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Asymmetries of the conductance matrix in a three-terminal quantum ring in the Coulomb blockade regime

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Abstract

The conductance matrix of a three-terminal quantum ring has been measured in the Coulomb blockade regime for strong coupling to the leads. In contrast to multi-terminal quantum dots in the weak coupling regime, we find that the conductance matrix is asymmetric. This is a direct evidence of the relevance of phase coherence in transport experiments through quantum dots strongly coupled to the leads.

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1. Introduction

Open mesoscopic rings are widely used as interferometers to study electronic phase coherence. In addition, a multi-terminal set-up is sensitive to the phase of the electrons travelling through the structure [1–4], which can be probed by the asymmetry of the conductance matrix of such a system at finite magnetic field. On the other hand, for sufficiently small rings, Coulomb blockade has been reported [5]. In the weak coupling regime, the incoherent sequential tunneling model [6], widely used to interpret Coulomb blockade experiments, assumes a 0-D state coupled to leads through tunnel barriers, which leads to a symmetric conductance matrix for a multi-terminal set-up. Here we address the question of the cross-over from closed to open quantum rings observed by the symmetries of the conductance matrix.

In previous work on a quantum dot weakly coupled to three leads we have shown that the conductance matrix is symmetric at finite magnetic field, as expected in the sequential tunneling model [7]. Interestingly, the validity of this model even for coherent transport has been confirmed theoretically [8]. Here we present measurements of the conductance matrix in a new geometry, a three-terminal quantum ring. Compared to a many-electron quantum dot, the ring shape allows a better control of the Aharonov–Bohm flux through the shape of its wave function, and of the electronic phase along the ring. In the strong coupling regime, we show that the conductance matrix is asymmetric, in contrast to weakly coupled quantum dots. This signature of the phase coherence in the quantum ring shows the importance of fully coherent models [8] for describing the transport through quantum dots in the strong coupling regime.

2. Sample and methods

The three-terminal ring shown in Fig. 1 has been fabricated by using an atomic force microscope (AFM) to oxidize locally the surface of a GaAs/AlGaAs heterostructure containing a two-dimensional electron gas (2DEG) 34 nm below the surface (electron density $4.5 \times 10^{15}/\mathrm{m}^2$, mobility $25\,\mathrm{m}^2/\mathrm{Vs}$) [9,10]. The 2DEG is depleted below the oxide lines, defining a ring structure connected to three leads through quantum point contacts. The coupling strengths from the ring to the leads is tuned

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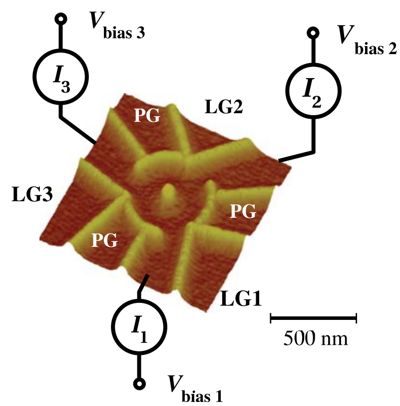

Fig. 1. Sample and experimental set-up. The picture is a micrograph of the oxide lines defining the three-terminal quantum ring, taken with an AFM after the oxidation. The properties of the ring are tuned by applying voltages on the gates LG1–LG3 and PG (see text). Bias voltages V_{bias1}–V_{bias3} are applied on the three leads, while the currents through each lead I_1–I_3 are measured.

by applying a voltage on the gates LG1–LG3, while the three plunger gates (PG) are used to tune the number of electrons in the ring. Measurements of Coulomb diamonds in the weak coupling regime reveal a charging energy $E_C \approx 1.2\,\text{meV}$ and an average single-particle level spacing $\Delta E \approx 200\,\mu\text{eV}$. In the open regime, the conductance shows Aharonov–Bohm oscillations as a function of the magnetic field, with a period of 75 mT, corresponding to a diameter of the ring of 270 nm. The measurements have been performed in a ^{3}He/^{4}He dilution refrigerator with an electron temperature less than 50 mK.

In order to measure the conductance matrix elements, a bias voltage of $10\,\mu\text{V}$ is applied on one lead, the two other leads being grounded (see Fig. 1). The current measured through the three leads, divided by the bias voltage, is then a direct measurement of the three elements of the first column of the conductance matrix. Applying the bias successively to the two other leads gives the other elements G_{ij} of the conductance matrix. For this experiment we have checked that the contact resistances are negligible. More details about the measurement method are given in Ref. [7].

3. Results

We have measured the conductance matrix of the quantum ring in the Coulomb blockade regime, for which $k_B T < \Delta E < E_C$ and the resistances of the tunnel barriers are larger than h/e^2. By tuning the gates LG1–LG3, we can tune the coupling of the ring to the leads in order to be in the strong coupling regime, for which the level broadening Γ is larger than the temperature, but still smaller than the mean level spacing: $k_B T < h\Gamma < \Delta E$. In this regime, Kondo physics is expected and has been observed in this system [11]. A trace of the conductance measured by applying a bias to lead 3 and measuring the current through lead 3, i.e., G_{33}, as a function of the plunger gate voltage V_{PG} is shown in Fig. 2. This trace shows broad conductance

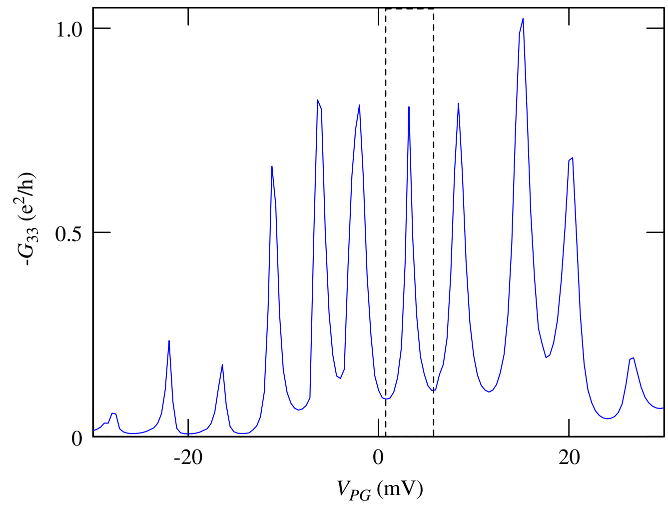

Fig. 2. Absolute value of the conductance matrix element G_{33} as a function of the plunger gate voltage V_{PG}. The two dashed lines enclose the conductance peak considered in the following.

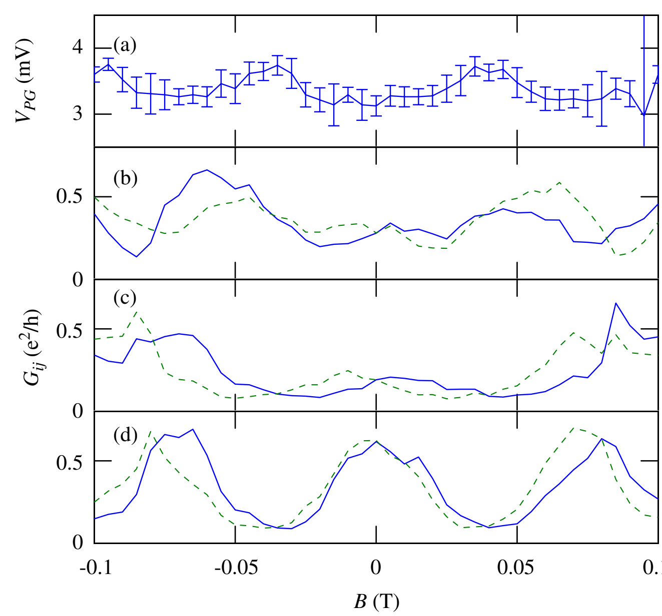

Fig. 3. Position and amplitude of one Coulomb peak in the strong coupling regime vs. magnetic field, obtained from a fit with a Lorentzian shape [12]. (a) Mean of the peak position. The error bars represent the standard deviation over the nine conductance matrix elements. (b)–(d) Off-diagonal conductance matrix elements taken for the maximum of the conductance peak. The elements are (b) G_{12} (solid line) and G_{21} (dashed line), (c) G_{23} (solid line) and G_{32} (dashed line), (d) G_{31} (solid line) and G_{13} (dashed line).

resonances, characteristic of coupling-broadened peaks [12].

The whole conductance matrix corresponding to the plunger gate range shown in Fig. 2 has been measured as a function of a magnetic field applied perpendicular to the 2DEG. The peaks have been fitted by a Lorentzian function [12] in order to determine their positions in V_{PG} and their magnitudes. In the following we focus on the peak enclosed in Fig. 2 by the two dashed lines, for which the fit gives a level broadening $h\Gamma = 110 \pm 20\,\mu\text{V}$, but all peaks measured in the same regime show similar width.

Fig. 3 shows the position and the magnitude of the peak marked in Fig. 2 as a function of the magnetic field. The off-diagonal conductance matrix elements are shown in Figs. 3(b)–(d). It is clear from these plots that the conductance matrix is not symmetric, i.e., $G_{ij}(B) \neq G_{ji}(B)$. But the traces follow the Onsager relations $G_{ij}(B) = G_{ji}(-B)$, as already observed for open systems [2].

4. Discussion

The asymmetry of the conductance matrix observed in our system in the strong coupling regime is clearly different than the symmetry reported in multi-terminal quantum dots in the weak coupling regime [7]. To the best of our knowledge, it is the first time that such an asymmetry is observed in a closed system, i.e., showing Coulomb blockade. Such an asymmetry could be attributed to a classical origin (Hall effect). But the fact that the asymmetry is observed at very low magnetic field and has the same period as the Aharonov–Bohm oscillations is a strong indication that it comes from coherent transport through the ring. In addition, we have checked that no asymmetry is observed in this system for weak coupling. We attribute it to the phase accumulated by electrons travelling through the ring, as already observed in multi-terminal open ring structures [1–4], but further experiments are needed to confirm this point.

This asymmetry cannot be explained with a incoherent sequential tunneling model [6], for which the conductance through the ring would be only determined by the tunneling rates from the ring to the leads. It is then a clear evidence that the phase coherence in quantum dots is not only relevant in phase measurements [13,3,4], but also directly in transport experiments. It points out the relevance of fully coherent models [8] for describing the transport through strongly coupled quantum dots.

To check further the consistency of the measurement, we have performed the following analysis. The conductance matrix can be split into a symmetric and an anti-symmetric part $\boldsymbol{G} = \boldsymbol{G}^{\mathrm{s}} + \boldsymbol{G}^{\mathrm{a}}$. For a three-terminal set-up, the sum rules impose that the symmetric part $\boldsymbol{G}^{\mathrm{s}}$ contains only three free parameters, and the anti-symmetric part $\boldsymbol{G}^{\mathrm{a}}$ only one:

$$\boldsymbol{G}^{\mathrm{s}} = \begin{pmatrix} -(G^{\mathrm{s}}_{12} + G^{\mathrm{s}}_{31}) & G^{\mathrm{s}}_{12} & G^{\mathrm{s}}_{31} \\ G^{\mathrm{s}}_{12} & -(G^{\mathrm{s}}_{12} + G^{\mathrm{s}}_{23}) & G^{\mathrm{s}}_{23} \\ G^{\mathrm{s}}_{31} & G^{\mathrm{s}}_{23} & -(G^{\mathrm{s}}_{31} + G^{\mathrm{s}}_{23}) \end{pmatrix} \quad (1)$$

and

$$\boldsymbol{G}^{\mathrm{a}} = \begin{pmatrix} 0 & G^{\mathrm{a}} & -G^{\mathrm{a}} \\ -G^{\mathrm{a}} & 0 & G^{\mathrm{a}} \\ G^{\mathrm{a}} & -G^{\mathrm{a}} & 0 \end{pmatrix}. \quad (2)$$

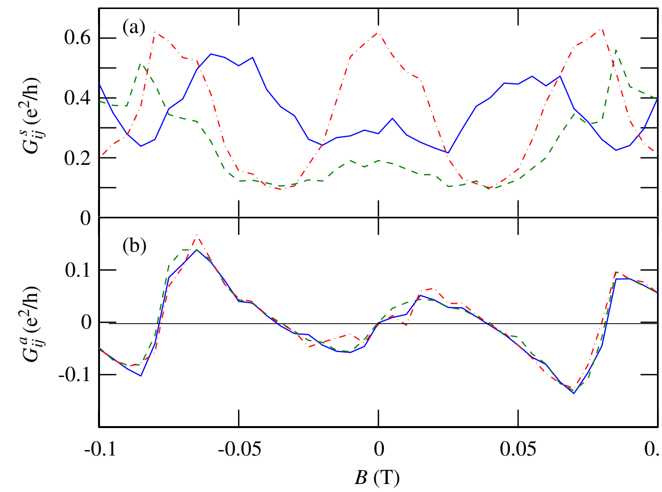

Fig. 4. Splitting up of the conductance matrix for the amplitude of one Coulomb peak in the strong coupling regime vs. magnetic field. (a) Elements of the symmetric part, G^{s}_{12} (solid line), G^{s}_{23} (dashed line), G^{s}_{31} (dash-dot line). (b) Elements of the anti-symmetric part, G^{a}_{12} (solid line), G^{a}_{23} (dashed line), G^{a}_{31} (dash-dot line).

Splitting of the conductance matrix for the amplitude of the peak marked in Fig. 2 is shown in Fig. 4. We note in particular that the anti-symmetric elements $G^{\mathrm{a}}_{12} = (G_{12} - G_{21})/2$, $G^{\mathrm{a}}_{23} = (G_{23} - G_{32})/2$ and $G^{\mathrm{a}}_{31} = (G_{31} - G_{13})/2$ all merge into the same curve, as expected from the sum rules.

In the weak coupling regime, the three independent elements of the symmetric part $\boldsymbol{G}^{\mathrm{s}}$ were shown to be related to the coupling strengths to the leads [7]. Whether this is still true in the strong coupling regime has to be investigated.

5. Conclusion

We have studied the conductance matrix of a three-terminal quantum ring in the Coulomb blockade regime for strong coupling to the leads. We show a clear asymmetry of the conductance matrix in this regime. This direct evidence of coherent transport through the quantum ring points out the relevance of fully coherent models to describe transport experiments in quantum dots.

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