Magnetic-field-induced resonant tunneling in parallel two-dimensional systems

T. Ihn, H. Carmona, P. C. Main, L. Eaves, and M. Henini

Department of Physics, University of Nottingham, Nottingham NG7 2RD, United Kingdom

(Received 1 March 1996)

The magnetoresistance at 4.2 K of two spatially remote two-dimensional electron gases is investigated. The magnetic field *B* is applied parallel to the planes of the two-dimensional electron gases (2DEG's). Tunneling between them occurs above a critical field B_c . In a configuration where the current *I* is perpendicular to *B*, a resistance resonance is observed at B_c , which is absent when *I* is parallel to *B*. This resistance resonance is due to a large difference in the mobilities of the two 2DEG's. A model calculation based on the Boltzmann equation and the symmetry properties of the electron subband dispersion curves reproduces the observed phenomena. [S0163-1829(96)50128-1]

In recent years there has been great interest in the electrical transport properties of parallel two-dimensional electron systems (2DES's) coupled by tunneling or by Coulomb interaction. Usually these systems are realized in narrow symmetric double quantum wells (DQW's) coupled by tunneling through the thin central barrier^{1–5} or in wide quantum wells, in which the 2DES's occur at the two interfaces.^{6,7} In this paper we report measurements of electrical transport on a wide asymmetric *single* quantum well structure.

Our work is related to the resistance resonance (RR) effect previously found in DQW samples.¹ An enhanced longitudinal resistance R_1 is observed when two electron subbands 1 and 2 in different wells have the same Fermi-wavevector k_F (resonance condition) and their transport scattering times τ_1, τ_2 differ strongly. If the resonance condition is not fulfilled, tunneling between the wells is suppressed since the subband states become localized in the individual wells and the conductivity is dominated by transport in the high mobility well. Under the resonance condition, however, the symmetric and antisymmetric states are extended over both wells and tunneling occurs on a characteristic time scale of $\tau \sim \hbar / \Delta_{SAS}$ (Δ_{SAS} is the symmetric antisymmetric splitting). If $\tau \ll \tau_1, \tau_2$ (strong coupling) the two states have essentially the same mobility determined by the low mobility well. It has also been shown that the resonant tunneling between the wells can be suppressed by a magnetic field parallel to the plane of the 2DEG's,² thus quenching the RR.³

In contrast to these experiments, we start off with a sample where the k_F of the two subbands are different (off resonance) and show how a parallel magnetic field can delocalize states that were confined to the opposite interfaces at B=0 T. A resistance resonance peak occurs at a field $B_c \approx \hbar (k_F^{(0)} - k_F^{(1)})/(ed)$, where *d* is the effective spatial separation of the two-dimensional states and $k_F^{(0,1)}$ are the respective Fermi wave vectors of the two subbands. The resonance is sensitive to the angle between the current and the magnetic field and relatively insensitive to temperature.

The samples are MBE-grown, modulation-doped structures incorporating a single 407-Å-wide QW embedded between Al_{0.3}Ga_{0.7}As barriers. Undoped spacer layers of 76 Å separate the doped regions ($N_D = 1.33 \times 10^{18}$ cm⁻³) from the QW. Analysis of the transport properties in a magnetic field perpendicular to the plane of the well similar to that in Ref. 8 reveals the occupation of three subbands, E_0 , E_1 , and E_2 with electron concentrations $n_0=1\times10^{12}$ cm⁻², $n_1=0.5\times10^{12}$ cm⁻², $n_2=0.11\times10^{12}$ cm⁻², respectively, and mobilities $\mu_0=1.0$ m²/V s, $\mu_1=20.4$ m²/V s. $\mu_2 < \mu_1$, but an accurate determination was not possible because this subband contributes little to the total conductivity. A self-consistent calculation of the subband structure is shown in the inset of Fig. 1. The E_0 and E_1 states are effectively separated by a distance d=32 nm.

Four-terminal measurements were performed on conventional Hall-bar structures using a standard low-frequency lock-in technique. Figure 1 shows the dependence of the longitudinal resistance R_I at 300 mK on the magnetic field B_{\parallel} in



FIG. 1. Longitudinal resistance as a function of the parallel magnetic field at 300 mK. The inset shows the asymmetric potential profile of the 40-nm-wide quantum well at zero magnetic field, the three energy levels, and the corresponding electron-density distributions.



FIG. 2. Angular dependence of the magnetoresistance with *B* in the plane of the quantum well. The angle-independent point at B=2 T is marked by an arrow. The fact that the curves do not exactly cross in one point is due to a small component of the magnetic field perpendicular to the plane of the 2DEG's.

the plane of the QW for the two current orientations, $B_{\parallel} \perp I$ (denoted by B_{\parallel}^{\perp}) and $B_{\parallel} \parallel I (B_{\parallel}^{\parallel})$. For B_{\parallel}^{\perp} the resistance shows a pronounced peak at $B_c = 1.6$ T with a relative amplitude $\Delta R/R = 26\%$. The peak is followed by a rapid decrease in resistance down to almost half of its zero-field value. Two shoulders can be seen, at $B_1 = 10.5$ T and $B_2 = 6$ T. Above 12 T the resistance increases with increasing field. The magnetoresistance for $B_{\parallel}^{\parallel}$ is very different below 5 T. With increasing *B*, a relatively smooth increase in the resistance saturates at about 3.5 T and no resistance peak is observed. Above 5 T, however, the general behavior resembles the B_{\parallel}^{\perp} case.

To clarify the nature of the sharp resistance peak, we studied the temperature dependence of the resistance between T=0.3 K and 42 K. For T<7 K, R_1 is not changed significantly. At higher temperatures the peak broadens and $\Delta R/R$ becomes smaller. This weak temperature dependence rules out a quantum explanation for the peak.

Figure 2 shows how the resistance peak depends on the angle between *I* and B_{\parallel} . We note two features: first, there is no shift of the peak position with angle; second, at 2 T the magnetoresistance is independent of angle. Such fixed points were also observed but not discussed in Ref. 4.

For an explanation, we note that in our three-subband sample the Fermi surface at B=0 consists of three concentric circles [see Fig. 3(a)]. For *B* in the plane of the QW, to first order the contours remain circular, but⁹ (i) the origin of each circle *i* shifts by $\Delta k_i = \langle z \rangle_i / l_c^2$ in the direction perpendicular to *B* (here *z* denotes the direction normal to the plane of the 2DEG's); (ii) each subband rises in energy by $\Delta E_i = m \omega_c^2 \langle (z - \langle z \rangle_i)^2 \rangle_i / 2$. Here $\langle \cdots \rangle_i$ denotes an expectation value in subband *i*, $l_c = (\hbar/eB)^{1/2}$ is the magnetic



FIG. 3. Fermi contour lines and wave-function extent Δz of the E_0 and E_1 subbands at different magnetic fields. (a) B=0 T. (b) $B=B_c=1.6$ T. (c) B=2 T.

length, ω_c is the cyclotron frequency, and *m* the effective mass of the electron. This diamagnetic shift depopulates successive subbands with increasing magnetic field, each time causing a stepwise decrease in R_1 .¹⁰ A self-consistent calculation for our structure, including the parallel magnetic field, identifies the shoulders at B_1 and B_2 in Fig. 1 as the result of the depopulation of the E_1 and the E_2 subband, respectively.

It is known that the shift of the origin of the Fermi circles can lead to a dramatic change in the topology of the whole Fermi surface.^{2-4,11} The two significant magnetic-field values in a two-subband system are those at which the two Fermi circles touch.^{2,5} This condition defines $B_c^{\pm} = \hbar |k_F^{(0)} \pm k_F^{(1)}|/(ed)$. If $B < B_c^-$ or $B > B_c^+$ the two Fermi circles are independent, whereas if $B_c^- \le B \le B_c^+$ they cross. If $k_F^{(0)} = k_F^{(1)}$ (see Ref. 3) then $B_c^- = 0$. Crossing of the circles gives rise to tunneling coupling of the subbands and an enhanced tunneling current has been observed in a direct tunneling experiment.² At $B = B_c^+$ a van Hove singularity in the density of states passes through the Fermi level, giving rise to a resistance enhancement in parallel transport experiments.⁴ In our system, however, $B_c^+ > 16$ T.

We find that in our sample the position B_c of the resistance peak coincides with B_c^- , the field at which the Fermi circles of the E_0 and E_1 subbands first come into contact. Figure 3(b) shows the Fermi surface at $B = B_c = B_c^-$ and also $\Delta z = \sqrt{\langle (z - \langle z \rangle_i)^2 \rangle_i}$, the extent of the wave function in the *z* direction, as a function of k_y . The curves were obtained from a self-consistent solution of the Schrödinger and Poisson equations in a parallel magnetic field. At this field, a large fraction of the states at the Fermi level are delocalized as a result of the tunnel coupling. The delocalized wave functions have approximately equal probability amplitude at either interface. Increasing *B* above B_c moves the two crossing points of the Fermi circles to larger $|k_x|$, thus decreasing the fraction of delocalized states at the Fermi level [see Fig. 3(c)].

We know also that we have very different mobilities in the two lowest subbands. In the RR effect in gated DQW's,¹ two factors determine $\Delta R/R$.¹² First, the scattering asymmetry $\beta = (\tau_1 - \tau_0)/(\tau_1 + \tau_0)$ contributes a factor $\beta^2/(1 - \beta^2)$, which becomes large as $\beta \rightarrow 1$. Second, an impairment due to disorder of the tunneling coupling reduces $\Delta R/R$ by $\Omega_T^2/(1 + \Omega_T^2)$, with $\Omega_T = \Delta_{SAS} \overline{\tau}/\hbar$ and $\overline{\tau} = 2\tau_1 \tau_0/2$ $(\tau_1 + \tau_0)$.¹² Using the parameters of our samples we would expect a contribution of 4.64 from the β -dependent factor and a reduction by 0.55 due to the partly supressed tunneling. This predicts $\Delta R/R = 2.55$, which we could verify, if we were able to balance the two interface states with a gate. We therefore attribute the strong resistance enhancement we do observe in a magnetic field to the same mechanism as in Ref. 1. Tunneling is "switched on" at $B_{\parallel} = 1.6$ T for some states, which can then scatter at both interfaces.

The observed behavior of R_l may be described by Boltzmann transport theory using the relaxation time approximation.¹⁴ The magnetic field enters into this description only via the dispersion relation $\mathcal{E}_{i\mathbf{k}}$. The *x* direction is the direction of the magnetic field as shown in Fig. 3. Since the dispersion in a magnetic field is then symmetric with respect to the *y* direction the conductivity can be described by a diagonal 2×2 matrix with elements

$$\sigma_{xx} = \frac{2e^2}{A} \sum_{i\mathbf{k}} \mathbf{v}_{i\mathbf{k}}^2 \cos^2 \theta_{i\mathbf{k}} \tau_{i\mathbf{k}} \bigg[-\frac{\partial f_i(\mathcal{E}_{i\mathbf{k}})}{\partial \mathcal{E}_{i\mathbf{k}}} \bigg],$$

$$\sigma_{yy} = \frac{2e^2}{A} \sum_{i\mathbf{k}} \mathbf{v}_{i\mathbf{k}}^2 \sin^2 \theta_{i\mathbf{k}} \tau_{i\mathbf{k}} \bigg[-\frac{\partial f_i(\mathcal{E}_{i\mathbf{k}})}{\partial \mathcal{E}_{i\mathbf{k}}} \bigg],$$

$$\sigma_{xy} = \sigma_{yx} = 0, \qquad (1)$$

where $\theta_{i\mathbf{k}}$ is the angle between $\mathbf{v}_{i\mathbf{k}}$ and the *x* axis, $\mathbf{v}_{i\mathbf{k}} = \hbar^{-1}(\nabla_{\mathbf{k}} \mathcal{E}_{i\mathbf{k}})$ is the velocity of an electron with wave vector \mathbf{k} , $\tau_{i\mathbf{k}}$ is the scattering time, $f_i(\mathcal{E}_{i\mathbf{k}})$ is the Fermi distribution function, and *A* is a normalization area. In the experiment, the current along the Hall-bar axis is in general at an angle α to B_{\parallel} , and we measure the components of the electric field parallel (giving R_i) and perpendicular to this direction (giving the transverse resistance R_i). In this "experimental" reference frame, where the current defines the x' direction (i.e., $j_{y'} = 0$) the conductivity tensor is no longer diagonal but has components

$$\sigma_{x'x'}(\alpha) = \sigma_{xx} \cos^2 \alpha + \sigma_{yy} \sin^2 \alpha,$$

$$\sigma_{y'y'}(\alpha) = \sigma_{xx} \sin^2 \alpha + \sigma_{yy} \cos^2 \alpha,$$

$$\sigma_{x'y'}(\alpha) = \sin \alpha \cos \alpha (\sigma_{yy} - \sigma_{xx}) = \sigma_{y'x'}(\alpha).$$
 (2)

Equations (2) immediately make clear that, whenever $\sigma_{xx} = \sigma_{yy}$, points independent of α occur in an angular dependent measurement. This explains the occurrence of fixed points in the $\Delta R/R$ versus B_{\parallel} plot in terms of the symmetry properties of the dispersion. If we make the approximation that all the current is carried by the high mobility subband, the summations over *i* in Eqs. (1) can be dropped and $\sigma_{xx} = \sigma_{yy}$ when the angle between the k_y axis and the vector from the origin of the Fermi circle to the crossing point is exactly $\pi/4$. The fixed point should then occur at about 2 T, in good agreement with experiment.

Equations (1) also give us some insight into the nature of the anisotropy of σ_{xx} and σ_{yy} . The derivative of f_i at low temperatures restricts the summation to the Fermi surface. The square of the velocity does not vary significantly within one subband, but τ_{ik} can be expected to change drastically in the vicinity of states that are delocalized in real space, since their mobility is roughly determined by the low mobility interface. However, the scattering times are weighted by $\cos^2 \theta_{i\mathbf{k}}$ and $\sin^2 \theta_{i\mathbf{k}}$, respectively, which means the tunneling coupling has a much stronger influence on $\sigma_{yy}(B_c)$ than on $\sigma_{xx}(B_c)$. Also, when *B* rises above B_c the crossing points of the Fermi circle of the high mobility subband move to smaller $\theta_{i\mathbf{k}}$, thus decreasing $\sigma_{xx}(B)$ but increasing $\sigma_{yy}(B)$. Looking at Fig. 3(b), this means that when the current is along the *y* direction (i.e., perpendicular to B_{\parallel}) there will be more effect of the scattering than when it is along the *x* direction (i.e., parallel to B_{\parallel}) because in the latter case the electrons scattered at the crossing point of the circles carry little current.

Significant changes in conductivity are expected if $kT > \Delta_{SAS}$. At these temperatures states localized at the interfaces that are more remote in energy from the tunneling gap start to contribute increasingly to the conductivity. As a result the RR smears out.

To illustrate this explanation, we calculated $\rho_l(B)$ using the first-order correction to the dispersion and neglecting the level repulsion between resonant states. This leads to a parabolic dispersion, which simplifies the summation over **k** in Eq. (1). We did not calculate the $\tau_{i\mathbf{k}}$ from the Boltzmann equation, but chose a Lorentz function with the width γ as a plausible functional dependence on **k**:

$$\tau_{i\mathbf{k}} = \tau_i + \frac{\overline{\tau} - \tau_i}{1 + (k_v - k_v^{(\text{res})})^2 \gamma^2}$$

Here τ_i denotes the transport scattering time at B=0 T and all k_y are measured from the center of the Fermi circle of subband *i*. The scattering time at the resonance point is taken to be $\overline{\tau} = \overline{\tau}/\delta$, where δ is an adjustable parameter. In the very strong tunneling limit ($\delta = 1$) $1/\overline{\tau}$ is the average of the two



FIG. 4. Calculated magnetoresistance at different angles between current and the magnetic field. The arrow marks the fixed points at B = 2 T and the resistance peak at $B_c = 1.6$ T.

scattering rates $1/\tau_1$ and $1/\tau_2$ since in this case an electron spends exactly the same time at either interface. The "tunneling strength parameter" δ is therefore allowed to vary with the restriction $\delta \leq 1$. The crossing point between the two circles $k_v^{(\text{res})}$ is given by

$$k_{y}^{(\text{res})} = \frac{2m\Delta E/\hbar^{2} \pm (d/l_{c}^{2})^{2}}{2d/l_{c}^{2}},$$

where ΔE is the energy separation of the bottoms of the two subbands, calculated from the measured density difference.

The result of the calculation at T=0 K is shown in Fig. 4 for $\gamma=27.6$ nm and $\delta=0.5$, corresponding to $\overline{\tau}=1.57$ ps. It resembles strongly the experimental data shown in Fig. 2. The value of δ implies that the system is not in the strong tunneling limit. This can be explained by a partial suppression of tunneling due to the disorder,^{12,13} which is also supported by $\delta \approx \Omega_T^2/(1+\Omega_T^2)$, the "tunneling strength parameter" of the RR theory in Ref. 12. Note that the magnetic fields at which the RR peak and the crossing point occur are mainly determined by the measured electron densities at the two interfaces. The two fitting parameters γ and δ are needed to adjust the resistance values at these two fields.

In summary, we have shown that in a system with two remote 2D subbands of different electron density, a parallel magnetic field opens a tunneling channel between the two subbands. If the mobilities of electrons in the two 2DEG's differ strongly, a RR marks the onset of the tunneling in a parallel transport experiment with the current perpendicular to the magnetic field. No resonance occurs if the current is parallel to the field. The existence of a magnetic field, for which the longitudinal resistance does not depend on the angle between current and field is explained by the symmetry properties of the dispersion relation.

This work was supported by the EPSRC (UK). L.E. thanks the Royal Society (UK), and H.C. the CNPq (Brazil) for financial support.

- ¹A. Palevski et al., Phys. Rev. Lett. 65, 1929 (1990).
- ²J.P. Eisenstein, T.J. Gramila, L.N. Pfeiffer, and K.W. West, Phys. Rev. B 44, 6511 (1991).
- ³Y. Berk et al., Phys. Rev. B 51, 2604 (1995).
- ⁴J.A. Simmons, S.K. Lyo, N.E. Harff, and J.F. Klem, Phys. Rev. Lett. **73**, 2256 (1994).
- ⁵S.K. Lyo, Phys. Rev. B **50**, 4965 (1994).
- ⁶Y.W. Suen et al., Phys. Rev. Lett. 72, 3405 (1994).
- ⁷T.S. Lay et al., Phys. Rev. B 50, 17 725 (1994).
- ⁸Th. Ihn, K.J. Friedland, R. Hey, and F. Koch, Phys. Rev. B 52, 2789 (1995).
- ⁹F. Stern, Phys. Rev. Lett. 18, 1687 (1968).

- ¹⁰J.C. Portal *et al.*, Solid State Commun. **43**, 907 (1982); Th. Englert, J.C. Maan, D.C. Tsui, and A.C. Gossard, *ibid.* **45**, 989 (1983); A. Zrenner *et al.*, Phys. Rev. B **33**, 5607 (1986).
- ¹¹D.R. Leadley, R.J. Nicholas, J.J. Harris, and C.T. Foxon, in *Physics of Semiconductors: Proceedings of the XX International Conference*, edited by J.D. Joannopoulos and E. Anastassakis (World Scientific, Singapore, 1990), Vol. 2, p. 1609.
- ¹²F.T. Vasko and O.E. Raichev, Zh. Éksp. Teor. Fiz. **107**, 951 (1995) [J. Exp. Theor. Phys. **80**, 539 (1995)].
- ¹³F.T. Vasko and O.E. Raichev, Phys. Rev. B **52**, 16 349 (1995).
- ¹⁴N.W. Ashcroft and N.D. Mermin, *Solid State Physics* (Holt, Rinehart and Winston, New York, 1976), Chap. 13, p. 250.