The pT distribution of Higgs production at next-to-leading order in α_s

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in collaboration with
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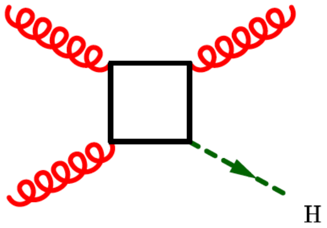
F. Moriello, G. Salvatori, G. Somogyi, F. Tramontano
J. Henn, L. Maestri, V. Smirnov

HP2 20 September 2022

Higgs p_T distribution at LHC

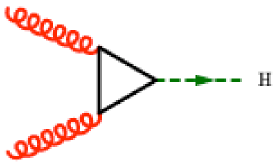


- high-p_T tail of the Higgs p_T distribution is sensitive to the structure of the loop-mediated Higgs-gluon coupling New Physics particles circulating in the loop would modify it

- QCD NLO corrections to the top- and *b*-quark loop contributions to the Higgs p_T distribution, in the on-shell and MSbar mass renormalisation schemes

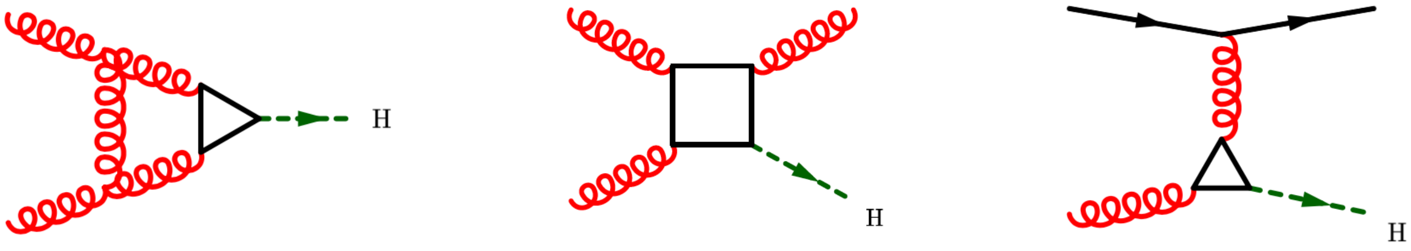
Higgs production at LHC

- In proton collisions, the Higgs boson is produced mostly via gluon fusion
  The gluons do not couple directly to the Higgs boson
  For matter, the coupling is mediated by a heavy quark loop
  The largest contribution comes from the top-quark loop
  The production mode is (roughly) proportional to the top Yukawa coupling y_t^2



- QCD NLO corrections (for any heavy quark mass)

Djouadi Graudenz Spira Zerwas 1991-1995



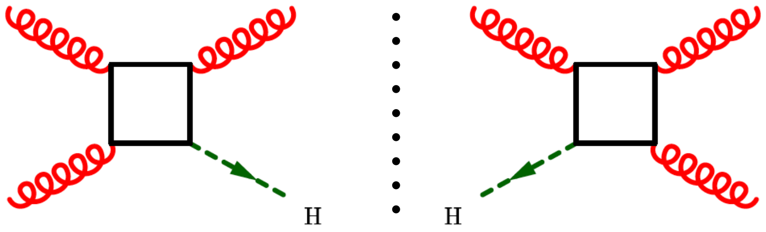
  - QCD NLO corrections are about 100% larger than leading order

- QCD NNLO corrections are known for the top-quark loop only

Czakon Harlander Klappert Niggetiedt 2021

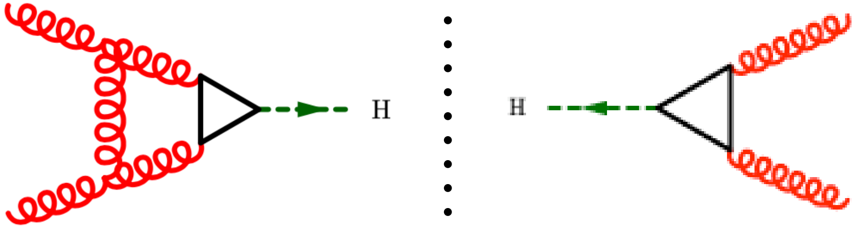
QCD NLO corrections

- real radiation



K. Ellis Hinchliffe Soldate van der Bij 1988

- virtual corrections

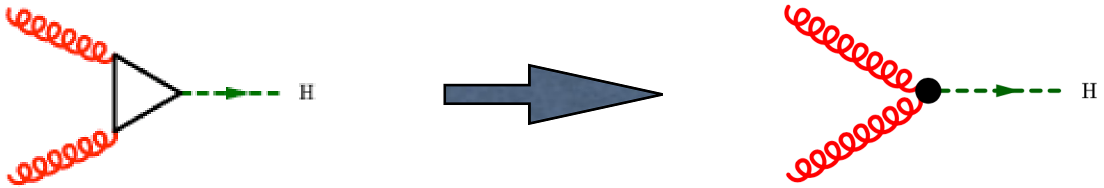


Djouadi Graudenz Spira Zerwas 1993
Anastasiou Beerli Bucherer Daleo Kunszt 2006
Aglietti Bonciani Degrassi Vicini 2006

} in terms of Harmonic Polylogarithms (HPL)

QCD NLO corrections

- $m_H << 2m_t$



all amplitudes are reduced by one loop

σ^{LO}_{EFT}	15.05 pb	σ^{NLO}_{EFT}	34.66 pb
$R_{LO}\,\sigma^{LO}_{EFT}$	16.00 pb	$R_{LO}\,\sigma^{NLO}_{EFT}$	36.84 pb
$\sigma^{LO}_{ex;t}$	16.00 pb	$\sigma^{NLO}_{ex;t}$	36.60 pb
$\sigma^{LO}_{ex;t+b}$	14.94 pb	$\sigma^{NLO}_{ex;t+b}$	34.96 pb
$\sigma^{LO}_{ex;t+b+c}$	14.83 pb	$\sigma^{NLO}_{ex;t+b+c}$	34.77 pb

	$\frac{\sigma_{t+b}}{\sigma_t} - 1$
LO $O(\alpha_s^2)$	- 6.6 %
NLO $O(\alpha_s^2)$ + $O(\alpha_s^3)$	- 4.5 %
NLO $O(\alpha_s^3)$	- 2.8 %

Anastasiou Duhr Dulat Furlan Gehrmann Herzog Lazopoulos Mistlberger 2016

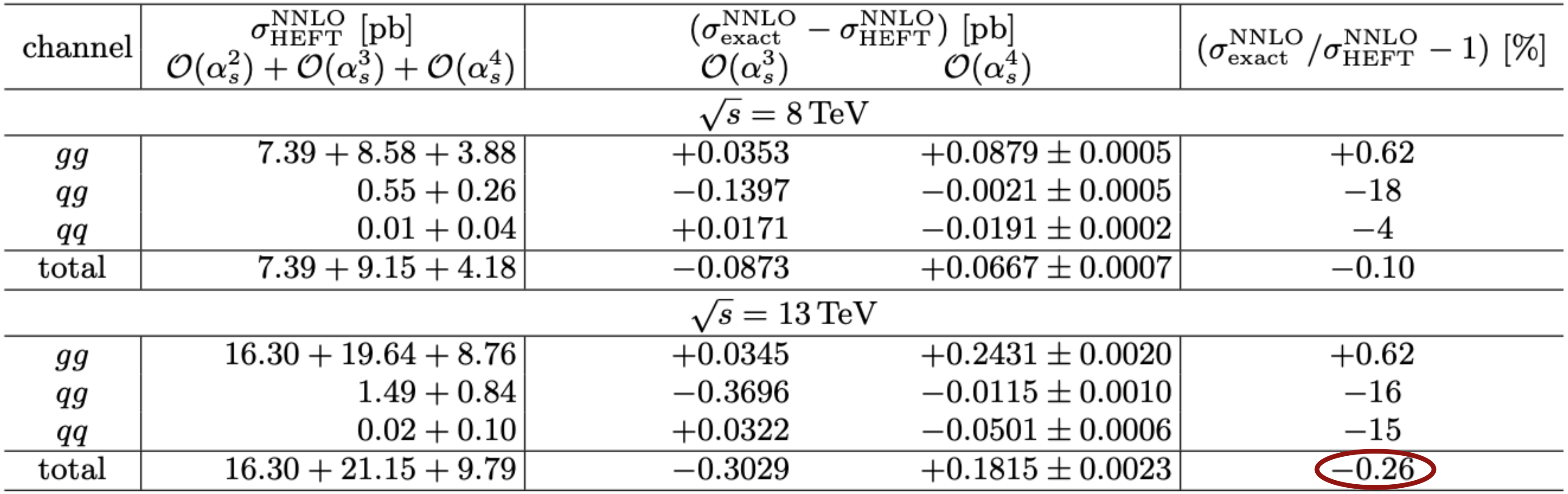
- $R_{LO} = \frac{\sigma^{LO}_{ex:t}}{\sigma^{LO}_{EFT}} = 1.063$

  rescaled HEFT (rHEFT) does a good job (< 1%) in approximating the exact (only top) NLO σ but misses the *t*-*b* interference

QCD NNLO corrections

- Top-quark mass corrections are known at NNLO

Czakon Harlander Klappert Niggetiedt 2021



channel	$\sigma^{\text{NNLO}}_{\text{HEFT}}$ [pb] $\mathcal{O}(\alpha_s^2)+\mathcal{O}(\alpha_s^3)+\mathcal{O}(\alpha_s^4)$	$(\sigma^{\text{NNLO}}_{\text{exact}}-\sigma^{\text{NNLO}}_{\text{HEFT}})$ [pb] $\mathcal{O}(\alpha_s^3)$	 $\mathcal{O}(\alpha_s^4)$	$(\sigma^{\text{NNLO}}_{\text{exact}}/\sigma^{\text{NNLO}}_{\text{HEFT}}-1)$ [%]
		$\sqrt{s}=8\,\text{TeV}$		
gg	$7.39+8.58+3.88$	$+0.0353$	$+0.0879\pm0.0005$	$+0.62$
qg	$0.55+0.26$	-0.1397	-0.0021 ± 0.0005	-18
qq	$0.01+0.04$	$+0.0171$	-0.0191 ± 0.0002	-4
total	$7.39+9.15+4.18$	-0.0873	$+0.0667\pm0.0007$	-0.10
		$\sqrt{s}=13\,\text{TeV}$		
gg	$16.30+19.64+8.76$	$+0.0345$	$+0.2431\pm0.0020$	$+0.62$
qg	$1.49+0.84$	-0.3696	-0.0115 ± 0.0010	-16
qq	$0.02+0.10$	$+0.0322$	-0.0501 ± 0.0006	-15
total	$16.30+21.15+9.79$	-0.3029	$+0.1815\pm0.0023$	-0.26

- HEFT not so good for *qg* and *qq* channels
- for top-quark mass, used $m_t^2/m_H^2 = 23/12$ (on-shell scheme)

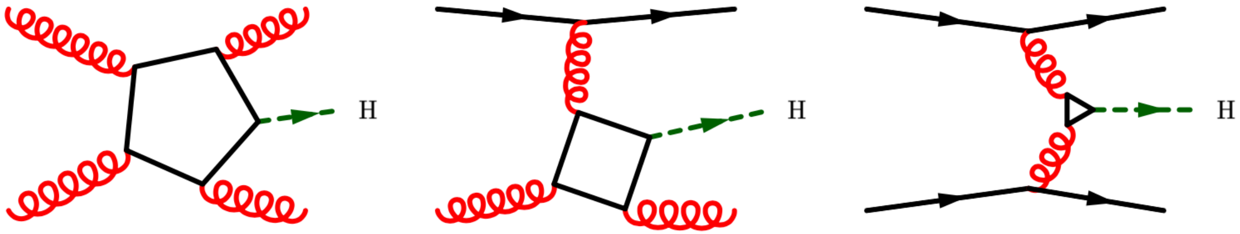
The main obstacle when calculating the total cross section with full top-mass dependence are the two-loop single-emission amplitudes.

Czakon Harlander Klappert Niggetiedt 2021

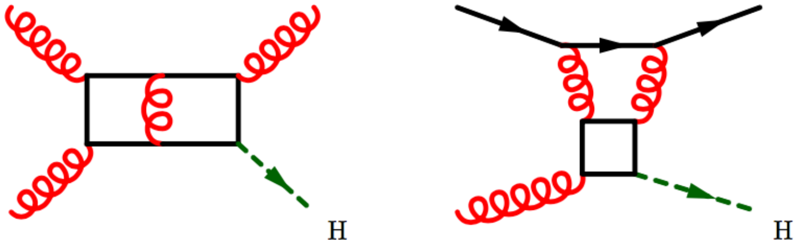
QCD NNLO corrections

- Higgs + 4-parton amplitudes at one loop

VDD Kilgore Oleari Schmidt Zeppenfeld 2001
Budge Campbell De Laurentis K. Ellis Seth 2020
OpenLoops



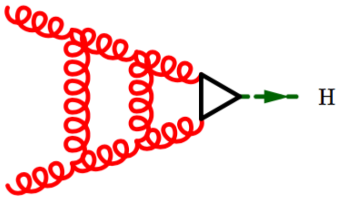
- Higgs + 3-parton amplitudes at two loops



top loop: Jones Kerner Luisoni 2018
Czakon Harlander Klappert Niggetiedt 2021

Bonciani VDD Frellesvig Moriello Hidding
Hirschi Salvatori Somogyi Tramontano 2022

- gg→Higgs amplitudes at three loops



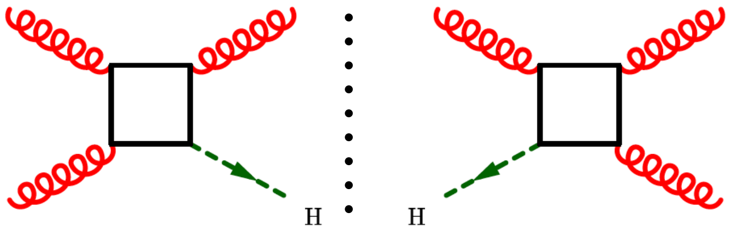
one scale: one & two top loops
one top loop + light-quark loop

Czakon Niggetiedt 2020
Harlander Prausa Usovitsch 2019

two scales: one top loop + *b*-quark loop

Higgs p_T distribution at LHC

- leading order



K. Ellis Hinchliffe Soldate van der Bij 1988

- high-p_T tail of the Higgs p_T distribution is sensitive to the structure of the loop-mediated Higgs-gluon coupling
New Physics particles circulating in the loop would modify it

- in high-p_T regime, clean signature of decay products ($H \to b\, b$)

- QCD NLO corrections
    - for the top-quark, with on-shell scheme

      Jones Kerner Luisoni 2018
      Chen Huss Jones Kerner Lang Lindert Zhang 2021

    - for the top-quark, with on-shell and MSbar schemes
      for top- and *b*-quarks (for any heavy quark mass), with MSbar scheme

      Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022

- HEFT $m_H << 2m_t$ and $p_T << m_t$

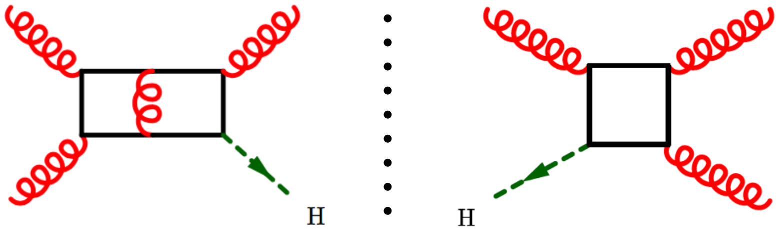
  Baur Glover 1990

  QCD corrections are known at NNLO in HEFT, and yield a 15% increase wrt NLO

  Boughezal Caola Melnikov Petriello Schulze 2015
  Boughezal Focke Giele Liu Petriello 2015
  Chen Cruz-Martinez Gehrmann Glover Jaquier 2016

Higgs p_T distribution at NLO

- virtual corrections



top-quark loop

Jones Kerner Luisoni 2018
Czakon Harlander Klappert Niggetiedt 2021

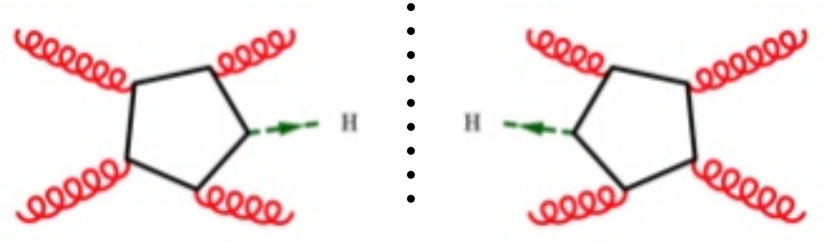
any heavy quark in the loop

Bonciani VDD Frellesvig Henn Moriello V. Smirnov 2016
all above + Hidding Maestri Salvatori 2019

Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022

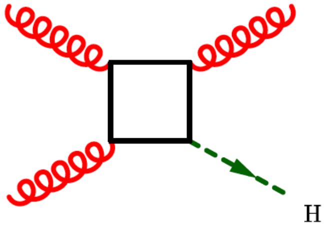
multi-scale problem with complicated analytic structure
elliptic iterated integrals appear

- real corrections



VDD Kilgore Oleari Schmidt Zeppenfeld 2001
Budge Campbell De Laurentis K. Ellis Seth 2020

- one-loop amplitudes for Higgs + 3-partons



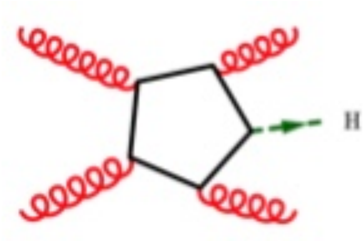
leading order: up to $O(\varepsilon^2)$

analytic: up to $O(\varepsilon^0)$ K. Ellis Hinchliffe Soldate van der Bij 1988

numeric: up to $O(\varepsilon^2)$

(numeric) derivative for mass renormalisation

- one-loop amplitudes for Higgs + 4-partons



NLO real corrections: up to $O(\varepsilon^0)$

analytic: unitarity-cut methods (taken from MCFM-9.1)

Budge Campbell De Laurentis K. Ellis Seth 2020

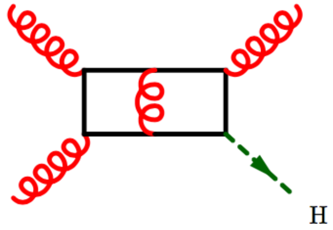
numeric: GoSam & MG5_aMC

run time

analytic: few ms/pt

numeric: O(100) times slower than analytic

two-loop amplitudes for Higgs + 3-partons



NLO virtual corrections

amplitude → form factors → scalar integrals → Master Integrals

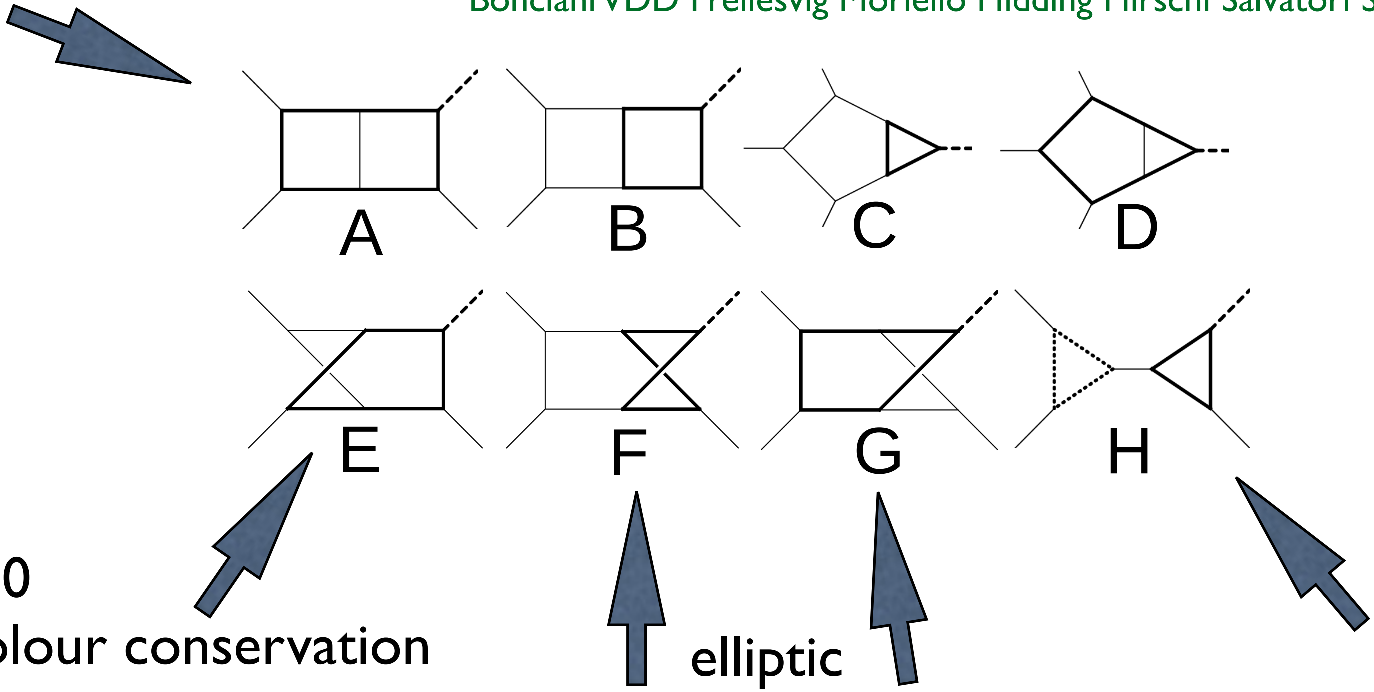
IBP

FIRE-KIRA

run time: 5 — 60 min/pt

4 scales, s, t, m_H, m_t → 3 external parameters
7 seven-propagator integral families

Bonciani VDD Frellesvig Henn Moriello Smirnov 2016 (A, B, C, D)
Bonciani VDD Frellesvig Henn Hidding Maestri Moriello Salvatori Smirnov 2019 (F)
Frellesvig Hidding Maestri Moriello Salvatori 2019 (G)
Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022 (H)



elliptic

= 0
colour conservation

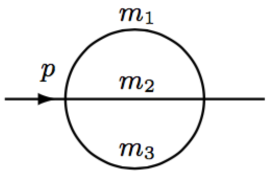
elliptic

two masses

\# MIs
A: 72
B: 5
C: 45
D: 17
F: 73
G: 84
H: 12

Elliptic iterated integrals

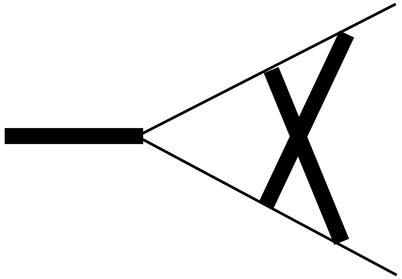
- 2-loop sunrise graph



Sabry 1962: …;Broadhurst 1989; …; Bloch Vanhove 2013; …
Brödel Duhr Dulat Penante Tancredi 2017-2019

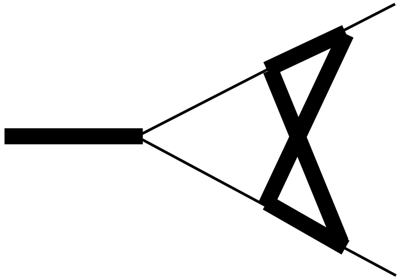
- 2-loop 3-pt functions

electroweak form factor



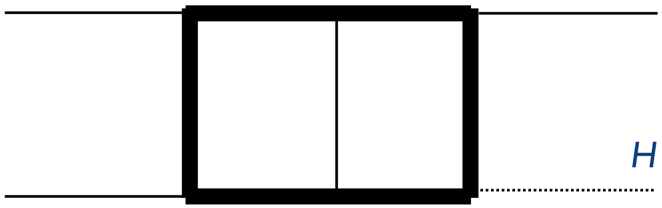
Aglietti Bonciani Grassi Remiddi 2007

t-*t*bar



von Manteuffel Tancredi 2017

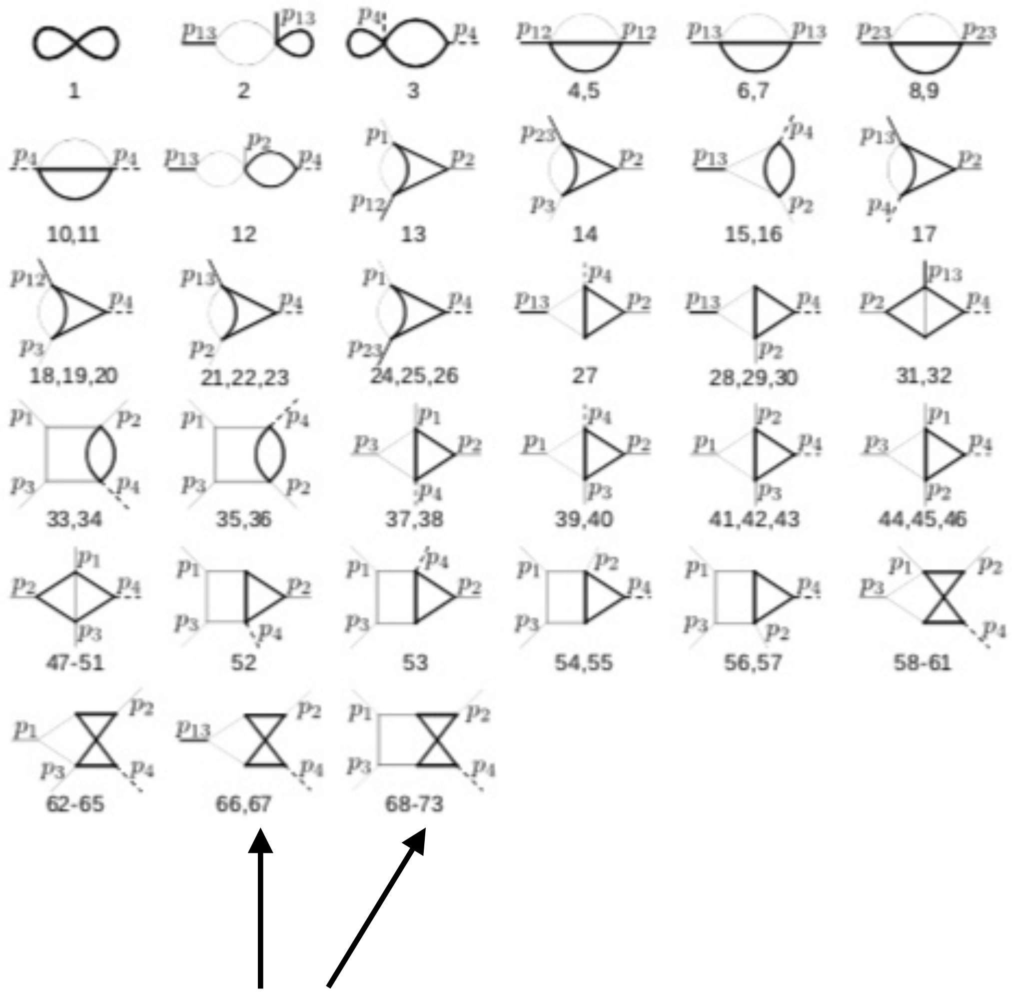
- 2-loop 4-pt function for Higgs + 1 jet



Bonciani VDD Frellesvig Henn Moriello Smirnov 2016

first instance of elliptic iterated integrals
in a genuine 4-pt topology

Family F: 73 MIs (65 in the polylogarithmic sector, 8 in the elliptic sector)
alphabet: 69 independent letters, with 12 independent square roots



elliptic

Differential Equations

- Differential Equation method to solve the MIs

  $\partial_i f(x_n;\varepsilon) = A_i(x_n;\varepsilon) f(x_n;\varepsilon)$

  f: N-vector of MIs, A_i: NxN matrix, *i*=1,…,n external parameters

  but in some cases ε-independent form

  $\partial_i f(x_n;\varepsilon) = \varepsilon A_i(x_n) f(x_n;\varepsilon)$ Henn 2013

  solution in terms of iterated integrals

- mass values are floating →
  DEs solved with 3 (top) or 4 (top and *b*) external parameters

DEs: Series Expansion Method

- Take two points (a_1, …, a_n) and (b_1,…, b_n) in the *n*-dim parameter space, and parametrise the contour $\gamma(t)$ that connects the two points

  $$\gamma(t) : t \to \{x_1(t), \ldots, x_n(t)\} \qquad \vec{x}(0) = \vec{a}\,, \qquad \vec{x}(1) = \vec{b}$$

  and write the differential equation with respect to *t*.
  Then find a solution about a point τ by series expanding the coefficient matrix *A* and then iteratively integrating it.
  The procedure works for both polylogarithmic and elliptic sectors

  Moriello 2019

- numerical solution of DEs through DiffExp:
  Mathematica implementation of Moriello's series expansion method

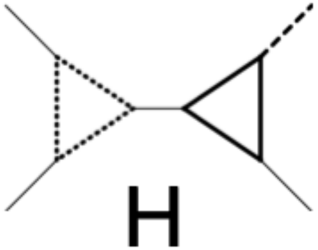
  Hidding 2021

- checked with AMFlow Liu Ma Wang 2018

two-loop amplitudes for Higgs + 3-partons: Renormalisation

Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022

- coupling constant: 5-flavour running in MSbar
- renormalisation:
  - top Yukawa coupling and top mass in OS scheme (massless *b*)
  - top Yukawa coupling and top mass in MSbar scheme (massless *b*)
  - top Yukawa coupling and top and *b* masses in MSbar scheme



massive *b* in Higgs-*b* loop
massless *b* in *b* loop

alternative:
massive *b* everywhere,
but requires 4-flavour running and including *gg* → *Hbb*

two-loop amplitudes for Higgs + 3-partons: validation checks

- IR poles

$$\mathcal{M}^{(2)}_{ij,IR} \propto I^{(1)}_{ij}(\{p\},\epsilon)\mathcal{M}^{(1)}_{ij}$$

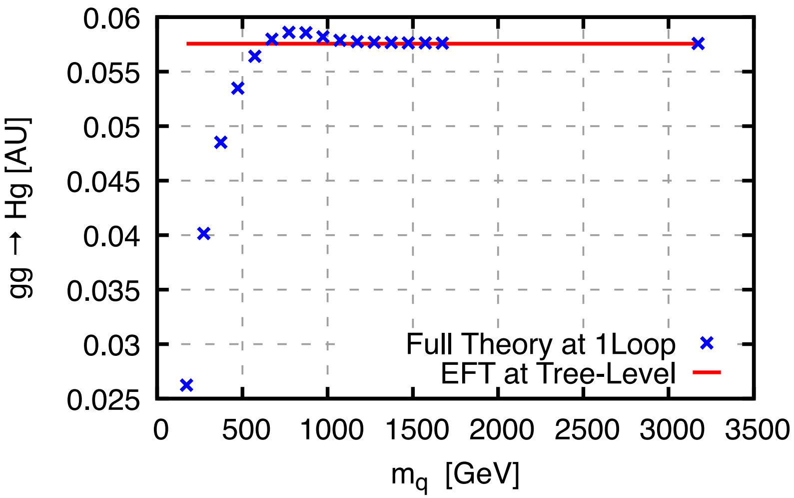
with insertion operators

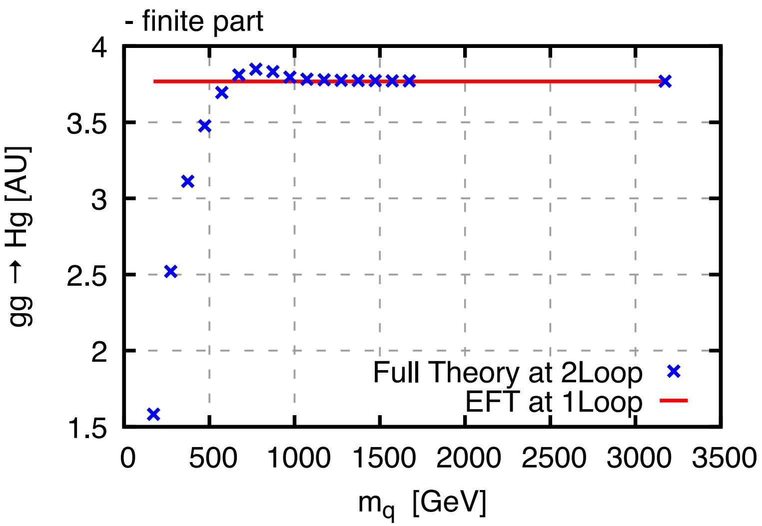
$$I^{(1)}_{gg}(\{p\},\epsilon) = -\frac{\alpha_S}{\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\frac{N_c}{\epsilon^2}+\frac{\beta_0}{\epsilon}\right)\left[\left(\frac{\mu^2}{-s}\right)^\epsilon + \left(\frac{\mu^2}{-t}\right)^\epsilon + \left(\frac{\mu^2}{-u}\right)^\epsilon\right]$$

$$I^{(1)}_{q\bar{q}}(\{p\},\epsilon) = -\frac{\alpha_S}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left\{-\left(\frac{N_c}{\epsilon^2}+\frac{3N_c}{4\epsilon}+\frac{\beta_0}{2\epsilon}\right)\left[\left(\frac{\mu^2}{-t}\right)^\epsilon + \left(\frac{\mu^2}{-u}\right)^\epsilon\right] + \frac{1}{N_c}\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right)\left(\frac{\mu^2}{-s}\right)^\epsilon\right\}$$

- agreement with HEFT limit

$$\mathcal{M} = \mathcal{M}_{HEFT} + \mathcal{O}\left(\frac{1}{M_t}\right)$$

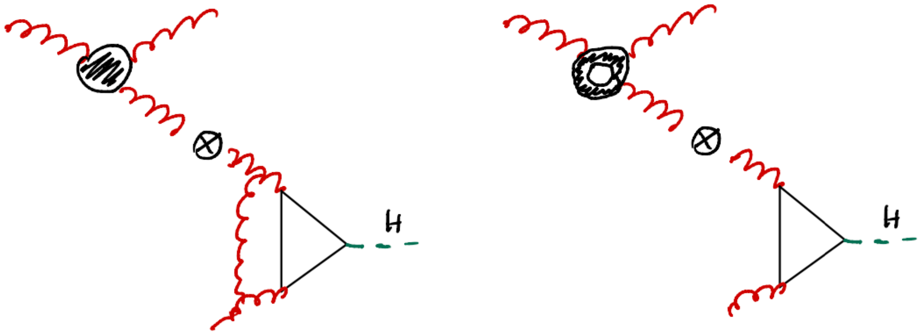




two-loop amplitudes for Higgs + 3-partons: validation checks

- soft and collinear limits

  (these are checks on real-virtual parts of NNLO cross section, however they are feasible on our two-loop amplitudes)



Aglietti Bonciani Degrassi Vicini 2006

one-loop 2-parton splitting functions

Bern Dixon Dunbar Kosower 1994
Bern Kilgore Schmidt VDD 1998-99
Kosower Uwer 1999

one-loop 1-soft-gluon factor

Bern Kilgore Schmidt VDD 1998-99
Catani Grazzini 2000

- checked also “two-loop photon correction”

Higgs p_T distribution at **NLO**: checks with previous results

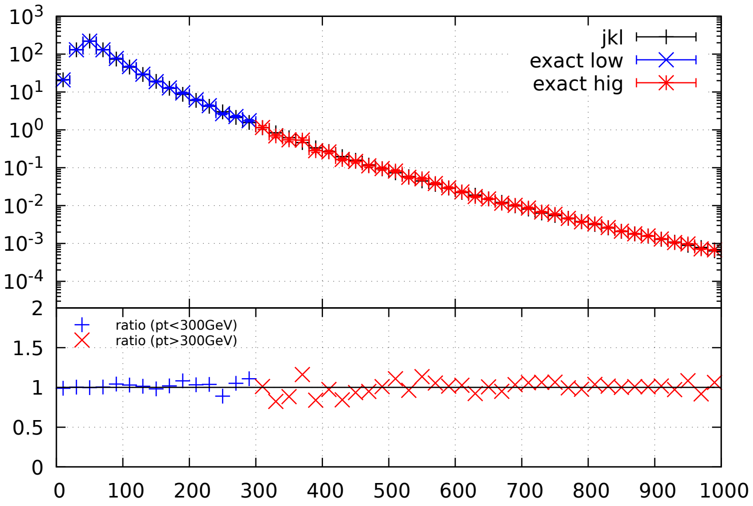
- inclusive p_T distribution ($p_{T,j} > 30$ GeV) with OS mass renormalisation

our result

$$\sigma_{NLO} = 14.37 \pm 0.05\,\text{pb}$$

Chen Huss Jones Kerner Lang Lindert Zhang 2021
(Jones Kerner Luisoni 2018-2021)

$$\sigma_{NLO} = 14.15 \pm 0.07\,\text{pb}$$



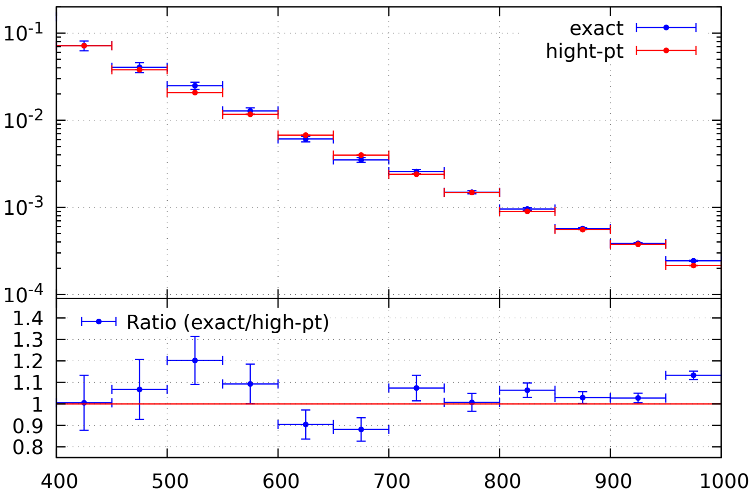
- high p_T tail of distribution

checked with approximate high-p_T distribution

Lindert Melnikov Kudashkin Wever 2018

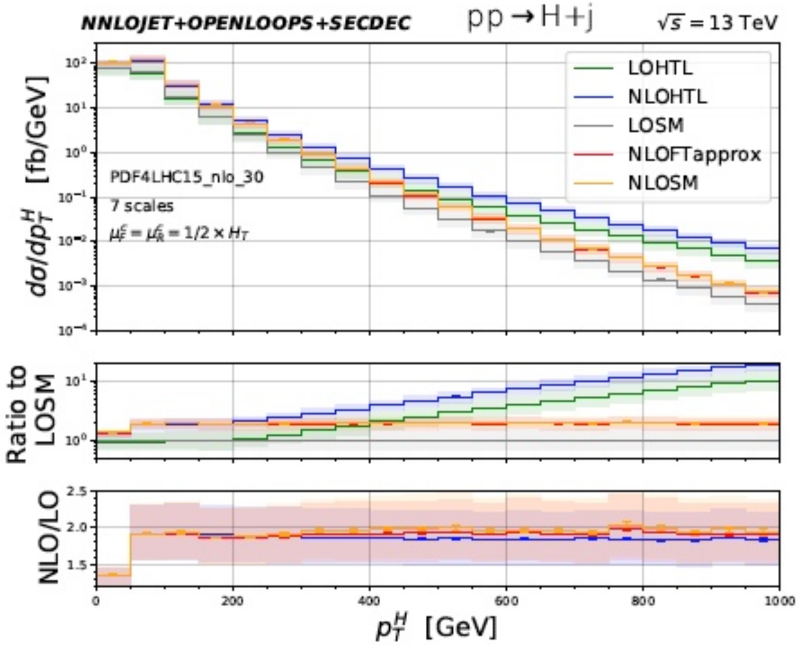
based on approximate high-p_T two-loop amplitudes

Melnikov Kudashkin Wever 2018



Higgs p_T distribution at LHC

- QCD NLO corrections for the top-quark (on-shell mass renormalisation)



Jones Kerner Luisoni 2018
Chen Huss Jones Kerner Lang Lindert Zhang 2021

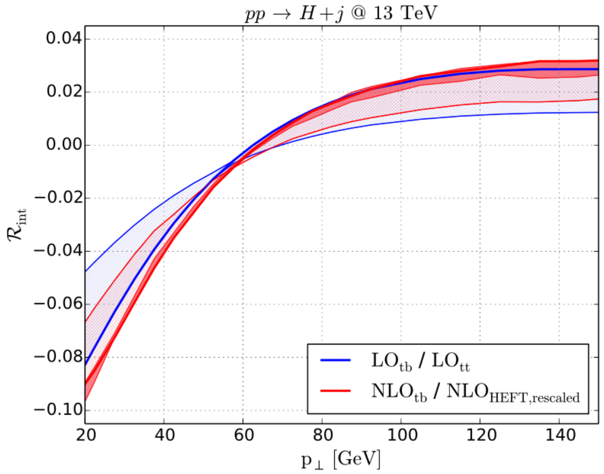
$\frac{d\sigma}{dp_T^2} \propto \frac{1}{p_T^2}$ in HEFT NLO corrections

$\frac{d\sigma}{dp_T^2} \propto \frac{1}{(p_T^2)^2}$ in top NLO corrections

NLO/LO in HEFT and top loop agree to O(10%)

- QCD NLO corrections to top-*b* interference, using top-quark loop in HEFT and *b*-quark loop in small m_b limit

Lindert Melnikov Tancredi Wever 2017



Higgs p_T distribution at NLO

- p_T distribution computed with
  CoLorFulNLO — Somogyi 2009
  dual subtraction — Prisco Tramontano 2020

- evaluated on:
  3x10⁴ pt for OS top (1.4x10⁴ pt on basic grid, 1.6x10⁴ pt on biased grid)
  9x10⁴ pt for MSbar top
  1.8x10⁵ pt for MSbar top and *b*

- set-up

$$\sqrt{s} = 13\,\mathrm{TeV}$$

$$m_H = 125.25\,\mathrm{GeV}$$

$$m_t^{\mathrm{OS}} = 172.5\,\mathrm{GeV}$$

$$m_t^{\overline{\mathrm{MS}}}(m_t^{\overline{\mathrm{MS}}}) = 163.4\,\mathrm{GeV}$$

$$m_b^{\overline{\mathrm{MS}}}(m_b^{\overline{\mathrm{MS}}}) = 4.18\,\mathrm{GeV}$$

$$G_F = 1.16639 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$$

NNPDF40_nlo_as_01180

$$p_{T,j_1} > 20\,\mathrm{GeV}$$

anti-kt algorithm with $R = 0.4$

7-pt scale variation about:

$$\mu_R^0 = \mu_F^0 = \frac{H_T}{2} = \frac{1}{2}\left(\sqrt{m_H^2 + p_T^2} + \sum_i |p_{T,i}|\right)$$

inclusive Higgs p_T distribution

- QCD NLO corrections Bonciani VDD Frellesvig Moriello Hidding Hirschi Salvatori Somogyi Tramontano 2022

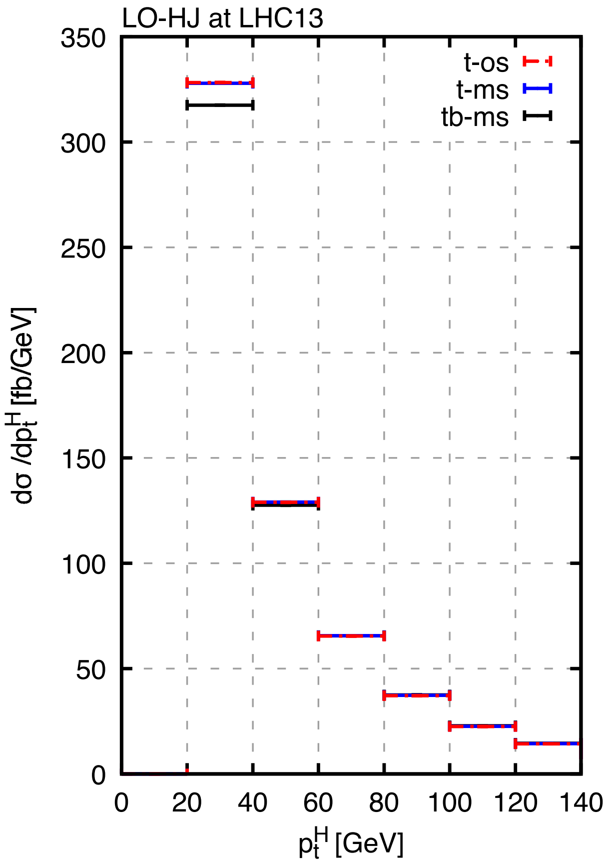
  for the top-quark, with on-shell and MSbar schemes
  for top- and *b*-quarks with MSbar scheme

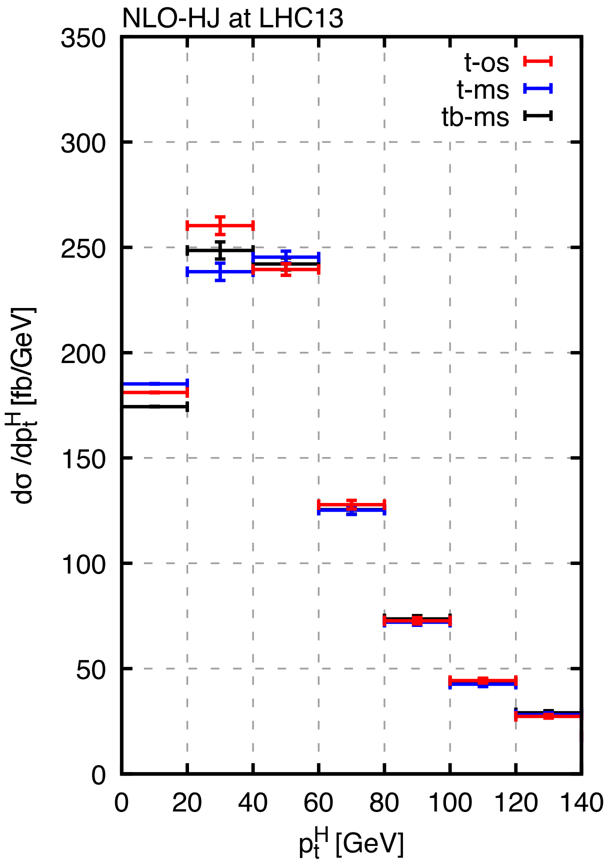
renormalisation of internal masses	σ_{LO} [pb]	σ_{NLO} [pb]
top+bottom–($\overline{\mathrm{MS}}$)	$12.318^{+4.711}_{-3.117}$	$19.89(8)^{+2.84}_{-3.19}$
top–($\overline{\mathrm{MS}}$)	$12.538^{+4.822}_{-3.183}$	$19.90(8)^{+2.66}_{-2.85}$
top–(OS)	$12.551^{+4.933}_{-3.244}$	$20.22(8)^{+3.06}_{-3.09}$

- from LO to NLO large *k* factor and reduction of scale uncertainty
- top-*b* interference is a negative correction at $O(\alpha_s^3)$ but positive at $O(\alpha_s^4)$
- effect of top mass renormalisation utterly negligible at LO
  but 15 times bigger at NLO

$$\frac{\sigma_{t(\mathrm{OS})}}{\sigma_{t(\overline{\mathrm{MS}})}} - 1 = \begin{cases} 0.1\% \text{ at LO} \\ 1.6\% \text{ at NLO} \end{cases}$$

Higgs p_T distribution at low-intermediate p_T





20-40 GeV bin

260^{+16}_{-83} fb/GeV

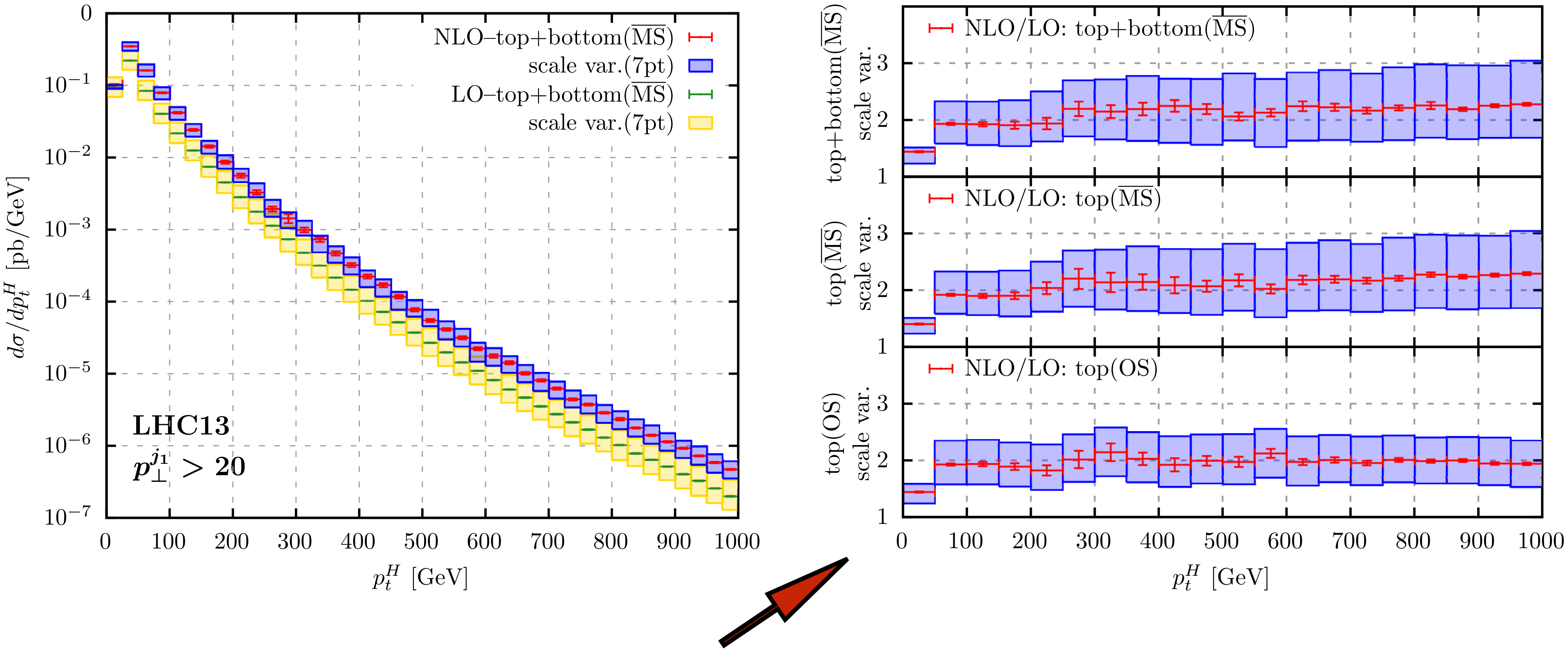
249^{+21}_{-65} fb/GeV

238^{+27}_{-98} fb/GeV

- at LO no events below 20 GeV since $p_{T,j} > 20$ GeV
- at LO no appreciable difference between t(OS) and t(MSbar)
- at NLO sizeable shape distortion in the lowest bins
- at NLO agreement (not shown) between exact and rHEFT in the low-middle p_T range

  HEFT $m_H << 2m_t$ and $m_b << p_T << m_t$
- scale uncertainty bands (not shown) are much larger than differences

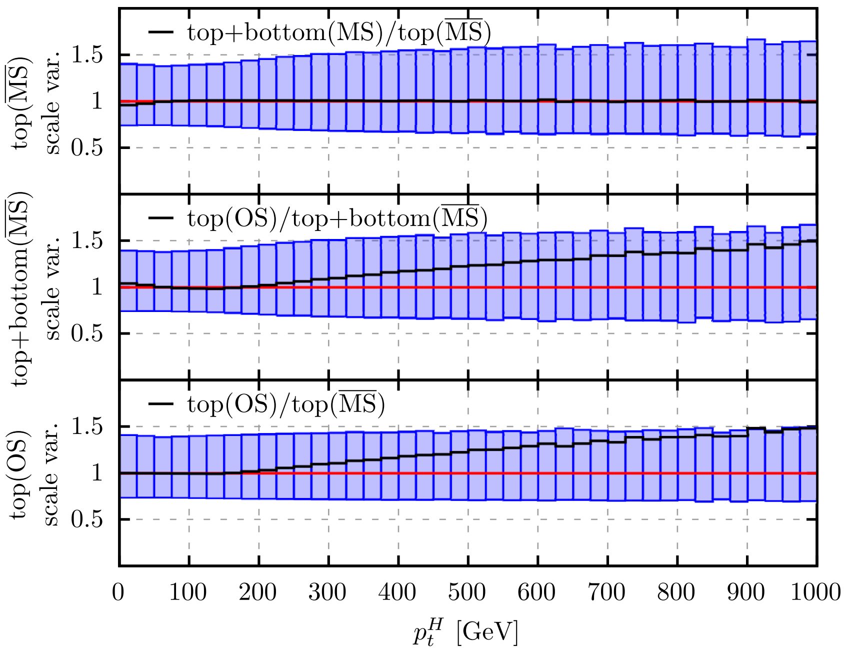
Higgs p_T distribution at LHC



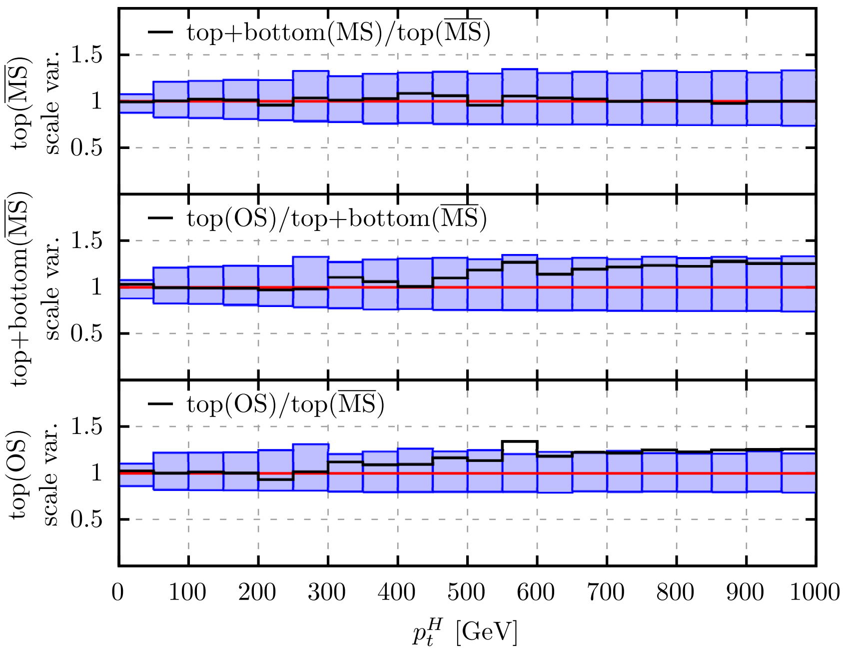
- scale uncertainty bands = ratio of bands at NLO over central value at LO
- *k* factor almost always larger than 2 for MSbar, and about 2 for OS

Ratios of **Higgs** p_T distributions

LO



NLO



- from LO to NLO, reduction of scale uncertainty and of mass renormalisation scheme dependence

- except in the lowest bins, no appreciable difference between *t*+*b*(MSbar) and *t*(MSbar) The *b* quark, and thus top-*b* interference, is negligible, except at low end of p_T range

- p_T distribution for *t*(MSbar) falls off faster than same for *t*(OS) as p_T increases because μ_R increases with p_T and so $m_t^{\overline{\rm MS}}(\mu_R)$ decreases

- mass renormalisation scheme difference between *t*(MSbar) and *t*(OS) is same size as scale uncertainty at high end of p_T range, both at LO and NLO

Conclusions

- we computed the Higgs p_T distribution at NLO in QCD
  including for the first time top and b quarks and the MSbar mass scheme
- computation has excellent numerical stability
- b quark, and thus top-b interference, is negligible,
  except at low end of p_T range, where it affects the shape of the distribution
- in the intermediate to high p_T range, use of top quark only is warranted,
  but sizeable dependence on mass renormalisation scheme
- p_T distribution can be improved:
  mixed QCD-EW corrections (we already have $gg \to Hg$),
  resummation,
  top-charm interference, …