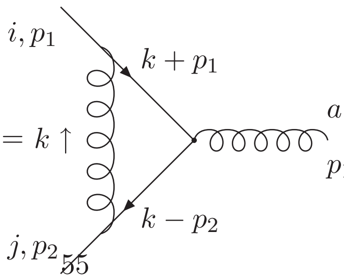
Introduction to QCD

FS 10, Series 5

Due date: 31.03.2010, 1 pm

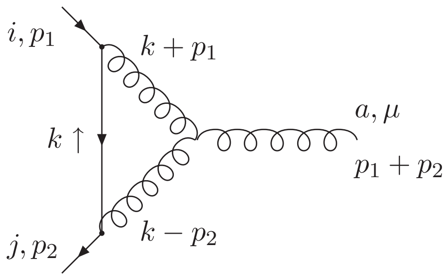
Exercise 1 In order to renormalise the $q\bar{q}g$ vertex in QCD we simply write down all the QCD corrections (to the order to which we want to renormalise the theory) to this vertex. Subsequently we absorb all the ultraviolet poles into the Z coefficients. Convince yourself that at the next to leading order the contribution is given by the following two diagrams.

$\delta V_{q\bar{q}g} =$



$= V_1$

$+$



$= V_2$

It should be noted that in general here the external particles will not be onshell and that the fermions can be massive.

a) Prove the identities

$$\begin{aligned} t^b t^a t^b &= (C_F - \frac{N_c}{2}) t^a \\ t^b t^c f^{abc} &= i\frac{N_c}{2} t^a \end{aligned} \tag{1}$$

where the t^a are generators in the fundemental representation of SU(N).

b) Use the QCD Feynman rules (omitting external spinors and polarisation vectors) in Feynman gauge to show that the above diagrams may be expressed as

$$\begin{aligned} V_1 &= -g^3 \mu^{3\epsilon} (t^b t^a t^b)_{ji} \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\nu (\not{k} - \not{p}_2 + m) \gamma^\mu (\not{k} + \not{p}_1 + m) \gamma_\nu}{[(k-p_2)^2 - m^2][k^2][(k+p_1)^2 - m^2]} \\ V_2 &= i g^3 \mu^{3\epsilon} (t^b t^c)_{ji} f^{abc} \\ & \int \frac{d^d k}{(2\pi)^d} \frac{\gamma^\nu (\not{k} - m) \gamma^\delta [g^\mu_\nu (k - p_1 - 2p_2)_\delta + g_{\nu\delta}(p_2 - p_1 - 2k)^\mu + g^\mu_\delta (k + 2p_1 + p_2)_\nu]}{[k^2 - m^2](k-p_2)^2 (k+p_1)^2}. \end{aligned} \tag{2}$$

c) Use power counting to deduce that only terms proportional to $O(k^2)$ in the numerator are necessary in order to isolate the UV singularities. Further more one can set the masses to zero, this does not change the behaviour of the Integrals when $k \to \infty$ and even more radical set the incoming momenta to zero. However this introduces further IR singularities which will come in through the Feynman parameter integrals. We must some sort of IR cutoff to omit these.

Hence show that

$$\begin{aligned} V_1^{UV} &= -g^3\mu^{3\epsilon}t^a_{ji}(C_F - \frac{N_c}{2})\int \frac{d^dk}{(2\pi)^d}\frac{(2-d)[2k^\mu \not{k} - k^2\gamma^\mu]}{(k^2)^3} \\ V_2^{UV} &= -g^3\mu^{3\epsilon}t^a_{ji}\frac{N_c}{2}\int \frac{d^dk}{(2\pi)^d}\frac{2[\gamma^\mu k^2 - (2-d)k^\mu \not{k}]}{(k^2)^3} \end{aligned} \tag{3}$$

must contain the correct UV poles.

d) Justify or proof that the following substitution can be used in V_1^{UV} and V_2^{UV}.

$$\not{k}k^\mu \to \frac{k^2\gamma^\mu}{d} \tag{4}$$

Use this to show that

$$\begin{aligned} V_1^{UV} &= (-igt^a_{ji}\gamma^\mu)\frac{\alpha_s}{4\pi}\Gamma(\epsilon)(4\pi)^\epsilon\left(C_F - \frac{N_c}{2}\right) + \textit{finite} \\ V_2^{UV} &= (-igt^a_{ji}\gamma^\mu)\frac{\alpha_s}{4\pi}\Gamma(\epsilon)(4\pi)^\epsilon\left(\frac{3N_c}{2}\right) + \textit{finite} \end{aligned} \tag{5}$$

and thus

$$V^{UV} = (-igt^a_{ji}\gamma^\mu)\frac{\alpha_s}{4\pi}\Gamma(\epsilon)(4\pi)^\epsilon\,(C_F + N_c) + \textit{finite}.$$

e) Use the result for V^{UV} in order to renormalise the $q\bar{q}g$ vertex. Do this by letting

$$g\bar{\psi}\not{A}\psi \to g\bar{\psi}\not{A}\psi Z_{1F}.$$

Why can we identify $Z_{1F} = Z_g Z_2 Z_3^{1/2} = 1 - \frac{\alpha_s}{4\pi}(C_F + N_c)\Gamma(\epsilon)(4\pi)^\epsilon$?
Given

$$\begin{aligned} Z_2 &= 1 - \frac{\alpha_s}{4\pi}C_F\Gamma(\epsilon)(4\pi)^\epsilon \\ Z_3 &= 1 - \frac{\alpha_s}{4\pi}\left(\frac{2n_f}{3} - \frac{5N_c}{3}\right)\Gamma(\epsilon)(4\pi)^\epsilon \end{aligned} \tag{6}$$

prove that $Z_g = 1 - \frac{\alpha_s}{4\pi}\left(\frac{11N_c}{6} - \frac{n_f}{3}\right)\Gamma(\epsilon)(4\pi)^\epsilon$.