Magnetization of a two-dimensional electron gas and the role of one-dimensional edge currents

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The influence of the sample edge for the de Haas–van Alphen oscillations in the magnetization of a two-dimensional electron gas is derived using a single-particle approach. The relation of the concept of one-dimensional edge channels, taken from the context of the quantum Hall effect, to the oscillatory magnetization is investigated. The possibility of a local measurement of the de Haas–van Alphen effect in a two-dimensional electron gas is discussed. [S0163-1829(99)09511-9]

The magnetization of a three-dimensional (3D) electron gas due to the orbital motion of the electrons is a quantum statistical problem which has been solved by Landau in a pioneering paper published in 1930.[1] In that paper the quantum-mechanical energy levels of the electron in a homogeneous magnetic field are calculated. Today these are referred to as Landau levels. It becomes clear that the magnetization, which turns out to be zero for classical electrons,[2] appears due to the presence of the sample boundary. Although Landau considered only the case of high temperatures ($\hbar\omega_c \ll kT$, with the cyclotron frequency ω_c and the temperature T), he was already aware of the oscillatory behavior of the magnetization, which was experimentally discovered in the same year in bismuth at 14.2 K by de Haas and van Alphen.[3]

The magnetization of two-dimensional electron gases at low temperatures and in a strong magnetic field H oriented normal to the plane of the electron motion was calculated along the same lines as in the work of Landau.[4] In contrast to the 3D case, there is no Landau diamagnetism but the magnetization vanishes as the magnetic field tends to zero. Similar to the 3D case, in 2D the $1/H$-periodic de Haas–van Alphen oscillations appear. The magnetization of two-dimensional electron gases (2DEGs) is very small and therefore difficult to measure. Early experiments relied on the signal from stacks of 2DEGs.[5,6] Recently several authors were able to measure the magnetization of a single-layer 2DEG using sophisticated experimental techniques.[7–9]

In this paper we show how our modern picture of one-dimensional edge channels can be used to obtain the boundary term of the magnetization of a 2D electron gas. First, we briefly repeat the standard derivation of the 2DEG magnetization from the free energy leading to a bulk term and an edge term. Then the physical origin of these two terms and their interplay are discussed. We then evaluate where the equilibrium bulk currents flow in a sample without disorder and show that conventional magnetization measurements measure mainly the magnetic moment originating from currents flowing at the edge of the sample. Finally, we discuss the possibility of a local measurement of the de Haas–van Alphen effect with local scanning techniques.

We consider a two-dimensional electron gas in the x-y plane with a circular boundary of radius R around the origin (see Fig. 1). The magnetic field, H, is applied in the z direction. Relevant quantities related to H are the cyclotron frequency $\omega_c = eH/m_e$, where m_e is the effective mass of the electron, and the magnetic length $a_H = \sqrt{\hbar/(eH)}$. Eigenenergies and wave functions are calculated in cylinder coordinates (ρ,φ). Each state is described by two integer numbers, $n \geq 0$, the radial quantum number, and m, the angular momentum quantum number. The eigenenergy of state $|n,m\rangle$ is given by $E_{n,m} = \hbar\omega_c[n+1/2+(m+|m|)/2]$. The set of states with energy $E_N = \hbar\omega_c(N+1/2)$ ($N \geq 0$ integer) is called a Landau level. Only states for which the condition $2n+|m|+1 \leq R^2/(2a_H^2)$ is fulfilled lie within the sample boundary. We restrict ourselves to the case of high magnetic fields where n is small and $R^2/(2a_H^2) \gg 1$ so that the requirement reduces to $|m| \leq R^2/(2a_H^2) =: L(H)$, where $L(H)$ is the number of states of a Landau level. The magnetic moment M is calculated from the free energy of the system,

$$F = \mathcal{N}\mu - kTL(H)\sum_{N=0}^{\infty} \ln\left[1+\exp\left(\frac{\mu - E_N(H)}{kT}\right)\right], \quad (1)$$

where μ is the chemical potential and $\mathcal{N}$ is the particle number. The magnetic moment is then given by $M = -\partial F/\partial H$, leading to

$$M = 2kT\frac{A}{\phi_0}\sum_{N=0}^{\infty} \ln\left[1+\exp\left(\frac{\mu - \hbar\omega_c(N+1/2)}{kT}\right)\right] \quad (2)$$

$$-2\hbar\omega_c\frac{A}{\phi_0}\sum_{N=0}^{\infty}\left[\left(N+\frac{1}{2}\right)f[\hbar\omega_c(N+1/2)]\right]. \quad (3)$$

Here $\phi_0 = h/e$ is the magnetic flux quantum, $A = \pi R^2$ is the sample area, and $f(E)$ is the Fermi-Dirac distribution function. The second term (3) originates from the statistical factor (bulk term) whereas the first one (2) arises due to the

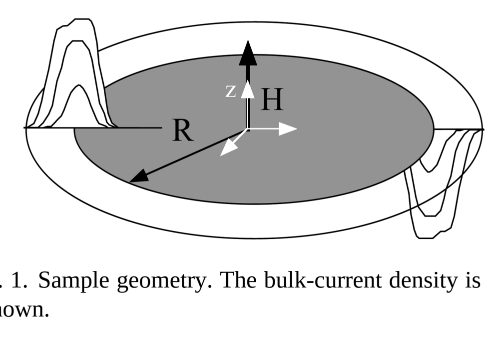

FIG. 1. Sample geometry. The bulk-current density is schematically shown.

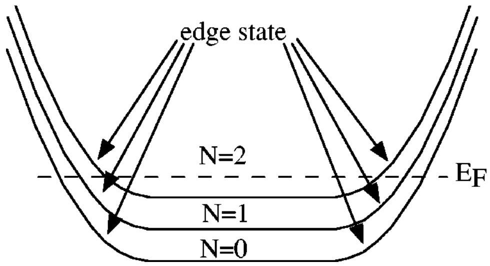

FIG. 2. Edge-state concept according to Halperin. The dispersion occurs as a result of the boundary potential.

existence of sample boundaries.[10] According to Ref. 10, the magnetic moment can alternatively be calculated by computing the statistically weighted sum over the magnetic moments of all states. Doing this, however, only the bulk term is found:

$$M_{\mathrm{bulk}}=2L(H)\sum_{N=0}^{\infty}\frac{\partial E_N}{\partial H}f(E_N)$$
$$=-2\hbar\omega_c\frac{A}{\phi_0}\sum_{N=0}^{\infty}\left[\left(N+\frac{1}{2}\right)f[\hbar\omega_c(N+1/2)]\right].$$

The boundary term can be recovered by taking the states which are affected by the presence of the boundary, i.e., the edge states, properly into account.

Although the importance of the sample edge was already realized in the early work on the orbital magnetism of free electrons, the concept of edge states has recently become of increasing importance in connection with the discovery and description of the quantum Hall effect.[11] Halperin laid the foundation for the edge-state concept in a pioneering paper.[12] The basic idea is that at the sample boundary of a two-dimensional electron gas, the degeneracy of individual Landau levels is lifted due to the boundary potential. The resulting edge states can still be thought of as originating from a certain Landau level with index n but have a one-dimensional dispersion as a function of angular momentum m. Figure 2 visualizes this idea. Later a self-consistent electrostatic model of edge channels was developed which led to the existence of compressible and incompressible stripes in the region where edge channels exist.[13] The detailed equilibrium current distribution at sample edges was predicted by Geller and Vignale.[14]

In the single-particle picture of Halperin,[12] the edge-state dispersion leads to a drift velocity for each state. All edge states near a given point at the sample boundary have a drift velocity in the same direction. The current carried by all occupied states originating from one Landau level N is given by

$$I_N=2e\int_{E_N}^{\infty}dE\,\frac{1}{h}\,\frac{\partial E_{n,m}}{\partial m}\,\frac{1}{\partial E_{n,m}/\partial m}f(E)$$
$$=2kT\frac{1}{\phi_0}\ln\left[1+\exp\left(\frac{\mu-E_n}{kT}\right)\right].$$

In a macroscopic sample the spatial extent of the edge states will be negligible compared to the sample dimensions and each edge state will encircle the same area A of the whole sample. The contribution of the edge currents to the magnetic moment of the sample can therefore be approximated as the sum of the magnetic moments of current loops around an area A leading to

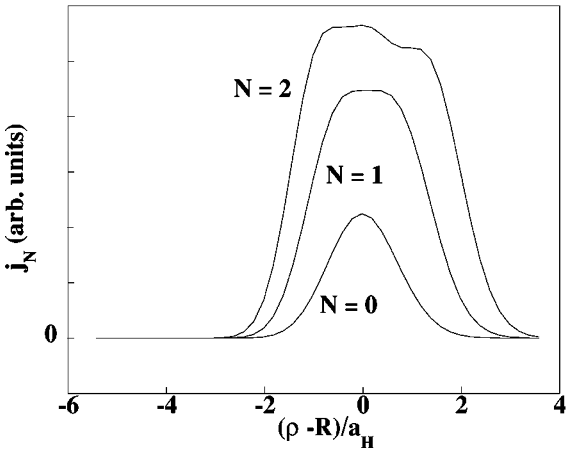

FIG. 3. Bulk current density distribution near the edge of the 2DEG for the lowest three Landau levels.

$$M_{\mathrm{edge}}=\sum_{N=0}^{\infty}I_NA=2kT\frac{A}{\phi_0}\sum_{N=0}^{\infty}\ln\left[1+\exp\left(\frac{\mu-E_n}{kT}\right)\right],$$

which is exactly the term (2) of the magnetic moment calculated from the free energy.

Now that we know where the currents creating M_{edge} flow, it is natural to ask which current density distribution leads to M_{bulk}. An approximate but still valuable answer can be obtained from quantum mechanics by calculating the sum over all currents associated with individual states. The current density of a state (n,m) is given by

$$j_{n,m}(x)=\frac{-e\hbar}{2\pi m_e a_H^3}\,\frac{n!(m+x)}{(n+|m|)!}\,\frac{x^{|m|}}{\sqrt{2x}}e^{-x}[L_n^{(|m|)}(x)]^2, \quad (4)$$

where $x=\rho^2/(2a_H)$ and $L_n^{(|m|)}(x)$ are the generalized Laguerre polynomials. The total bulk current density of a Landau level can now be numerically calculated by summing over all quantum numbers representing states within the sample boundaries. An example depicted in Fig. 3 and schematically transferred into Fig. 1 shows the main features of the bulk current density of individual Landau levels: first, the current flows only in the vicinity of the sample boundary, and second, the spatial extent of the current density distribution is of the order of the magnetic length for the lowest Landau level and increases with Landau-level index. This shows that at sufficiently high magnetic fields, when only a few Landau levels are occupied, the bulk contribution to the magnetic moment M_{bulk} is created by currents flowing at the sample edge, like the edge contribution M_{edge}. The de Haas–van Alphen effect therefore arises from the subtle interplay of bulk and edge currents which both flow at the sample boundary. Regions away from the boundary do not contribute to the magnetic moment.

This single-particle picture contains neither any self-consistency for the determination of the actual boundary potential[13] nor any exchange-correlation effects.[14] However, our aim is to clarify with a straightforward and transparent approach why the sample edge is so important for current-related phenomena occurring in 2DEGs subject to magnetic fields. In their quite involved approach, Geller and Vignale[14] have shown that the current in compressible and incompressible stripes flows in opposite directions. They distinguish between an ''edge'' current flowing in the compressible regions, which is proportional to the density gradient at the sample edge, and a ''bulk'' current flowing in the incompressible regions, which is proportional to the gradient of the self-consistent potential. Coming from our approach, it is immediately clear why edge and bulk currents play a role at the sample edge and their diamagnetic and paramagnetic effect becomes obvious. Of course, the full treatment in the current-density functional theory is necessary to obtain the detailed spatial arrangement of the current contributions.

Advances in scanning probe techniques have made it possible to measure local magnetic stray fields of a specimen down to length scales below 100 nm.[15] These techniques will be of increasing importance for the investigation of near-surface magnetic properties of submicron structures. For the local measurement of the de Haas–van Alphen effect, one would have to probe stray fields at the boundary of a 2DEG. We estimate typical equilibrium currents to be of the order of $M(H)/A$, which can be around 250 nA in typical semiconductor heterostructures at magnetic fields of 5–10 T. The stray fields which can be measured depend strongly on the distance of the probe to the 2DEG. Realistic values are around 100 nm. Since the oscillations are most pronounced at high magnetic fields, the employed sensor has to be sensitive under these conditions and it must be able to detect extremely small variations in the local magnetic stray field on the background of a huge external field. In order to estimate the typical size of the stray fields, we assume that the equilibrium current flows in a filament with no extent in the ρ and z directions. The stray field at a distance d from the 2DEG plane is then given by

$$H_\rho = \frac{-Id}{2\pi[(\rho-R)^2+d^2]},$$

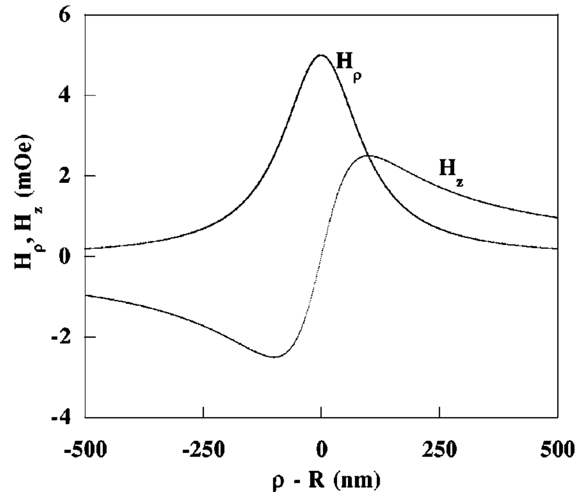

FIG. 4. Magnetic stray fields in a plane 100 nm above the 2DEG. A total edge current of 250 nA was assumed.

$$H_z = \frac{I(\rho-R)}{2\pi[(\rho-R)^2+d^2]}.$$

These functions plotted in Fig. 4 give typical stray fields of 0.005 Oe to be detected on a background of, say, 50 kOe. This means that a resolution $\Delta H/H$ better than 10^{-7} would be required for such a measurement.

In conclusion, we have shown how the interplay of bulk and edge contributions to the total current at the boundary of a two-dimenional electron gas leads to the de Haas–van Alphen oscillations in the magnetic moment. In contrast to earlier derivations of the effect, the edge contribution was introduced using the concept of one-dimensional edge states, which play an important role for the description of the quantum Hall effect. The local measurement of the de Haas–van Alphen effect may be in experimental reach.

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