $g \cdot \cos(\theta_C)$ whereas for the leptons it’s $g$.

39. Neutrinos are *not* massless, as can be seen by neutrino oscillations (which can be observed by $\beta$ decay of tritium).

40. Continuous energy spectrum for neutrinos in $\beta$ decay.
41. Charge density distribution of various materials ($He$ and $Pb$ (Fermi-distribution and Gauss Distribution)).

42. HERA experiment

43. Weinberg angle??

44. Memorize the Rosenbluth formula.

45. The potential for protons and neutrons is different in the Fermi gas model??

46. masses of different particles.

47. weak isospin! (all interactions conserve weak isospin, all interactions go from one state of weak isospin to another).


49. Bjorken scaling variable / scale “breaking??”

50. Higgs particle.

51. In Deep Inelastic Scattering, what is the significance of the deflection angle of the electron to the nucleon?

52. How to measure the mass of the $Z^0$?

53. Memorize the CKM matrix.

54. the spin of the $W$ and how to measure it!

55. Nuclear _fusion_.

56. Main cycle in the sun.

57. How to measure $e^-$? Crystals (scintillators or sandwiches, sandwiches have more fluctuations).

58. Different resonances of $e^- - e^+$ collision?

59. $Z^0$ production.

60. Why are there long lived and short lived vomitters? Because of available phase space.

61. Open questions of the standard model? (dahak dark matter, neutrinos’ masses)

62. Where does the mass difference between the proton and the neutron stem from (if one ignores the electric charge difference between the u and d quarks)? Mass difference between u and d’s, plus u and d interaction.

63. Which symmetries are exact and which are approximate in the STANDARD MODEL?
64. What’s “special” about the $J/\psi$ particle?

65. Propagator for each Feynman-diagram vertex.

66. Can scattering and annihilation be distinguished?

67. “Fermi-point” interaction was the model before the weak interaction.

68. Mixing of $B^0$ and $W^0$ with the WEINBERG angle.

69. $e = g \cdot \sin (\theta_W)$

70. Why can’t free quarks be observed? (only white color, or colorless, shit, can be observed)

71. How does the electron lose energy in a material? (brehmsstrahlung, ionization)

72. Feynman diagram for BREMSSTRAHLUNG.

73. How does the photon lose energy in a material? (photo electric effect, compton scattering, pair-production)
74. The reason for introducing the weak isospin.

75. Coherence length.

76. The weak decay is so slow not because the weak coupling constant is small, but rather, because the weak gauge boson are so massive, and their masses enter in the inverse form to the propagator.

14.1 Experimental Setups

14.1.1 Experimental Designs

14.1.1.1 Cowan-Reines Experiment  Discovered the continuous spectrum of electrons in beta decay, and thus, neutrinos.

14.1.1.2 Cherenkov Experiment

14.1.2 Actual Experiments

14.1.2.1 DESY  Typical beam energies.

- Protons $900\text{GeV}$
- $e^- 30\text{GeV}$

What does the experimental setup look like?

14.1.2.2 LHC  How are $\nu$ detected?

14.2 Particle Physics

14.2.1 Feynman Diagrams

Each vertex adds a factor of $\sqrt{\alpha}$
14.2.2 Interactions

14.2.2.1 EM Interactions

14.2.2.2 Weak Interactions Charged and neutral currents in $e - p$ scattering????

14.2.2.2.1 $\beta$ decay

14.2.2.3 Strong Interactions

14.2.3 Scattering Cross-Sections

14.2.3.1 Born Approximation Which assumptions for each model (spin / recoil / form factors)?

14.2.3.1.1 Magnetic Form Factor

14.2.3.1.2 Electric Form Factor

14.2.3.2 Parton Model

14.2.4 CP Violation in $K_0$
Eigenstates of the CP operator... decay into 2 $\pi$ or 3 $\pi$ states..

14.2.5 $Z_0$ resonance
One could count the number of $\nu$ families. “WG Breit-Wigner Distribution”.

14.3 Nuclear Physics

14.4 Early Universe

- Number of $\nu$ per cubic centimeter??