

Chapter 8

Polarimetry

8.1 Description of polarized radiation

The polarization of electromagnetic radiation is described by the orientation of the wave's electric field vector. There are two different cases of polarization:

- linear polarization: the electric field is oriented in a single direction
- circular polarization: the electric field vector rotates

In general, light is a combination of linearly and circularly polarized light. The polarization of light can be described by the Stokes vector:

$$\mathbf{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I \\ I_0 - I_{90} \\ I_{45} - I_{135} \\ I_l - I_r \end{pmatrix} = \begin{pmatrix} \text{intensity} \\ \text{linear polarization } 0^\circ/90^\circ \\ \text{linear polarization } 45^\circ/135^\circ \\ \text{circular polarization} \end{pmatrix}, \quad (8.1)$$

with the following definitions:

- I_x is the linear intensity with the polarization direction x in degrees,
- I_l and I_r are the left and right handed circular polarization respectively.

Often a normalized Stokes polarization vector is used to describe the polarization:

$$\begin{pmatrix} Q/I \\ U/I \\ V/I \end{pmatrix} \quad (8.2)$$

The individual parameters Q/I , U/I , and V/I behave like vectors components where the length of the vector is the fractional polarization:

$$p = \sqrt{(Q/I)^2 + (U/I)^2 + (V/I)^2} \leq 1. \quad (8.3)$$

If only the linear polarization is considered then the fractional polarization is

$$p_{Q,U} = \sqrt{(Q/I)^2 + (U/I)^2} \quad (8.4)$$

and the orientation of the linear polarization θ :

$$\theta = 0.5 \arctan(U/Q). \quad (8.5)$$

One can also write $Q/I = p \cos 2\theta$ and $U/I = p \sin 2\theta$.

Some examples and special cases for the Stokes vector :

(1, Q/I, U/I, V/I)	description
(1, 0, 0, 0)	unpolarized light: $I_0 = I_{90}$, $I_{45} = I_{135}$, $I_l = I_r$, $p = 0$
(1, 1, 0, 0)	100 % linear polarization in the positive Q -direction: $I_0 = I$, $I_{90} = 0$, $I_{45} = I_{135}$, $I_l = I_r$, $p = 1$ and $\theta = 0^\circ$
(1, 0, -1, 0)	100 % linear polarization in the 135° direction : $I_0 = I_{90}$, $I_{45} = 0$, $I_{135} = I$, $I_l = I_r$, $p = 1$ and $\theta = 135^\circ$
(1, 0.15, 0.26, 0)	30 % linear polarization in the 30° direction
(1, 0.001, 0, 0.01)	1 % circular polarization with a 0.1 % component in linear polarization

In the photon picture one can treat a polarization measurement as a difference of the two count measurements of fully polarized photons in two opposite polarization modes,

- N_0 and N_{90} for a Stokes Q measurement,
- N_{45} and N_{135} for a Stokes U measurement,
- N_l and N_r for a Stokes V measurement.

An isotropic thermal source emits unpolarized light. Thus, both channels register the same photon numbers within the accuracy of the Poisson statistics, e.g. for Q : $N_0 = N_{90} = N/2$ and similar for U and V.

8.2 Astronomical applications

Polarization can be produced by anisotropic radiation processes which can be related to a few basic phenomena, some of which are described below. Although polarized radiation is a widespread phenomenon it is often hard to observe because the polarization is averaged down to a level below the detection limit for observations of unresolved structures and objects. For example the sun shows a linear limb polarization and circular and linear polarization of spectral lines which depend on the small scale magnetic field structures. Observed like an unresolved star both polarization effects are averaged down to a level ($p < 0.001$ %) which is essentially undetectable.

Therefore astronomical polarimetry has “only” a few, but important applications where the polarization signal is strong and provides interesting information about certain objects:

Light scattering. Light scattering produces linearly polarized light because the scattering particle is put into oscillations in a direction perpendicular to the incoming light, while the polarization of the scattered photon is predominately parallel to the acceleration direction. Scattering processes occur everywhere in astronomy but a strong asymmetry in the scattering geometry is required for the production of a strong net polarization signal. Phenomena with strong scattering polarization are:

- *Scattered solar light*: scattered sunlight from planets, moons, asteroids, comets and other solar system objects is polarized linearly at a level of 0.1 – 50 %,
- *circumstellar scattering* on dust and gas for stars undergoing mass loss or stars in formation produces linear polarization of up to ≈ 50 %,
- *circumnuclear scattering in AGN* (active galactic nuclei) produces strong linear polarization due to the material near the accreting super-massive black hole.

Zeeman effect. The Zeeman effect describes the line emission and absorption of atoms and molecules in magnetized plasmas. The energy level of atoms and molecules are splitted due to their angular momentum direction. The different “magnetic” sublevels produce emissions and absorptions with opposite polarization signals. The typical polarimetric line profile due to the Zeeman effect is a blue line wing with one circular polarization and a red line wing with the opposite circular polarization. The linear polarization signal is usually about an order of magnitude weaker than circular polarization. The Zeeman effect is observed in the following cases:

- *magnetized solar atmosphere*, where the small scale structure of the magnetic fields, for example of solar spots, can be investigated in detail,
- *magnetized stars*, with strong and large scale magnetic fields, like a global dipole field, or a stellar spot which dominates the surface. Magnetic A stars and white dwarfs have often a strong global field while rapidly rotating solar type stars show often Zeeman splitting due to one dominant stellar spot.
- *interstellar magnetic fields* can be measured from the polarization of emission lines.

Synchrotron emission. Synchrotron and cyclotron radiation by fast moving particles (mostly electrons) in magnetic field produces linear and circular emission. The orientation of the polarization depends on the orientation of the magnetic field:

- *stellar coronae* produce relativistic electrons triggered by the magnetic activity of the star and they emit strongly variable, polarized synchrotron radio emission,
- *relativistic electrons in the interstellar and intergalactic medium* emit highly polarized $\approx 30\%$ circular and linear polarization in the radio range. This polarized radiation is present in strongly shocked regions, like supernova remnants, or on large scales as diffuse radio emission of the Milky Way and other galaxies.
- *relativistic jets from quasars and gamma ray bursts* produce polarized light of up to 30% linear polarization and up to a few % in circular polarization in the radio range but also in the visual.

Dichroic absorption. The absorption of light is polarization dependent if the orientation of anisotropic absorbers has a predominant direction. This polarizing effect occurs for the reflection from surfaces of solids, the transmission through structured materials, but also for gas where the absorbing gas particles have a predominant direction due to the alignment by magnetic fields.

- *interstellar dust* is magnetically aligned by galactic magnetic fields and they can produce a linear polarization of a few % for background sources which are affected by interstellar absorption. This effect is useful for the study of the magnetic field structure in the Milky Way.

Birefringence. Birefringence is the polarization dependent refraction by a medium because of a predominant orientation of the medium structure. This effect is used to analyze the polarization of light with special crystals. For plasma gas a magnetic field with a predominant direction can produce a circular birefringence (Faraday rotation).

- *Interstellar Faraday rotation*: the interstellar medium has a circular birefringence proportional to the product of the line of sight magnetic field and the electron column density $B_{\parallel} \cdot N_e ds$. This product is the so-called rotation measure, which

defines the rotation of the linear polarization of background radio sources. Since the rotation is wavelength dependent the measurement of the rotation at two frequencies yields the rotation measure.

8.3 Scientific requirements

The scientific requirement for polarimetric observations are dictated by the science goals. Of specific interest for polarimetry are the following issues:

- linear or circular polarimetry,
- polarimetry or spectropolarimetry,
- required polarimetric sensitivity for the normalized polarization,
- is an absolute polarimetric calibration required,
- the spatial resolution for imaging polarimetry,
- spectral resolution for spectropolarimetry.

Single target polarimetry: Instead of taking polarization measurements of a target in several filters one should consider the advantages and disadvantages of low resolution spectropolarimetry, which provides the polarization signal in a broad wavelength band with a single observation. Spectropolarimetric data can be very useful to investigate and distinguish between the polarization from different emission components and the contribution of the interstellar polarization.

Faint targets: The precision of polarimetric observations of faint targets, e.g. AGN, SN, GRB, is often just dictated by the photon statistics. Thus the predominant requirement for better polarimetric data is simply: **more photons**.

Spatial resolution: The spatial resolution of polarimetric observations is essential for spatially extended targets. Since the polarization can have a positive or a negative value the signal of unresolved structures with opposite polarization components just cancels and no net polarization remains. Thus a target with a strong polarization pattern consisting of positive and negative features may show only a very low polarization signal in low resolution observation.

In Zeeman spectropolarimetry one can often use the continuum as a zero polarization reference. Thus one should achieve a very high polarimetric sensitivity while the absolute calibration of the data is less important. The spectral resolution is very critical in Zeeman spectropolarimetry. Since a line is composed of positive and negative polarization components, insufficient spectral resolution results in the cancellation of the signal.

8.4 Basic measuring principle

In polarimetry one measures the normalized Stokes parameters Q/I , U/I , and V/I , which are the differential intensity signals between two opposite linear polarization modes.

There are two ways to measure the polarization:

- *single beam:* measuring the two polarization modes consecutively (e.g. I_0 and I_{90}) with an instrument that can measure different polarization states.
- *double beam:* measuring the two polarization modes (e.g. I_0 and I_{90}) simultaneously with an instrument which can split the opposite polarization modes.

If more than one Stokes parameter is measured then also more measurements are required: at least three measurements for Q/I and U/I and at least four measurements if also V/I is determined.

8.4.1 Polarizers, polarization beam splitters, and retarder plates

Dichroism. In dichroic materials the absorption is polarization dependent. Usually one direction of linear polarization is absorbed stronger than the opposite direction.

Wire grid polarizers: In wire grid polarizers the light with the E-vector parallel to the wires is absorbed because charge motions are induced, while a wave with perpendicular E-vectors passes through. The spacing between the wires must be less than the wavelength. Wire grid polarizers are now also available for visual light, thanks to the advances in micro- and nano-technology.

Polaroid: Polaroid films are made of a stretched plastic so that the molecules act like the wires in the wire grid polarizer. They are cheap to manufacture and are used in many applications like sunglasses, photographic filters, liquid crystal displays, and optics.

Birefringence. Birefringence or double refraction is the decomposition of light into two rays in structured materials, usually crystals, for which the refractive indices are different for different polarization directions. For uniaxial materials (single axis of anisotropy = optical axis) the birefringence magnitude is defined by

$$\Delta n = n_e - n_o$$

where n_e and n_o are the refractive indices for the polarization parallel (extraordinary) and perpendicular (ordinary) to the axis of anisotropy. A very important birefringent material is Calcite CaCO_3 , but also other materials like quartz SiO_2 or MgF_2 are birefringent:

Uniaxial birefringent materials (at 590 nm):

Material	n_o	n_e	Δn
Calcite	1.658	1.486	-0.172
Quartz	1.544	1.553	+0.009
MgF_2	1.380	1.385	+0.005

Birefringent materials can be used to change the polarization state of an electromagnetic wave and to separate (split) efficiently opposite polarization modes. There are many interesting applications of the birefringence effect in optics. We discuss here only retarder plates and polarization beam splitters:

Half wave and quarter wave retarder plates. These plates retard one polarization direction by a half or a quarter of a wavelength. Retarder plates can be made from quartz and other crystals, but also from special foils.

Half wave plates can be used to flip the polarization direction of linearly polarized light if the optical axis is rotated with respect to the polarization direction. An angle of 45°

switches the polarization by 90° , equivalent to an exchange of the $+Q$ and $-Q$ polarization modes.

Quarter wave plates convert linear polarization into circular polarization and vice versa if the orientation of the plate is ± 45 degrees with respect to the linear polarization direction.

Polarization beam-splitters. Polarization beam splitters are often made of calcite due to its strong birefringence. If two prisms are put together with optical axes perpendicular to each other, then one polarization direction sees an interface from n_o to n_e and the other n_e to n_o . Thus one beam will be refracted away from the surface normal, and the other towards the surface normal. If the angle of incidence onto the interface is large enough then one polarization direction is transmitted while the other is reflected due to total internal reflection. Two examples for polarization beam splitters are the Wollaston prism and the Glan prism.

8.4.2 Different instrument types

Single beam instrument. A single beam instrument is performing measurements in two or more alternating polarization modes. Such an instrument achieves a precision like for absolute photometry in different filters. Thus, a single beam polarimeter, like in HST, uses just a set of polarization filters which can be introduced in the beam.

For ground based observations the main problem are the variations introduced by the scintillation due to the seeing which is at a level of about 1 %. Another challenge is the photometric calibration of the individual polarization modes. With the single beam method it is difficult to achieve an accuracy of better than $\Delta p \approx \pm 3$ % with ground based observations.

With space observations, e.g. with HST, the atmospheric scintillation problem is absent and the achieved polarimetric precision is about $\Delta p \approx \pm 1$ %. The precision is limited by the uncertainties in the calibration of the individual polarization modes because of not well known polarization effects introduced by the telescope and other components of the system.

A fast modulation single beam instrument. Fast polarimetric modulation can solve the atmospheric variability and instrument drift problems. This instrument consists of a fast electro-optical polarization modulator which changes either the retardation or the orientation of the retardation. Combined with a subsequent polarizer the polarization signal is converted into an intensity modulation. There is no intensity modulation if the incoming light is unpolarized and a strong modulation for strongly polarized light. The detector must be able to measure the fast intensity changes between the two modulation modes. The Zurich Imaging Polarimeter (ZIMPOL) system is based on this principle.

Important is that the measurement between the two polarization modes is switched faster than the intensity variations introduced by the atmosphere or the instrument. A fast modulation (> 1 kHz) is possible with electro-optical components. With a fast modulation a very high sensitivity of $|\Delta p| < 0.001$ % can be achieved. Since the same detector elements are used for the opposite polarization modes also many flat-fielding and calibration issues are solved.

Fast modulation creates quite some challenging requirement for the imaging detector for measuring the differential polarization signal. This problem has been solved with the ZIMPOL system which charge shifting on the CCD.

The absolute calibration of such high sensitivity measurements is still a problem and is often not better than $\Delta p \approx \pm 0.1$ %. This is, however, no problem if one needs to measure the polarization signal only relative to a reference signal, like Zeeman profiles with respect to the stellar continuum.

A double beam instrument. If both polarization directions are measured simultaneously then atmospheric changes cancel out in the subtraction (e.g. $I_0 - I_{90}$). The two channels must then be calibrated (flat-field and zero point) relative to each other. The measuring precision which can be achieved with this mode is about $\Delta p \approx \pm 1$ %. The precision is mainly limited by the the calibration of the two channels.

Thus a double beam instrument uses a beam splitter which splits the parallel and perpendicular polarization direction as defined in the instrument coordinate system. The whole instrument can then be rotated to measure another linear polarization direction. Circular polarization can be measured with a quarter-wave retarder plate inserted in front of the beam splitter.

A double beam instrument with beam switch. Beam switching reduces strongly the calibration problems of a double beam instrument. With beam switching a polarimetric sensitivity of $\Delta p \approx \pm 0.1$ % can be achieved. The limiting factor in such systems are usually the calibration of the polarization effects introduced by the telescope and the optical components in front of the polarimeter.

In this system the first polarimetric component is a rotatable retarder, either a half-wave plate (HWP or $\lambda/2$), which rotates the linear polarization into a particular orientation or a quarter wave plate (QWP or $\lambda/4$) which converts circular polarization into measurable linear polarization. The Wollaston then splits the $I_{\perp}$ and $I_{\parallel}$ polarization direction as defined by the orientation of the Wollaston into two images separated on the CCD by about $10''$ or $20''$. Thanks to the special aperture masks the signals from the two Wollaston beams do not overlap. Rotating the retarder between the two exposures, e.g. from 0° to 45° for Stokes Q , allows to swap the two polarization images, so that differential effects in the two Wollaston beams cancel out in the polarization signal (including the

individual pixel efficiencies of the CCD). Thus two images taken with different retarder plate orientations yield one normalized Stokes parameter corrected for differential effects.

8.5 Telescope and instrument polarization

The polarimeter measures the polarization at the location of the analyzing polarization components. This means that any polarization effect introduced by the telescope or optical components in front of the polarization components have to be calibrated. Basically all inclined surfaces introduce instrument polarization. For example a Nasmyth mirror (telescope mirror M3) with an Al coating which is inclined by 45° affects the incoming light by introducing an instrument polarization of about 5 % and causing a retardation which converts about 10 % of the linear Stokes U component of the light into circular polarization Stokes V. Therefore one should:

- place a polarimeter at a polarization free focus, like the Cassegrain or Gregory focus of the telescope,
- place no inclined surfaces in front of the polarization optics.

Unfortunately this is not always possible. For example the E-ELT has no polarization free focus, or for 8 m telescopes the instruments are so large that they cannot be mounted on the Cassegrain focus (e.g. SPHERE, VLT planet finder). FORS is a good example for an ideal double beam instrument with beam switch located at the Cassegrain focus of the telescope.

8.5.1 Mueller matrix model.

The polarization effects of an instrument can be modelled with Stokes vectors and Mueller matrices. Mueller matrices $\mathbf{M}$ describe the effect of optical components on light described as Stokes vector $\mathbf{I}$. The resulting Stokes vector is then

$$\mathbf{I}' = \mathbf{M} \cdot \mathbf{I} \quad (8.6)$$

The polarization effects of the instrument can be characterized with different elements of the matrix $\mathbf{M}$:

$$\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} = \begin{pmatrix} I \rightarrow I & Q \rightarrow I & U \rightarrow I & V \rightarrow I \\ I \rightarrow Q & Q \rightarrow Q & U \rightarrow Q & V \rightarrow Q \\ I \rightarrow U & Q \rightarrow U & U \rightarrow U & V \rightarrow U \\ I \rightarrow V & Q \rightarrow V & U \rightarrow V & V \rightarrow V \end{pmatrix}.$$

For example the elements $I \rightarrow Q, U, V$ describe the instrument polarization introduced by the telescope and instrument, and $Q \leftrightarrow U, Q \leftrightarrow V, U \leftrightarrow V$ are the polarization cross talks. The Mueller matrix for a perfect instrument would be just the unit matrix

$$\mathbf{M}_{i,t} \cong c_{i,t} \cdot \mathbf{1} \quad (8.7)$$

multiplied with the transmission $c_{i,t}$. The Cassegrain instruments FORS is close to this ideal case.

Reflections from inclined surfaces are often the cause for instrument polarization. The effect depends on the component's surface coating and on incidence angle and wavelength.

For an Al-coated mirror with an incidence angle of 45° (Nasmyth mirror) the Mueller matrix is for the I-band (800 nm) and H-band ($1.6 \mu\text{m}$):

$$\mathbf{M}_{t(\text{I})} = \begin{pmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.05 & 0.95 & 0 & 0 \\ 0 & 0 & -0.94 & -0.14 \\ 0 & 0 & 0.14 & -0.94 \end{pmatrix}, \quad \mathbf{M}_{t(\text{H})} = \begin{pmatrix} 0.99 & 0.01 & 0 & 0 \\ 0.01 & 0.99 & 0 & 0 \\ 0 & 0 & -0.99 & -0.08 \\ 0 & 0 & 0.08 & -0.99 \end{pmatrix} \quad (8.8)$$

For Nasmyth instruments we have to consider the polarization introduced by the inclined mirror M3 and the field rotation. Thus rotation matrices $\mathbf{R}(\theta)$ have to be introduced in the Mueller calculus:

$$\mathbf{R}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (8.9)$$

The full Mueller matrix for the telescope and instrument polarization for a Nasmyth instrument is given by:

$$\mathbf{M} = \mathbf{M}_i \cdot \mathbf{R}_2^{Q,U}(p, z) \cdot \mathbf{R}_1(z) \cdot \mathbf{M}_t \cdot \mathbf{R}_0(p) \quad (8.10)$$

The different matrices stand for:

- $\mathbf{R}_0(p)$: rotation for the parallactic angle p (telescope orientation),
- $\mathbf{M}_t$: Mueller matrix for telescope (essentially mirror M3),
- $\mathbf{R}_1(z)$: rotation for the zenith angle,
- $\mathbf{R}_2^{Q,U}(p, z)$: rotator position for Nasmyth instrument,
- $\mathbf{M}_i$: Mueller matrix for Nasmyth instrument.

It should be noted that the parallactic angle p and the zenith angle z change with the telescope pointing direction. Thus, the instrument polarization $\mathbf{M}$ has to be known for a large range in p and z in order to be able to calibrate polarimetric measurements with a high accuracy. The calibration has to be achieved with measurements of polarization standard stars. The telescope and instrument matrices, $\mathbf{M}_t$ and $\mathbf{M}_i$, have to be determined, while the rotation matrices are defined by the telescope pointing direction.

8.5.2 Polarimetric calibration measurements

The measured polarization signal needs to be calibrated and converted into the sky polarization system. The sky polarization is defined by:

- the N-S direction for the zero point for the orientation of the linear polarization
- the polarization position angle θ is measured from N over E
- an unfortunate confusion exists for the circular polarization, because radio astronomer and optical astronomers used traditionally opposite definitions. It is necessary to specify and check in each paper the definition of right handed or left handed circular polarization.

For the conversion of the measured signal with a given instrument one has to calibrate or check the instrument specific parameters with standard stars measurements:

- the determination of the amount of instrument polarization p_{inst} with observations of zero-polarization standard stars,

- the measurement of the zero point of the polarization angle θ_{inst} with observations of polarized standard stars,
- the polarization efficiency of the instrument with measurements using a polarizer as calibration component or a check with a polarized standard star for which one can measure the instrumental reduction of the resulting polarization.

Standard stars. Popular lists of polarized standard stars are Hsu & Breger (1982) or Turnshek et al. (1991).

Zero polarization standard stars. Most nearby solar type stars are good zero-polarization standard stars. In order to be sure that a given star is unpolarized one should, however, consider the existing standard star lists.

High linear polarization standard stars. Since the interstellar polarization introduces a linear polarization for distant objects there exist a reasonable number of well measured high polarization standard stars. One should notice that these stars are all distributed along the galactic plane and not available for all sky regions. Further the measuring precision for the polarization position angle is even for these standard stars not significantly better than $\Delta\theta = \pm 1^\circ$.

Circular polarization standard stars. For circular polarization one should check with a zero polarization standard star the instrument polarization and with a linearly polarized standard stars the cross talk effect, the conversion from $Q, U \rightarrow V$. Circularly polarized standard stars (which deserve the name standard star) are essentially not existing. For Zeeman spectropolarimetry, it is recommended to measure always a comparison star with known magnetic field orientation in order to be sure about the sign $+/-$ of the measured V-signal.

Example: FORS For FORS the instrument polarization is $p_{\text{inst}} \cong 0.1\%$ near the optical axis. It has been noticed that there exists a centro-symmetric instrument polarization for off-axis positions. For FORS p_{inst} increases steadily from zero to about $p_{\text{inst}} \approx 1.5\%$ at the edge of the field of view at $d = 5'$.

For FORS the zero point angle θ_{inst} varies slightly with wavelength due to the color dependent orientation of the optical axis of the HWP (see FORS web page).

8.6 Data reduction

In principle polarimetry or spectropolarimetry is simply differential photometry or spectroscopy. Thus the following standard steps are required:

- *detector effects:* subtraction of detector bias (or dark) and bad pixel correction
- *flat-fielding:* pixel to pixel flat-fielding may not be required for the polarimetric (= differential) image, provided that the $I_\perp$ and $I_\parallel$ signal for a pair of retarder positions (e.g. 0° and 45°) are located at the same position on the detector. Thus it can be important for high precision polarimetry that seeing conditions are stable and that the telescope tracking is good. Nonetheless, flat-fielding may be useful to reduce detector effects. It is hard to predict whether flat-fielding produced better polarization results or not and one has to carry out the reduction of the data in both ways and choose in the end the better result. Of course flat-fielding is required for the intensity spectrum or image, which is obtained at the same time with the polarimetric result.

- *background subtraction*: background subtraction can be critical for faint sources. Scattered moon-light can be highly polarized and the intensity and level of this polarization signal varies during the night.

8.6.1 Calculating the polarization

There exist different possibilities for the polarimetric data reduction.

Photo-polarimetry. For photometrically calibrated single measurements one can deduce the individual Stokes parameters according to:

$$\begin{aligned} Q &= I_0 - I_{90} \\ U &= I_{45} - I_{135} \\ V &= I_l - I_r \\ I &= I_0 + I_{90} \quad \text{or} \quad I_{45} - I_{135} \quad \text{or} \quad I_l - I_r. \end{aligned}$$

For observations based on such measurements it is always possible to deduce normalized Stokes parameters.

For linear polarization there exists an alternative method for the determination of the polarization using three observations with photometrically calibrated polarizers with orientations of 0° , 60° , and 120° :

$$\begin{aligned} I &= \frac{2}{3}(I_0 + I_{60} + I_{90}) \\ Q &= \frac{2}{3}(2I_0 - I_{60} - I_{90}) \\ U &= \frac{2}{\sqrt{3}}(I_{60} - I_{120}). \end{aligned}$$

The Hubble Space Telescope uses this method for imaging polarimetry.

These methods require photometric conditions. If the transmission of the atmosphere or the instrument are variable then it is not clear whether a differential signal is due to the polarization or due to variations in the measuring efficiency.

Double beam. Normalize Stokes parameters can be determined under non-photometric atmospheric conditions using the a double beam or a fast modulation method:

$$\begin{aligned} \frac{Q}{I} &= \frac{I_0 - I_{90}}{I_0 + I_{90}} \\ \frac{U}{I} &= \frac{I_{45} - I_{135}}{I_{45} + I_{135}} \\ \frac{V}{I} &= \frac{I_l - I_r}{I_l + I_r} \end{aligned}$$

This method requires a relative calibration of the efficiency between the two channels or between the two modulation states. If also a flux calibration can be carried out then the intensity I and the polarization flux Q , U and V can be determined. The advantage of this method is that it depends not on photometric efficiency variations (atmospheric and instrument transmission).

Double beam with beam switch. The precision of the measurement of normalized Stokes parameters can be significantly enhanced with a beam switch because the efficiency of each channel is (self)-calibrated in the measurement. For Stokes Q/I measurements there are 2 two channel observations: channels $\parallel$ and $\perp$ and beam switch positions HWP0 and HWP45 providing in total 4 measurements:

$$\begin{aligned} I_{\parallel, \text{HWP0}} &= 0.5(I + Q) \cdot G_{\parallel} \cdot T_{\text{HWP0}} \\ I_{\perp, \text{HWP0}} &= 0.5(I - Q) \cdot G_{\perp} \cdot T_{\text{HWP0}} \\ I_{\parallel, \text{HWP45}} &= 0.5(I - Q) \cdot G_{\parallel} \cdot T_{\text{HWP45}} \\ I_{\perp, \text{HWP45}} &= 0.5(I + Q) \cdot G_{\perp} \cdot T_{\text{HWP45}}, \end{aligned} \tag{8.11}$$

where $I_0 = 0.5(I + Q)$ and $I_{90} = 0.5(I - Q)$, $G_{\parallel}$ and $G_{\perp}$ are the photon efficiencies of the channels $\parallel$ and $\perp$, and T_{HWP0} and T_{HWP45} the atmospheric and instrument transmission for the two HWP configurations. Using a double ratio the G and T factors drop out in the data reduction process. The corresponding formula for the derivation of Q/I is:

$$\frac{Q}{I} = \frac{R - 1}{R + 1} \quad \text{where} \quad R^2 = \frac{(I_{\perp}/I_{\parallel})_{\text{HWP0}}}{(I_{\parallel}/I_{\perp})_{\text{HWP45}}} \tag{8.12}$$

and similar for U/I (HWP 22.5° and 67.5°), or V/I (QWP -45° and $+45^\circ$). As in the previous method a flux calibration can be made in addition if the data were taken under photometric conditions.

Aperture polarimetry. For unresolved sources one can measure the signal $I_{\perp}$ and $I_{\parallel}$ within adequate apertures and just apply the formula given above to the net counts in the apertures.

Imaging polarimetry. For spatially resolved data (extended sources) the reduction is applied to 2-dimensional pixel arrays. It is of importance to achieve a good alignment of the signal in the $I_{\perp}$ and $I_{\parallel}$ beams. This can be difficult, because the Wollaston introduces some differential chromatic aberrations, so that the two images are not exactly identical.

Spectropolarimetry. For spectropolarimetry the formulas given above can be applied to the one-dimensional (extracted) $I_{\perp}$ and $I_{\parallel}$ spectra. Important is that the wavelength calibration is accurate (done separately for the two beams). For Zeeman spectropolarimetry of spectral lines a bad wavelength calibration results in a spurious detection of a magnetic field. Particular care is required for this type of data analysis.

8.6.2 Correcting for polarization offsets

The basic polarimetric data reduction provides the polarization signal as measured by the polarization analysis components in the instrument. In a final step these data must still be corrected for the instrumental effects introduced by the optical components upstream of the polarization analysis.

Instrumental polarization effects. Corrections for the instrumental polarization include:

- polarimetric efficiency of the polarization analysis,

- introduced polarization by the telescope and instrument,
- induced polarization cross talk by the telescope and instrument, for linear polarization this is a rotation of the zero point of polarization position angle.

The instrumental effects can be determined by the calibration measurements with standard polarization stars. All science data must be corrected for these instrumental effects.

Interstellar polarization. For measurements of the linear polarization one should always check for contributions from the interstellar polarization. The interstellar polarization can be up to several % for targets near the galactic plane. For a galactic latitude $|b| > 30^\circ$ the interstellar polarization is less than 1 %, typically even less than 0.2 %.

Corrections in the $Q/I - U/I$ plane. The corrections for instrumental polarization effects but also the contribution of the interstellar polarization are best carried out in the Q/I and U/I plane.

- the polarimetric efficiency corresponds to a multiplication of the Q/I and U/I parameters
- an instrument polarization or an interstellar polarization component are equivalent to a shift of the Q/I , U/I points
- the position angle zero point calibration is equivalent to a rotation in the $Q/I - U/I$ plane.