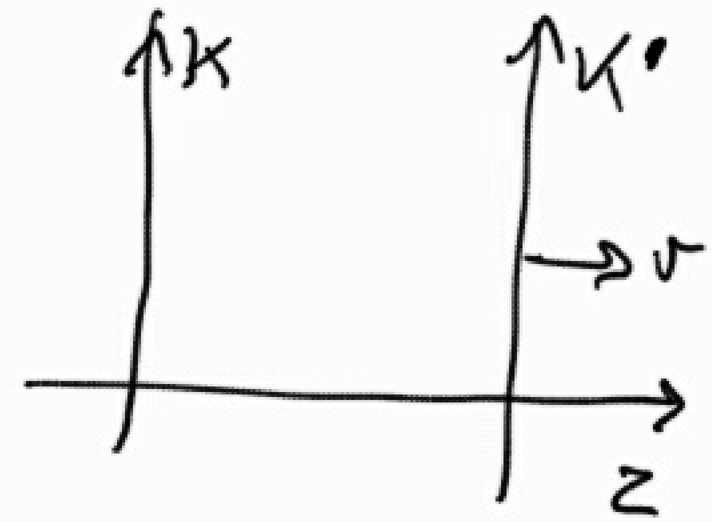
Ex. 1:

We consider two inertial frames in relative motion along the beam axis z



The Lorentz transformation between the 2 frames is

$$\begin{cases} E' = E\cosh y - p_z \sinh y \\ p_z' = p_z \cosh y - E \sinh y \\ p_x' = p_x \\ p_y' = p_y \end{cases}$$

where y is the rapidity : $\cosh y = \gamma = \dfrac{1}{\sqrt{1-v^2}}$

If in the frame K', the particle moves only perpendicula[r]

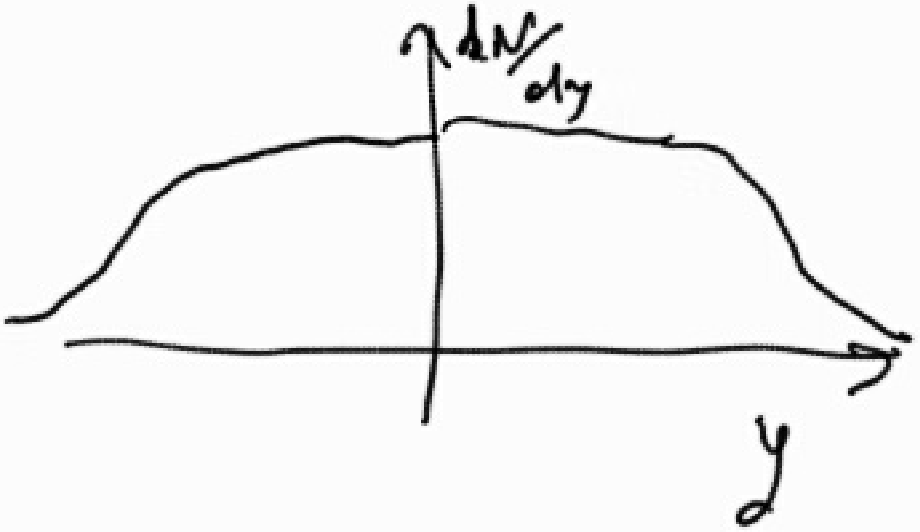
to the beam, show that $\tanh y = \dfrac{p_z}{E}$

and in particular that $\begin{cases} E = m_\perp \cosh y \\ p_z = m_\perp \sinh y \end{cases}$

with $m_\perp = \sqrt{m^2 + p_\perp^2}$ the transverse mass

and $p_\perp = \sqrt{p_x^2 + p_y^2}$ the transverse momentum

The multiplicity distribution $\frac{dN}{dy}$ is boost invariant.



Consider the pseudo-rapidity η, defined through

$p_{\parallel} = p_{\perp} \sinh \eta$

For low $p_{\perp}$ massive particles, with $p_{\perp} \ll m$,

what does the multiplicity spectrum $\frac{dN}{d\eta}$ look like?

Solution :

we have $p_z' = 0$ $\quad p_x'^2 + p_y'^2 = p_x^2 + p_y^2 = p_\perp^2$

$$(E')^2 = (\vec{p}\,')^2 + m^2 = p_\perp^2 + m^2 \qquad \text{so } E' = m_\perp$$

$$p_z' = 0 \Rightarrow p_z \cosh y - E \sinh y = 0 \Rightarrow \tanh y = \frac{p_z}{E}$$

$$E' = m_\perp \rightarrow E \cosh y - p_z \sinh y = m_\perp$$

solving wrt E : $E\left(\cosh y - \dfrac{\sinh^2 y}{\cosh y}\right) = m_\perp \Rightarrow E = m_\perp \cosh y$

solving wrt p_z : $p_z\left(\dfrac{\cosh^2 y}{\sinh y} - \sinh y\right) = m_\perp \Rightarrow p_z = m_\perp \sinh y$

Low $p_\perp$ massive particles have $p_\perp \ll m$.

Since $\sinh y = \frac{p_\parallel}{m_\perp}$ and $\sinh \eta = \frac{p_\parallel}{p_\perp}$

if $p_\perp \ll m$ then $\eta \gg y$

so the particles are pushed away from the central region $\eta \simeq 0$

$\Rightarrow$