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Coherent nonlinear transport in quantum rings

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Abstract

While equilibrium properties of mesoscopic systems are well understood, many questions are still debated concerning the non-equilibrium properties, which govern nonlinear transport. Nonlinear transport measurements have been performed on two-terminal semiconductor quantum rings in the open regime, where the rings are used as electron interferometers and show the Aharonov–Bohm effect. We observe a magnetic field asymmetry of the nonlinear conductance, compatible with the non-validity of the Onsager–Casimir relations out-of-equilibrium. In particular, the voltage-antisymmetric part of the nonlinear conductance of these two-terminal devices is not phase rigid, as it is the case for the linear conductance. We show that this asymmetry is directly related to the electronic phase accumulated by the electrons along the arms of the ring and can be tuned using an electrostatic gate.

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Keywords: Aharonov–Bohm effect; Mesoscopic transport; Nonlinear transport

1. Introduction

Since the discovery of the Aharonov–Bohm (AB) effect, ring structures have played a major role in the development of mesoscopic physics. Transport experiments have been done in the open regime, where the ring is used as an electron interferometer, as well as on nearly isolated small rings, showing Coulomb blockade and the Kondo effect. The equilibrium properties of these systems have been studied extensively both theoretically and experimentally, and are generally understood. However, several questions have arisen recently about the non-equilibrium properties of these systems.

Many experiments have proven the coherent nature of transport in a semiconductor quantum ring. In the open regime, when the conductance through the ring is larger than e^2/h, the AB effect leads to oscillations of the conductance as a function of a magnetic flux enclosed by the ring, due to the successive constructive and destructive interference between both electron paths going around the ring [1]. In the closed regime, semiconductor quantum rings have shown effects of quantum confinement in the Coulomb blockade regime [2]. One of the best proofs of coherent transport through a closed quantum ring is the observation of the Kondo effect [3,4], where correlations between electrons in the leads and in the ring produce an enhanced density of states aligned with the chemical potential of the leads. Nonlinear transport measurements in a three-terminal ring in the Kondo regime have been used to probe the out-of-equilibrium Kondo density of states [14].

Recently it has been shown that nonlinear transport in a mesoscopic system can lead to intriguing effects [5–13]. In particular, properties usually known in the linear regime, such as Onsager–Casimir relations for the transport, are not valid any more. For a two-terminal system, this will induce a magnetic field asymmetry of the nonlinear conductance, as it has been demonstrated theoretically [5–8], and reported recently in various mesoscopic systems [9–13]. In the following we will show that an AB ring can show such an asymmetry, and that the asymmetry can be tuned by changing the electrostatic potential around the ring, suggesting the importance of the electronic phase in

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both the origin of the nonlinear transport and the magnetic field asymmetry.

2. The samples

The samples are fabricated from a GaAs/AlGaAs heterostructure containing a two-dimensional electron gas 34 nm below the surface. The surface of the heterostructure is locally oxidized using an atomic force microscope, thereby defining depleted lines in the 2DEG underneath the oxide lines [15,16]. The results presented here were obtained on two samples. Both are rings, connected to three terminals in case of the sample shown in Fig. 1(a) and labeled *ring A*, and to two terminals for the one shown in Fig. 1(b) and labeled *ring B*. From the period of AB oscillations, we determine the effective diameter of *ring A* to be 260 nm, and that of *ring B* to be 460 nm. Both values are compatible with the lithographic characteristics. Additional in-plane gates allow to tune the couplings of the leads to the ring or the Fermi energy in the ring.

In the following, the gate voltages are chosen in order to tune both rings in the open regime, with a conductance of the order of e^2/h. For the case of *ring A*, the third contact is closed by applying a negative voltage on the gate LG3, which leads to an effective two-terminal measurement. This is confirmed by checking that the current through lead 3 is always zero within measurement accuracy (see Fig. 2). The d.c. I–V characteristics are measured by applying a bias voltage between leads 1 and 2, i.e., $+V/2$ on lead 1 and $-V/2$ on lead 2, while the current is measured. The experiment has been done in a pumped ^{4}He cryostat with a base temperature of 1.7 K.

3. Magnetic field symmetries of the nonlinear conductance

A typical I–V trace is shown in Fig. 2 for the case of *ring A*. It is already clear from this measurement that nonlinearities occur above a bias voltage of around 100 μV, as seen when comparing the experimental curve

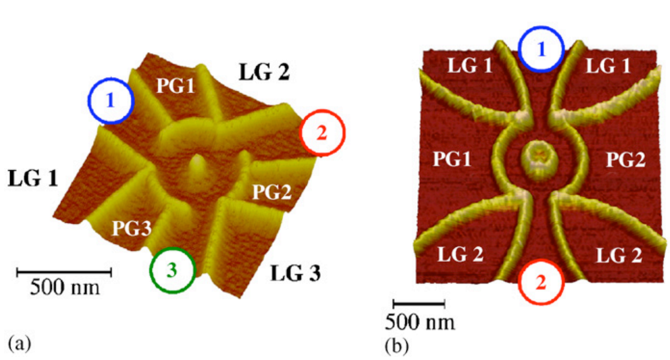

Fig. 1. Topographic images of the two samples, taken with an AFM just after the oxidation process: (a) *ring A*, which is initially connected to three terminals, but in the following the lead 3 is kept closed; (b) *ring B*, connected to two terminals. The gates labelled LG1, LG2 and LG3 are used to tune the openings of the rings, while the gates labelled PG1, PG2 and PG3 tune the Fermi energy in the ring.

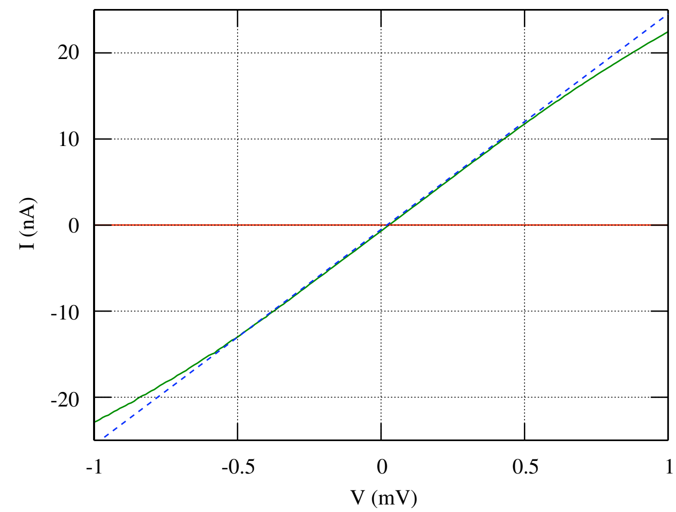

Fig. 2. Current–voltage characteristics measured on *ring A* at a magnetic field $B = 57$ mT (solid line). The dashed line shows a linear dependence with a slope corresponding to the linear conductance. The horizontal line at zero current is the measured current through lead 3.

(solid line) with a linear fit of the low-voltage part of the curve (dashed line). The nonlinear conductance, $g(V,B) = dI(V,B)/dV$, is extracted from the d.c. I–V curves by numerical differentiation. The voltage-symmetric part, $g_s(V,B) = \frac{1}{2}[g(V,B) + g(-V,B)]$, and voltage-antisymmetric part, $g_a(V,B) = \frac{1}{2}[g(V,B) - g(-V,B)]$, are plotted in Fig. 3. A clear difference is found between both plots. The voltage-symmetric part is symmetric in magnetic field, clearly shows h/2e oscillations around zero magnetic field, and is strongly suppressed at high bias voltage. The voltage-antisymmetric part is asymmetric in magnetic field, shows mainly h/e oscillations, almost no h/2e oscillations, and its magnitude is preserved at large bias voltage.

The magnetic field asymmetry of the nonlinear conductance can be seen more clearly when expanding the nonlinear conductance in powers of the applied voltage. We have fitted the I–V curves with a polynomial:

$$I(V,B) = \sum_{n=1}^{N} G^{\{n\}}(B)(V - V_0). \quad (1)$$

The order of the polynomial, $N = 5$ or 7, has been chosen in order to obtain the best fit of the curve. We have checked that changing this order does not change significantly the low-order coefficients, in which we are interested.

The coefficients of the Taylor expansion are shown in Fig. 4 as a function of magnetic field. We clearly see in this representation that the odd coefficients, which represent the Taylor expansion of g_s, are symmetric in magnetic field within experimental accuracy. On the contrary, the even coefficients are not symmetric in magnetic field, and show oscillations with a phase shift intermediate between 0 and π. The strong suppression of h/2e oscillations in the even coefficients compared to the odd ones is also remarkable.

These features of the nonlinear conductance have been obtained in a particular configuration of gate voltages, i.e.,

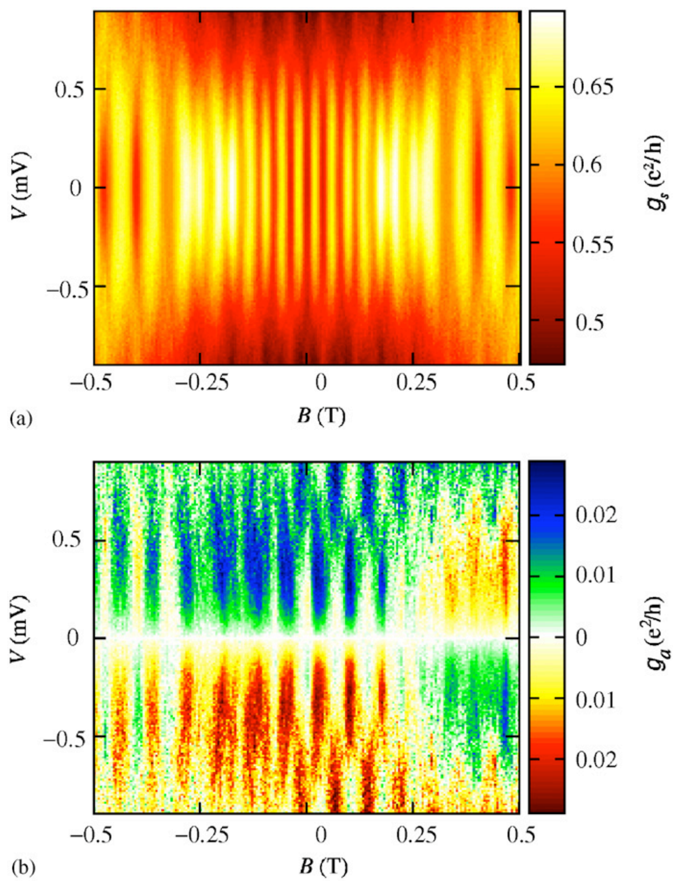

Fig. 3. Parts of the conductance which are (a) symmetric in bias voltage, g_s, and (b) antisymmetric in bias voltage, g_a, measured on *ring A* with lead 3 closed, and plotted as a function of the magnetic field and the bias voltage.

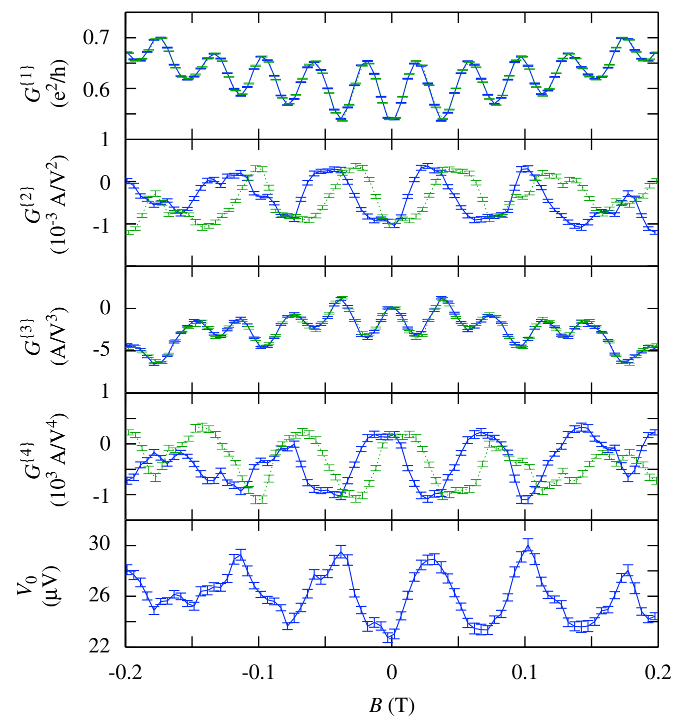

Fig. 4. Magnetic field dependence of the nonlinear conductance coefficients measured on *ring A*, and corresponding to the same measurement as in Fig. 3. Only the four first coefficients are shown here, while the I–V curves have been fitted with a fifth order polynomial. On the four upper curves, the solid line is the original set of data, while the dashed line is the same data set inverted with respect to $B = 0$. The bottom curve is the offset voltage, V_0, also deduced from the fit with Eq. (1).

particular values of the conductance at the quantum point contacts at the input and the output and particular electrostatic potential around the ring. We are unfortunately not able to characterize in detail this configuration, but the measurements obtained in *ring A* have been reproduced in *ring B* as shown in the next part, which proves the universality of this effect.

4. Modulation of the phase using a lateral electrostatic gate

To further check the origin of both the nonlinear conductance and the magnetic field asymmetry of the nonlinear conductance, we have varied the gate voltage $PG1$ on *ring B*, and measured I–V traces as a function of the magnetic field for each gate voltage. The gate is expected to modify the Fermi energy in one of the arms of the ring, thereby modifying the phase accumulated by the electron passing through this arm [17–21].

The results for $G^{\{1\}}$ and $G^{\{2\}}$ are shown in Fig. 5. The linear conductance shows clear phase jumps from 0 to π when the gate voltage is changed, as already observed for other two-terminal AB rings [19,21]. This plot shows that the electronic phase changes by more that 2π within the full gate voltage range.

The behavior observed in $G^{\{2\}}$ is much more surprising. The phase of the oscillations varies linearly with the gate voltage, as clearly seen for negative voltages (see white solid lines in Fig. 5(b)), and compatible with the results already reported in *ring A* [13]. This suggests that the rectifying part of the conductance can be controlled by changing the electronic phase accumulated along the arms of the ring. In addition, we show here that the nonlinear conductance is not phase rigid, as it is the case for the linear conductance. In particular, the linear change of the phase with gate voltage is very similar to the measurement done with open AB interferometers [20,22,23], where it has been shown that the phase of the AB oscillations is a direct measurement of the phase difference between the two electron paths around the ring.

In addition to the strong oscillations in $G^{\{2\}}$, one can see also weaker oscillations with a phase varying exactly in the opposite direction compared to the strong oscillations (see black dashed lines in Fig. 5(b)). These additional lines are stronger around zero gate voltage.

5. Discussion

Here we would like to discuss the main results that have been presented above. These are the nonlinear transport, in particular the even coefficients of the conductance expansion, the magnetic field asymmetries of these coefficients,

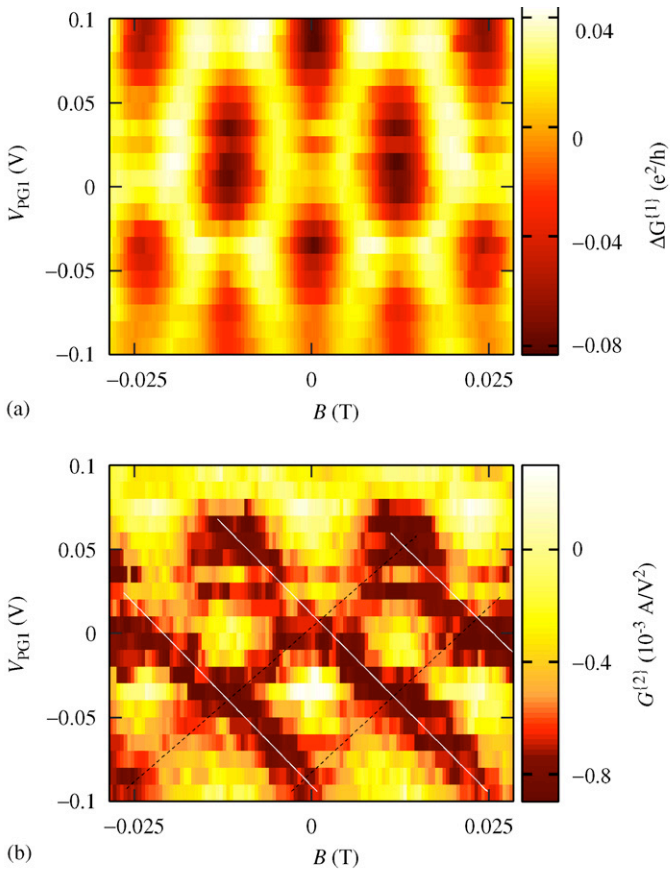

Fig. 5. (a) Oscillating part of the linear conductance, $\Delta G^{\{1\}} = G^{\{1\}} - \langle G^{\{1\}} \rangle$, and (b) quadratic conductance, $G^{\{2\}}$, measured on *ring B* as a function of the magnetic field and the voltage applied on PG1. The AB oscillations observed in $G^{\{2\}}$ for large negative gate voltage are clearly asymmetric in magnetic field, and the phase of these oscillations vary linearly as a function of the gate voltage (see plain lines). In addition, weaker oscillations with an exactly opposite phase can be seen (see dashed lines).

which can be tuned with an electrostatic gate, and the strong suppression of h/2e oscillations in the even coefficients.

Basic effects, such as electron heating or voltage induced decoherence, could explain nonlinear transport in mesoscopic systems. These effects are, however, expected to be independent of the sign of the current. They could therefore explain the voltage-symmetric part of the conductance, g_s, shown in Fig. 3(a). In particular, the suppression of the AB oscillations in g_s at high bias is indeed compatible with a decoherence mechanism or energy averaging, which could be induced either directly by the bias voltage, or by an increase of the temperature.

However, these effects are not expected to contribute to g_a, which involves only effects antisymmetric in the voltage direction. Nonlinearities could originate from the quantum point contacts (QPCs) at the input or output of the ring, or from non-ideal contacts. We have checked on a separate QPC made on the same wafer that nonlinearities originating from the contacts or the QPCs depend only weakly on the magnetic flux, on the scale of the flux quantum through the ring. Finally, the fact that both the voltage-symmetric and voltage-antisymmetric parts of the conductance have different magnetic field symmetries excludes spurious circuit-induced nonlinear effect to explain the voltage-antisymmetric part of the conductance: a change of the conductance at large bias due to self-gating effect would produce a nonlinear conductance with the same magnetic field symmetry as for the linear conductance.

A rectifying behavior has already been observed experimentally in mesoscopic systems [24–27], and was explained in terms of the scattering theory with a bias voltage dependent transmission [28–30]. Such nonlinear behavior is expected for a bias voltage larger than E_T/e, where E_T is the Thouless energy. In *ring A*, one can estimate the Thouless energy from the "Thouless number" $g = G^{\{1\}}/(e^2/h) = E_T/\Delta$ [31], where $\Delta \approx 200\,\mu eV$ is the mean level spacing, determined from measurements in the Coulomb blockade regime. This estimate gives $E_T \approx 100\,\mu eV$, which is of the order of the applied voltage. We are therefore in the right regime for such nonlinearities to occur.

In these models, even coefficients in the nonlinear conductance are expected if the system is asymmetrically coupled to the reservoirs, via scattering or capacitive coupling. This is very likely to be the case in our system, where the openings to the leads can be independently tuned. It is interesting to note that such nonlinear conductance has been observed only for particular values of the gate voltages.

These models were developed for independent electrons, and only effects symmetric in magnetic field were predicted. A nonlinear conductance asymmetric in magnetic field has been suggested only very recently [5–8], and reported in a couple of experiments [9–12]. The asymmetry comes from the screening response of the system [5–8], which is in general not symmetric in magnetic field, and is a direct demonstration of strong electron–electron interactions in these systems. Such an asymmetry has not been reported for previous experiments on rings [26,27], in particular due to the multi-terminal nature of these experiments. It is not clear in our system what the importance of electron–electron interactions is. In particular, the low number of modes in the ring would suggest a weak screening regime, and a microscopic model would be necessary to understand the origin of the nonlinear transport in our system.

Another effect of the applied bias voltage could be the electrostatic AB effect: the applied bias voltage creates an electric field along the arms of the ring, which changes the electronic phase [32,33]. Both screening effect and electrostatic AB effect could be present in our system, but we were not able to distinguish between them.

The lack of h/2e oscillations in the even coefficients is a surprising result, which cannot directly be understood with the scattering theory for nonlinear mesoscopic transport [5,13]. h/2e oscillations in the linear conductance have been attributed to interference between two time-reversed paths enclosing once the magnetic flux, the so-called Altshuler–Aronov–Spivak (AAS) oscillations [34]. Another possible origin is the standard AB effect between paths enclosing twice the magnetic flux. Interestingly, AAS oscillations are not sensitive to local variations of the electronic phase along the ring, since both time-reversed paths will feel the same phase. Our results would suggest that AAS oscillations are also not sensitive to the bias voltage, at least for the voltage-antisymmetric nonlinear conductance. Unfortunately we were not able to check the origin of h/2e oscillations in these devices, and this question remains to be investigated.

Finally it is interesting to discuss the offset voltage V_0. A constant offset voltage is expected from the amplifiers and thermal voltages. However, we see in the lower panel of Fig. 4 a strong oscillatory contribution. Since this oscillatory part has the same asymmetry than the coefficient $G^{\{2\}}$, it could come from the rectification of an a.c. noise signal by the voltage-antisymmetric part of the conductance.

6. Conclusion

In conclusion, we have made nonlinear transport measurements in two-terminal rings. The nonlinear conductance shows AB oscillations as a function of the magnetic field, and can be decomposed into a voltage-symmetric and a voltage-antisymmetric part. The voltage-symmetric part is also symmetric in magnetic field, and can be possibly attributed to heating, energy averaging, or voltage-induced decoherence. The voltage-antisymmetric part of the conductance is asymmetric in magnetic field, and this asymmetry can be controlled directly by changing the electronic phase along the arms of the ring. It shows that the voltage-antisymmetric part of the nonlinear conductance is not phase-rigid. The main symmetries observed can be understood within the scattering theory for nonlinear mesoscopic transport. However, the microscopic origin of the nonlinear transport, as well as the strong suppression of h/2e oscillations of the voltage-antisymmetric part of the nonlinear conductance are still a puzzle.

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