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Microelectronic Engineering 63 (2002) 47–52

MICROELECTRONIC ENGINEERING

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# Energy spectra of quantum rings

A. Fuhrer[a], S. Lüscher[a], T. Ihn[a], T. Heinzel[a,d], K. Ensslin[a,*], W. Wegscheider[b], M. Bichler[c]

[a] *Solid State Physics Laboratory, ETH Zürich, 8093 Zürich, Switzerland*
[b] *Angewandte und Experimtelle Physik, Universität Regensburg, 93040 Regensburg, Germany*
[c] *Walter Schottky Institut, TU München, 85748 Garching, Germany*
[d] *Fakultät für Physik, Universität Freiburg i. Br., 79104 Freiburg, Germany*

**Abstract**

High quality quantum rings are fabricated by nano-lithography with a scanning force microscope. The electron occupancy as well the tunneling contacts to source and drain are controlled by in-plane and top gate voltages. Coulomb blockade oscillations in the conductance through the quantum ring give information about the energy spectrum and the wave functions. The Coulomb blockade peak positions as well as the amplitudes oscillate periodically as a function of magnetic field. The period is given by an Aharonov–Bohm-type argument, i.e., one flux quantum h/e through the area defined by the ring. Some states show a pronounced zig-zag behavior as a function of field in agreement with the energy spectrum of a perfect ring. Other states display a rather flat magnetic field dispersion. We argue that the magnetic field behavior of a given state depends on the degree to which its wave function is localized or extended along the circumference around the quantum ring.


*Keywords:* Single electron transistor; Quantum rings; Persistent currents

## 1. Introduction

Single electron transport through semiconductor quantum dots allows to investigate quantum mechanical energy levels and wave functions in a controlled way (for a review, see Ref. [1]). Most semiconductor nanostructures on AlGaAs/heterostructures are fabricated by electron beam lithography followed by suitable etching or lift-off processes. We focus on the definition of nanostructures with the help of a scanning probe microscope [2]. This method allows one to combine small lateral depletion lengths with smooth confining potentials (for a review, see Ref. [3]). High-quality quantum dots have been fabricated [4] and single-level transport has been demonstrated [5].

*Corresponding author. Tel.: +41-1-633-2209; fax: +41-1-633-1146.
*E-mail address:* ensslin@solid.phys.ethz.ch (K. Ensslin).

0167-9317/02/$ – see front matter 
PII: S0167-9317(02)00598-1

Here we set out to explore quantum rings that have been envisioned by Aharonov and Bohm [6] in order to validate the influence of the magnetic vector potential on experimentally measurable quantities. The resistance of an open ring was predicted to oscillate periodically as a function of a magnetic field piercing the ring, since the interference condition of the two partial waves in the two arms of the ring changes with the magnetic flux enclosed. Such effects have been experimentally observed in metal [7] and semiconductor [8] rings. Open rings have also been used in order to demonstrate that electron tunneling through a single electron transistor placed in one arm of the ring is at least partially phase coherent [9]. The energy spectra of closed perfect rings have been calculated [10] and this has lead to the prediction of persistent currents [11]. The experimental determination of ring spectra was so far limited to optical experiments [12,13].

We measure the conductance of our quantum rings in the Coulomb blockade regime. The conductance peak width is smaller than the single-particle level spacing $D$ and dominated by temperature kT. This allows us to investigate the magnetic field dependence of the peak amplitude and position in the single level regime and we find that both properties oscillate periodically as a function of magnetic field. Details of the ring spectrum are discussed in view of the properties of the realistic ring.

## 2. Experiment

Fig. 1 (left hand side) presents an SFM (scanning force microscope) picture of the surface of an AlGaAs/GaAs heterostructure which was patterned by oxide lines using SFM lithography [2]. The two-dimensional electron gas which is located only 34 nm below the sample surface has a 4.2 K density of $5 \times 10^{11}$ cm$^{-2}$ and mobility of 900,000 cm$^{2}$/V s. The electron gas is depleted below the oxidized lines and we can therefore use the areas marked ‘‘QPC’’ and ‘‘plunger’’ as in-plane gate

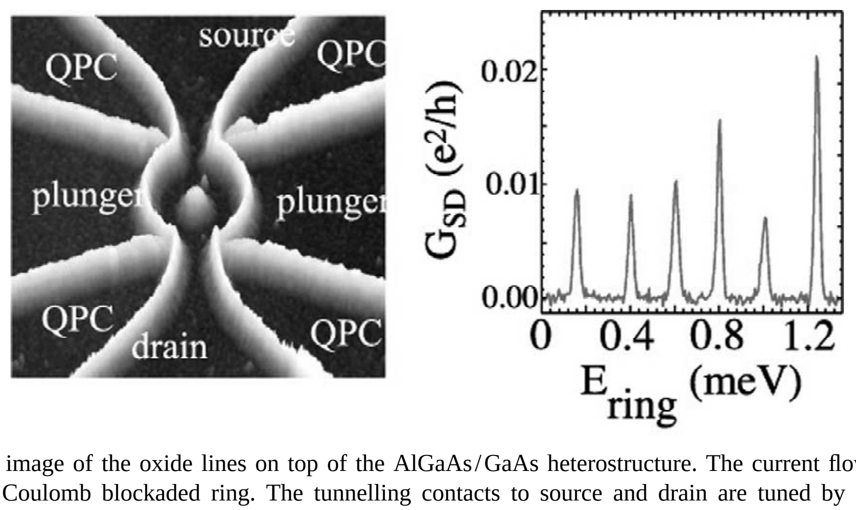


Fig. 1. Left: SFM image of the oxide lines on top of the AlGaAs/GaAs heterostructure. The current flows from source to drain through the Coulomb blockaded ring. The tunnelling contacts to source and drain are tuned by the in-plane gates marked ‘‘QPC’’. The electron occupancy is controlled by the lateral ‘‘plunger’’ gate electrodes. Right: Conductance measured at source-drain voltage $V_{SD} = 20$ mV through the quantum ring at properly adjusted ‘‘QPC’’ and top gate voltages as a function of energy.

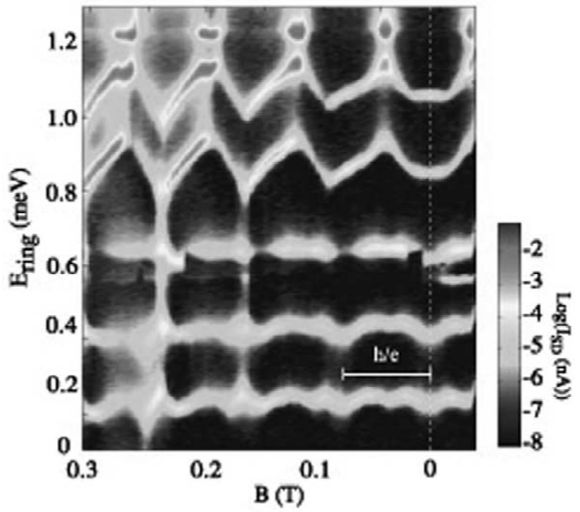


Fig. 2. The evolution of Coulomb-blockade resonances with magnetic field results in the addition spectrum. The regions of high current (bright/dark) mark configurations in which a bound state in the ring aligns with the Fermi level in source and drain. The Aharonov–Bohm period expected from the ring geometry is indicated by the white horizontal bar.

electrodes [5]. After the oxidation process the sample is covered with a homogenous top gate electrode in order to enhance the tunability of the ring.

The right hand side of Fig. 1 shows Coulomb blockade oscillations as a function of plunger gate voltage which has been converted into a relative energy scale. From corresponding measurements of the Coulomb blockade diamonds we determine a charging energy $E_c = e^2/C_S \approx 190$ μeV. The observed discrete level spacings after subtraction of $E_c$ within the constant interaction model [1] can be as large as $D \approx 180$ μeV. From a simple capacitance model taking the ring geometry into account we estimate that about 200 electrons are distributed on 2–3 radial subbands. The peak width corresponds to a temperature broadening of about 100 mK, which roughly coincides with the base temperature of the cryostat.

Fig. 2 presents a plot of the current through the quantum ring as a function of top gate voltage and magnetic field *B* [applied normal to the two-dimensional electron gas (2DEG) plane]. The position as well as the amplitude of the Coulomb blockade peaks oscillate as a function magnetic field with a period of DB = 75 mT which is exactly the Aharonov–Bohm period for this ring. By opening the point contacts (not shown) we find the well-known Aharonov–Bohm oscillations in the conductance with the same DB.

## 3. Discussion

Let us start from the energy spectrum of a perfect ring which can be calculated analytically:

$$E_{m,l} = \frac{h^2}{2m^* r_0^2}(m + l)^2$$

Here $r_0$ is the radius of the infinitely thin ring enclosing $m$ flux quanta, $m^*$ is the mass of the particle, and $l$ is the angular momentum quantum number. For a given angular momentum state the energies as a function of magnetic field (or flux $m$) lie on a parabola with its apex at $m = -l$, as depicted in Fig. 3. According to this simple picture a single Coulomb blockade peak should oscillate as a function of $B$ along a zig-zag line (see Fig. 3) with the Aharonov–Bohm period $\Delta$B. Comparison with the measurement in Fig. 2 shows that indeed some peaks move along a zig-zag line.

How can we understand the $B$-periodic oscillations of the Coulomb blockade amplitude? For the perfect ring the wave functions are plane waves running around the circumference of the ring. The probability density of each state has therefore no azimutal dependence at $B = 0$ as well as at finite

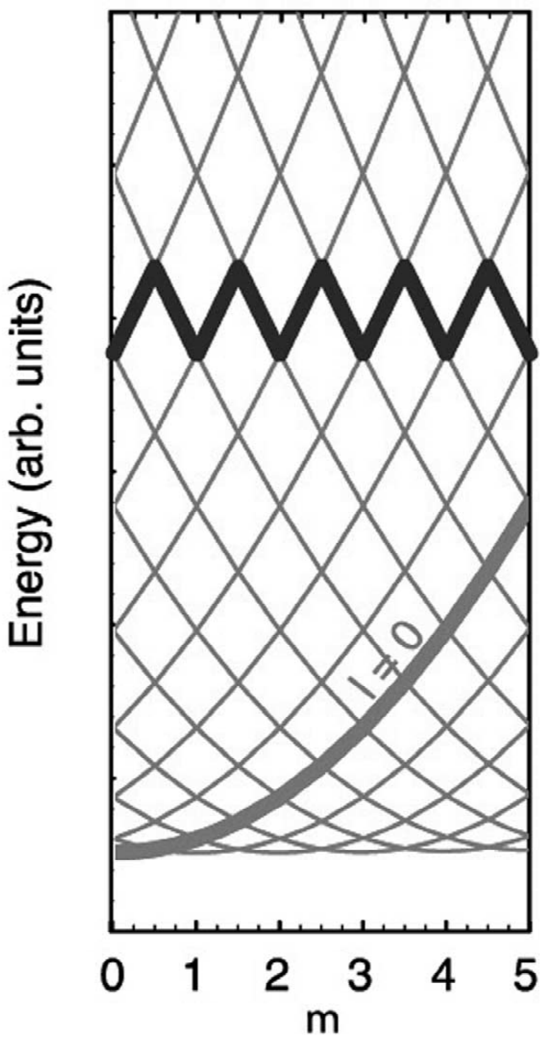


Fig. 3. Theoretical spectrum of a single mode ring. The parabolas with constant $l$ (bold line) have a minimum at $l = -m$. The bold zig-zag line corresponds to a Coulomb peak after the charging energy has been subtracted in the constant interaction model.

magnetic field. The amplitude of a Coulomb blockade peak depends on the overlap of the wavefunction in the ring with the wave functions in source and drain. In this picture we therefore do not expect to observe an oscillating Coulomb blockade amplitude.

The situation changes if one considers perturbations of the perfect ring potential which will already arise by the pure presence of source and drain. In this case the probability densities start to oscillate around the ring's circumference and a $B$-periodic Coulomb blockade amplitude can be understood this way.

In reality the ring contains 2–3 radial modes and the ring potential will deviate from perfect circular symmetry. A more refined model [14] shows that the resulting states are superpositions of few or many angular momentum states. If only few angular momentum states of similar $l$ contribute, the resulting level oscillates periodically as a function of magnetic field in a zig-zag like fashion which closely resembles the states of a perfect ring. The corresponding wave function has a rather smooth azimutal dependence. For a superposition of many angular momentum states the magnetic field dispersion of the resulting state can be become relatively flat, as can be seen for the lower three states in Fig. 2. A more refined discussion of this situation is presented in Ref. [14].

## 4. Conclusions

These experiments represent a breakthrough in the experimental detection of quantum ring energy spectra. The question of persistent currents can now be investigated more directly on the level of the magnetic field dispersion of the energy levels. In a more general sense our quantum rings are many electron quantum dots whose energy levels can be understood at least for part of the spectrum in a quantitative manner, a procedure so far limited to few electron quantum dots. Our lithographic approach offers the possibility to tune the potentials in such a way that geometries can be realized which are classically predominantly chaotic or regular (for more details see Ref. [14]). Research on quantum rings is therefore just at its beginning and several interesting fundamental physics questions await their understanding.

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